Global Value Chains and Effective Exchange Rates at the Country-Sector Level

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Disclaimer

- Views are those of the authors and do not necessarily correspond to the institutions they are affiliated with
  - ADB, BIS, USITC

- This paper:
  - New model of REER to better capture movements in competitiveness
  - Create database of country and sector level REERs using data from World Input Output Database (WIOD)

Introduction

- Real effective exchange rate (REER)
  - Measure of competitiveness
  - Many applications
    - ⋆ currency misalignment and manipulation
    - ⋆ vulnerability to crises (Chinn, 2000; Goldfajn and Valdés, 1999; Gagnon 2012)
- *"..we look at several variables, but certainly we've looked, as I think I've said in the last press conference, at the exchange rate in effective terms;"*
  - Mario Draghi, January 2016.
- *"..net exports are being held down by weak economic growth in several of our major trading partners and the appreciation of the dollar."*
  - Janet Yellen, July 15, 2015

The Concept of REER

$$\triangle REER_J = \triangle V_J = \sum_{i=1}^{n} w_{Ji} \triangle p_i$$

- V_J: value added by country J
- w_{Ji} are exchange rate weights
- $\triangle$ denotes log change from steady state

- Partial equilibrium concept
  - Primitive shocks not modeled
  - No restrictions on trade balance
- All papers (including ours) work in this setting

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Problems With Conventional REERs (like IMF)

1. Ignore intermediate inputs
   - Both offshoring and domestic outsourcing are ignored
   - In the data, around 60 percent of world trade comprises of trade in intermediate goods
2. Ignore sectoral heterogeneity within countries
   - Strong evidence documenting sectoral heterogeneity(Wang, Wei and Zhu ,2014)
3. Assume elasticity of substitution in production and preferences are the same and equal to 1

Importance of Considering Trade in Intermediates

..→[Japan] → **[China]** → **[US]**

- Standard real exchange rate measures(IMF, BIS ,Fed etc):
  - Classify iPhone as China's product(produced entirely in China)
  - China competes with other smart phone manufacturers
  - Decrease in Japanese prices reduce Chinese competitiveness
- In reality:
  - China is just the final assembly point for iPhones
  - Competes with other providers of these "assembly services"
  - Decrease in Japanese prices may increase China's competitiveness

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Importance of Incorporating Sectoral Heterogeneity

Example: Two sector Chinese Economy

	Electronics Sector	Non traded goods sector
%Domestic value added	34-40 percent	100 percent
Fraction Exported	high(˜90percent)*	Low(˜0)

(*source: Koopman, Wang and Wei,2012)

- Fraction of value added in aggregate "Chinese good" >40 percent
  - Over-predicts Chinese value added in exports

More on Sectoral Heterogeneity within Countries

China 2002			
IO Industry	%value added	% processing exports	GVC position
Electronic computer	19.3	99.1	Downstream
Textiles production	75	24	Upstream
(source: Koopman, Wang and Wei,2012)			

Germany 2011		
WIOD sector	Intermediate exports(% of total)	GVC position
Basic metals(C12)	93	Upstream
Machinery(C13)	45	Downstream
(source: Wang, Wei and Zhu 2013)		

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Need for Sector Level REERs

- World economy increasingly marked by specialization and global value chains
- Different sectors within a country can show very different behavior with regard to competitiveness

This Paper

- New model of REER
  - Construct sector and country level REER weights
  - Construct and incorporate sector level price indices
  - Estimate and incorporate different elasticities of substitution in production and consumption
- Focus on short run changes in competitiveness
  - Take GVC and production outsourcing pattern as given

Related Literature

- REER

	IMF	Fed	BIS	BJ	BST	BZ	This paper
Value added competitiveness				✓			✓
Sector level heterogeneity						✓	✓
Trade in intermediate goods				✓	✓		✓
Heterogenous elasticities							✓

*BJ: Bems and Johnson (2012); BST:Bayoumi et al. (2013);BZ:Bennett and Zarnic (2009)

- Global Value Chains /Export Accounting/Vertical Specialization:
  - Auer, Levchenko and Saure (2016), Borin and Mancini (2015), Koopman, Wang and Wei (AER2014, NBER wp 2014,), Wang, Wei and Zhu(2014), Hummels, Ishii and Yi (2001) etc.

Roadmap

- Brief sketch of the model
- Illustrative examples with stylized three country global value chain
- Data and empirical results
- Application: Bilateral Exchange rates
- Conclusion

Model

Model: Features

- Multi-Country multi-sector model allowing for arbitrary input-output linkages and global value chains.
- n countries and m sectors within each country
- Each country-sector is a production entity endowed with its own unique production technology
  - Takes inputs from (potentially) all other entities and combines with own value added
- n representative consumers -one for each country
  - Consumption bundle is an aggregate of nm goods.
- Partial Equilibrium framework
  - Prices are exogenous
  - Output (endogenous) is a function of prices

Model Setup: Production

- n countries, m sectors

$$Q_I^c = \left[(w_I^{vc})^{1/\sigma^3} (V_I^c)^{\frac{\sigma^3-1}{\sigma^3}} + (w_I^{Xc})^{1/\sigma^3} (X_I^c)^{\frac{\sigma^3-1}{\sigma^3}} \right]^{\frac{\sigma^3}{\sigma^3-1}}$$

Aggregate	Components	Elasticity	
X_I^c	$\{X_{sI}^c\}_{s=1}^m$	σ^2	
X_{sI}^c	$X_{sI}^{cc}, X(f)_{sI}^c$	σ^{1h}	Macro-Elasticity
$X(f)_{sI}^c$	$\{X_{sI}^{ic}\}_{i=1,i\neq c}^n$	σ^1	Micro-Elasticity

▸ full expressions

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Final Demand and Market Clearing

Aggregate	Components	Elasticity	
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${F_s}^c$	$F_s^{cc}, F(f)_s^c$	θ^{1h}	Macro Elasticity
$F(f)_s^c$	$\{F_s^{ic}\}_{i=1,i\neq c}^n$	θ^2	Micro Elasticity

▸ full expressions

▸ elasticity estimates

Market Clearing Condition:

$$Q_l^c = \sum_{i=1}^{n} F_l^{ci} + \sum_{j=1}^{m}\sum_{k=1}^{n} X_{lj}^{ck}, \forall(c,l)$$

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Model Solution and REER Expressions

$$\left(\hat{V}\right)_{nm} = \underbrace{\left[W_V\right]_{nmXnm}\left(\hat{p^v}\right)_{nm}}_{GVC-REER} + \left[W_{FV}\right]_{nmXnm}\left(\hat{F}\right)_{nm}$$

$$\left(\hat{Q}\right)_{nm} = \underbrace{\left[W_Q\right]_{nmXnm}\left(\hat{p^v}\right)_{nm}}_{Q-REER} + \left[W_{FQ}\right]_{nmXnm}\left(\hat{F}\right)_{nm}$$

Relationship to Other Measures in the Literature

Proposition 1	Expressions in our notation	Sufficient Conditions for equivalence
IMF manufacturing weights	$W^{ij}_{imfm} = \sum_k \left(\frac{sales^{ik}}{\sum_n sales^{in}}\right)\left(\frac{sales^{jk}}{\sum_l sales^{lk}}\right)$	CE,$A = 0$
Goods-REER(BST)	$(W_{imf})_{nXn} \underbrace{[B]'\,[Diag(v_i)]\,(\hat{p^v})}_{\hat{p^Q}}$	CE,$A = 0$
VAREER(BJ)	$W^{ij}_{BJ} = \sum_k \left(\frac{p^{iv}V^{ik}}{P^{iv}V^{i}}\right)\left(\frac{p^{jv}V^{jk}}{P^{k}F^{k}}\right)$	$m = 1$,CE

Key: CE: constant elasticity, A: global input output matrix, m= number of sectors in each country

Example 1

..→[Japan] → **[China]** → **[US]**

	J	C	U	J final	C final	U final	Total output
J	0	1	0	1	0	0	2
C	0	0	0	0.1	0.1	1	1.2
U	0	0	0	0	0	1	1
Value added	2	0.2	1				
Total output	2	1.2	1				

Example 1. Cont

	J	C	U	J f	C f	U f	
J	0	1	0	1	0	0	2
C	0	0	0	0.1	0.1	1	1.2
U	0	0	0	0	0	1	1
Value added	2	0.2	1				

σ=1	Elasticity of substitution between inputs in production
θ=5	Elasticity of substitution between components of final demand

	GVC-REER	VAREER (BJ)	**QREER**	GOODS-REER (BST)	IMF Weights (BLS)
W_{JC}	-0.04	0.19	-0.04	1.0	1.00
W_{CJ}	**-0.25**	**0.54**	**-4.07**	**-3.40**	**0.26**

Example 1. Cont

	J	C	U	J f	C f	U f	
J	0	1	0	1	0	0	2
C	0	0	0	0.1	0.1	1	1.2
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Example 2: Aggregation

		J		C		U		JFinal	CFinal	Ufinal	total output
		J1	J2	C1	C2	U1	U2				
J	J1	0	0	0	2	0	0	1	0	0	3
	J2	0	0	0	0	0	0	1	0	0	1
C	C1	0	0	0	0	0	0	0	2	0	2
	C2	0	0	0	0	0	0	0.5	0.5	2.5	3.5
U	U1	0	0	0	0	0	0	0	0	2	2
	U2	0	0	0	0	0	0	0	0	1	1
VA		3	1	2	1.5	2	1				
total output		3	1	2	3.5	2	1				

	J	C	U	J final	C final	U final	Total output
J	0	2	0	2	0	0	4
C	0	0	0	0.5	2.5	2.5	5.5
U	0	0	0	0	0	3	3
Value added	4	3.5	3				
Total output	4	5.5	3				

Example 2: Aggregation

		J		C		U		JFinal	CFinal	Ufinal	total output
		J1	J2	C1	C2	U1	U2				
J	J1	0	0	0	2	0	0	1	0	0	3
	J2	0	0	0	0	0	0	1	0	0	1
C	C1	0	0	0	0	0	0	0	2	0	2
	C2	0	0	0	0	0	0	0.5	0.5	2.5	3.5
U	U1	0	0	0	0	0	0	0	0	2	2
	U2	0	0	0	0	0	0	0	0	1	1
VA		3	1	2	1.5	2	1				
total output		3	1	2	3.5	2	1				

	J	C	U	J final	C final	U final	Total output
J	0	2	0	2	0	0	4
C	0	0	0	0.5	2.5	2.5	5.5
U	0	0	0	0	0	3	3
Value added	4	3.5	3				
Total output	4	5.5	3				

Example 2.Cont.

	GVC-REER	A-REER
W_{JC}	0.18	0.27
W_{JU}	0.19	0.12
W_{CJ}	0.20	0.32
W_{CU}	**0.16**	**0.25**
W_{UJ}	0.25	0.16
W_{UC}	0.19	0.28

key	
GVC-REER	Our benchmark measure
A-REER	computed from the aggregate (country level)IO table

- GVC-REER assigns a lower value to W_{CU}
  - Intuition: the smaller sector of C competes with U

Data

- Source: World Input-Output Database(WIOD), augmented by the Asian Development Bank (ADB) statistics group
- Nature:
  - Inter-Country Input-Output (ICIO) tables at the country-sector level
  - 40 countries and 35 sectors within each country
  - Available in both current and previous year prices
  - Sample: 1995-2011(1996-2009 for previous year prices)
  - Detailed description in Timmer et. al (2012) , (2015).

Results

1. Elasticity estimation
2. REER weights
3. REER indices

Estimation of Elasticity of Substitution

- Basic framework: Feenstra (1994)
  - Subsequently developed by Broda and Weinstein (2006) and Soderbery (2014)
  - We modify the framework to allow for elasticities<1
  - Use hybrid grid search constrained LIML estimator

details..

- Consistency relies on $T \to \infty$
- Right now we have T$=$14
- Take steps to deal with small sample
  - LIML instead of 2SLS
  - Winsorize data at 10 and 90 percentile

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Elasticity Estimates: Medians Across Different Groups

Median Consumption Elasticities			
	θ^1	θ^{1h}	θ^2
Full Sample	**16.05**	**6.06**	**1.93**
OECD(28)	14.84	5.46	2.13
Non-OECD(13)	21.49	9.00	1.32
Asia (7)	22.32	2.40	2.15
Europe (29)	15.088	6.06	1.16
Americas (4)	14.23	7.31	1.65

Median Production Elasticities				
	σ^1	σ^{1h}	σ^2	σ^3
Full Sample	**16.94**	**8.93**	**4.26**	**0.93**
primary(2)	15.96	13.20	6.13	0.84
secondary(15)	16.19	5.33	5.08	0.94
tertiary(18)	17.74	10.59	3.79	1.02

REER Weights

	Dimension	Elasticity	
		Uniform	Heterogenous
country by country	n by n by T	✓	
country by country-sector	n by nm by T	✓	✓
country-sector by country	nm by n by T	✓	✓
country-sector by country-sector	nm by nm by T	✓	✓

Biggest Competitors for Japan in 2007 based on different REER weighting schemes

Rank	**GVC-REER(CE)**	VAREER	Goods-REER	IMF	**Q-REER**
1	**'ROW'**	'ROW'	'ROW'	'ROW'	**'USA'**
2	**'USA'**	'USA'	'China'	'China'	**'China'**
3	**'China'**	'China'	'USA'	'USA'	**'ROW'**
4	**'Germany'**	'Germany'	'Korea'	'Korea'	**'Germany'**
5	**'Korea'**	'Korea'	'Taiwan'	'Taiwan'	**'Korea'**
6	**'Australia'**	'Australia'	'Germany'	'Germany'	**'UK'**
7	**'UK'**	'UK'	'Australia'	'Australia'	**'France'**
8	**'Russia'**	'Taiwan'	'UK'	'UK'	**'Italy'**
9	**'Taiwan'**	'France'	'Russia'	'Russia'	**'Taiwan'**
10	**'France'**	'Russia'	'Indonesia'	'Indonesia'	**'Russia'**

Comparison of biggest sector level competitors for China in 2007

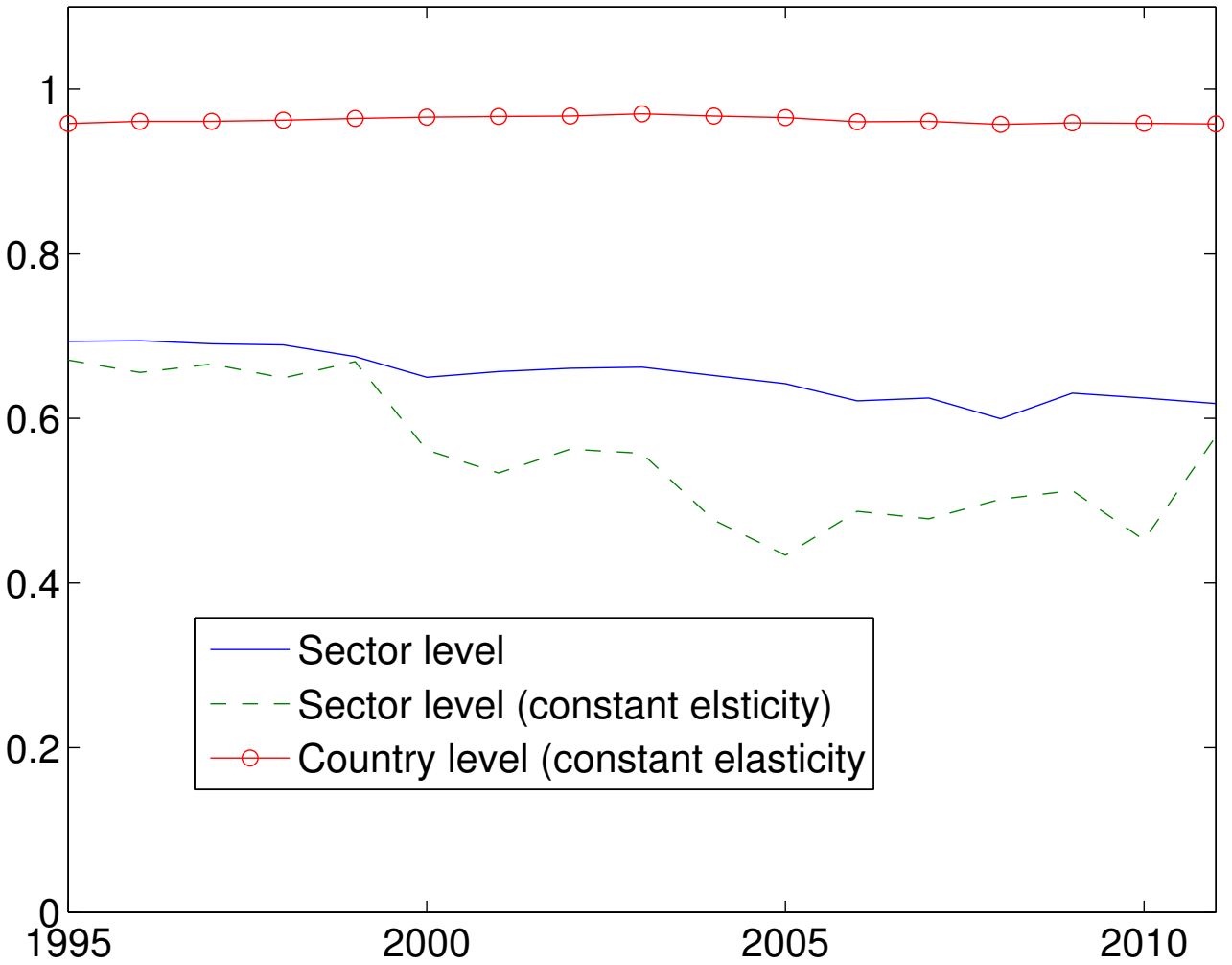
		Constant Elasticity
1	'ROW'	'Mining and Quarrying'
2	'USA'	'Renting of M&Eq and Other Business Activities'
3	'USA'	'Public Admin and Defense; Compulsory Social Security'
4	'USA'	'Real Estate Activities'
5	'USA'	'Financial Intermediation'
6	'ROW'	'Agriculture, Hunting, Forestry and Fishing'
7	'USA'	'Wholesale Trade and Commission Trade, Except auto'
8	'ROW'	'Wholesale Trade and Commission Trade, Except auto'
9	'USA'	'Retail Trade, Except auto; Repair of Household Goods'
10	'USA'	'Health and Social Work'

Biggest Competitor (countries) for Largest Sector of the US in 2011

US Sector: Renting of M&Eq and Other Business Activities Share of GDP in 2011:14%		
Rank	Heterogenous Elasticity	Constant Elasticity
1	'ROW'	'Canada'
2	'China'	'ROW'
3	'Canada'	'China'
4	'Germany'	'United Kingdom'
5	'Japan'	'Germany'

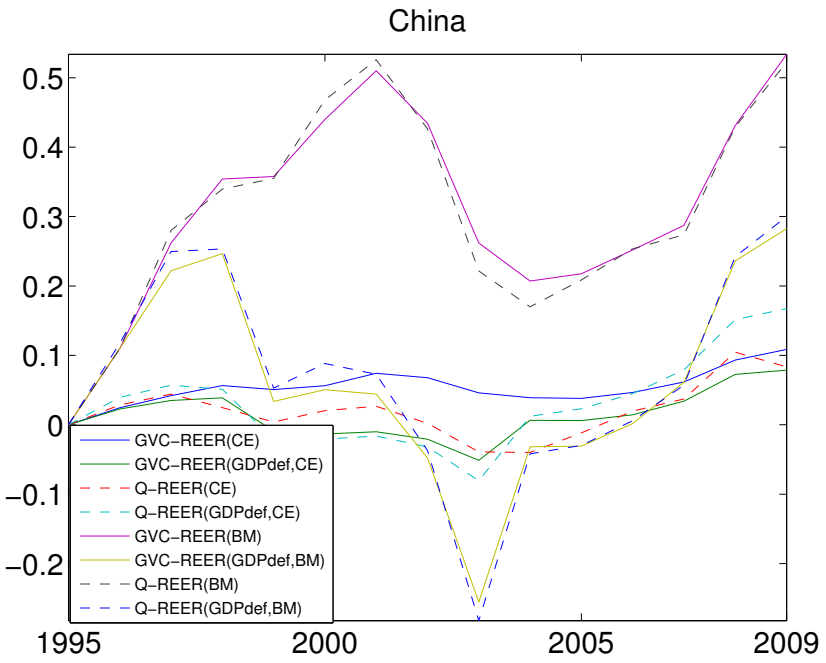
Heterogenous Elasticity		
Rank		
1	'Canada'	'Mining and Quarrying'
2	'Canada'	'Electrical and Optical Equipment'
3	'Germany'	'Agriculture, Hunting, Forestry and Fishing'
4	'Canada'	'Renting of M&Eq and Other Business Activities'
5	'Canada'	'Wholesale Trade and Commission Trade, Except autos'

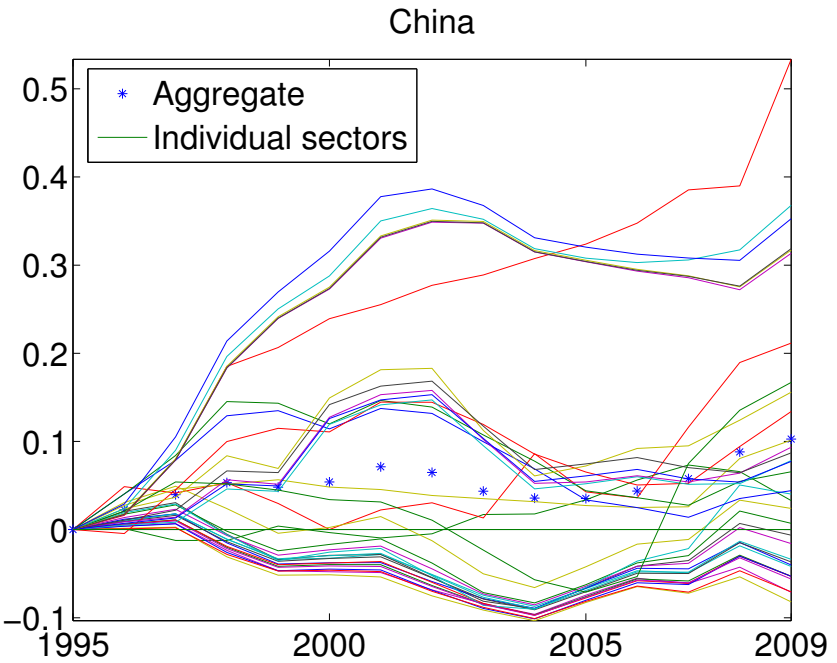
Correlation between Value added and Gross Output REER Weights



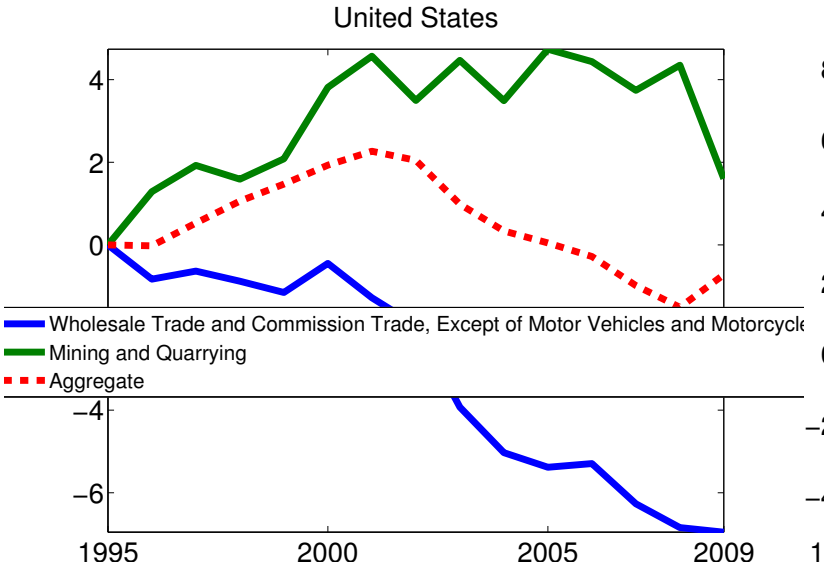
From Weights to REER Indices: China

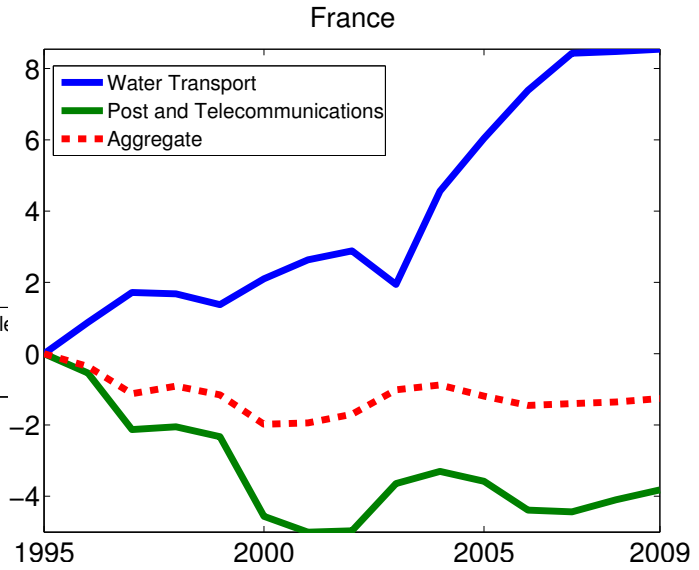
- 8 REER indices for each country..
- ..and 35 sector level indices for each of the 8:

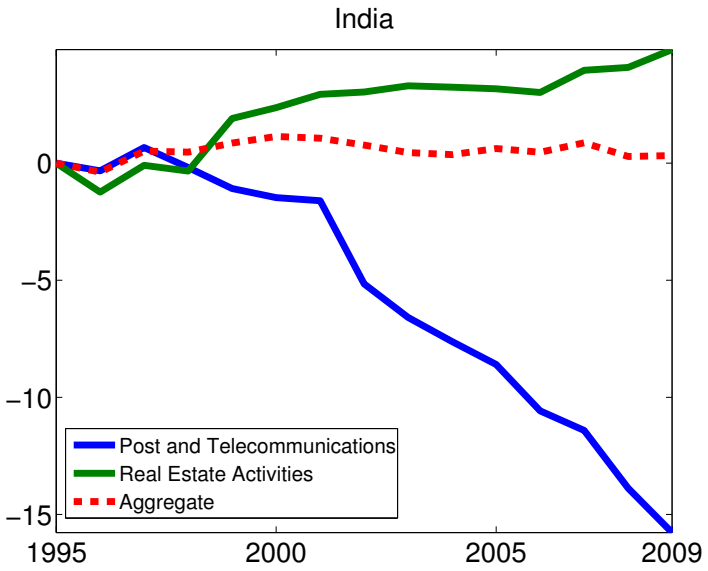


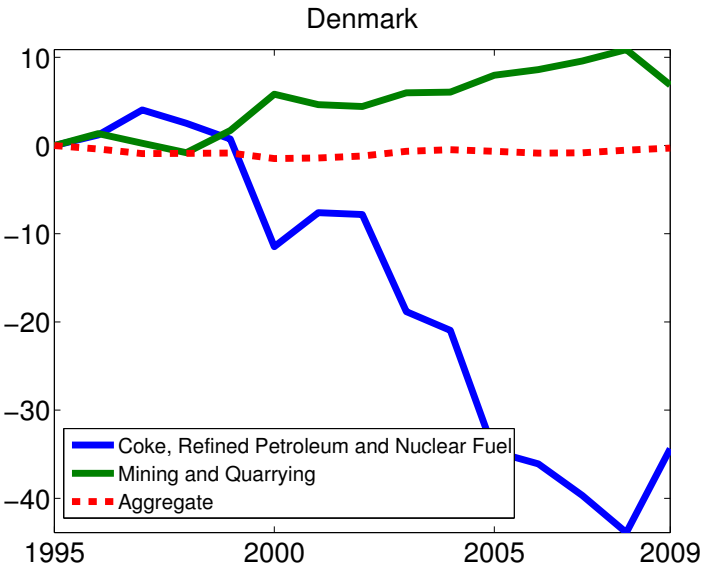


Sector Level Exchange Rates: Select Examples









Countries with highest and lowest divergence of REERs across sectors

High Dispersion		Low Dispersion	
Denmark	0.48	Malta	0.05
Czech Republic	0.24	Ireland	0.05
Bulgaria	0.23	UK	0.07
Australia	0.19	Spain	0.08
Brazil	0.18	France	0.08

Note: The dispersion is computed as the average standard deviation of REER movements within a country (i.e an average of 14 observations on the standard deviation for each time period).

Role of Elascities: How often do Elasticities lead to a sign reversal in REER Movement?

level of Aggregation	Country	Country-Sector
Sample size	41	1435
$Mean(d^e)$	0.10	0.24
$Median(d^e)$	0.07	0.21
$Stdev(d^e)$	0.07	0.14
$min(d^e)$	0	0
$max(d^e)$	0.29	0.79

Application: Bilateral RER

- Typically defined as: $RER^{hf} = \hat{p}(V)^f - \hat{p}(V)^h$
- This is appropriate for some purposes (like comparison of cost of living across countries)
- But is misleading as a gauge of competitiveness
- We propose a reweighing similar to the multilateral weights

Example: Bilateral RER

C_1: Traded good; C_2: non traded good

		C		U		CFinal	Ufinal	total output
		C1	C2	U1	U2			
C	C1	0	0	0	0	1	1	2
	C2	0	0	0	0	3	0	3
U	U1	0	0	0	0	0	1	1
	U2	0	0	0	0	0	1	1
VA		2	3	1	1			
total output		2	3	1	1			

$$\hat{p^v}(c_1) = -0.01, \hat{p^v}(c_2) = 0.02, \hat{p^v}(u_1) = 0, \hat{p^v}(u_2) = 0$$

- Given the price changes, $\hat{RER}_{us}$ indicates an increase in competitiveness of the US ($\hat{RER}_{us}$=0.008)
  - This is misleading since US competes only with C1 and $p^V(c_1) \downarrow$
  - The conceptually correct should indicate a fall in competitiveness of the US

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- Given the price changes, $\hat{RER}_{US}$ indicates an increase in competitiveness of the US ($\hat{RER}_{US}$=0.008)
  - This is misleading since US competes only with C1 and $p^V(c_1) \downarrow$
  - The conceptually correct should indicate a fall in competitiveness of the US

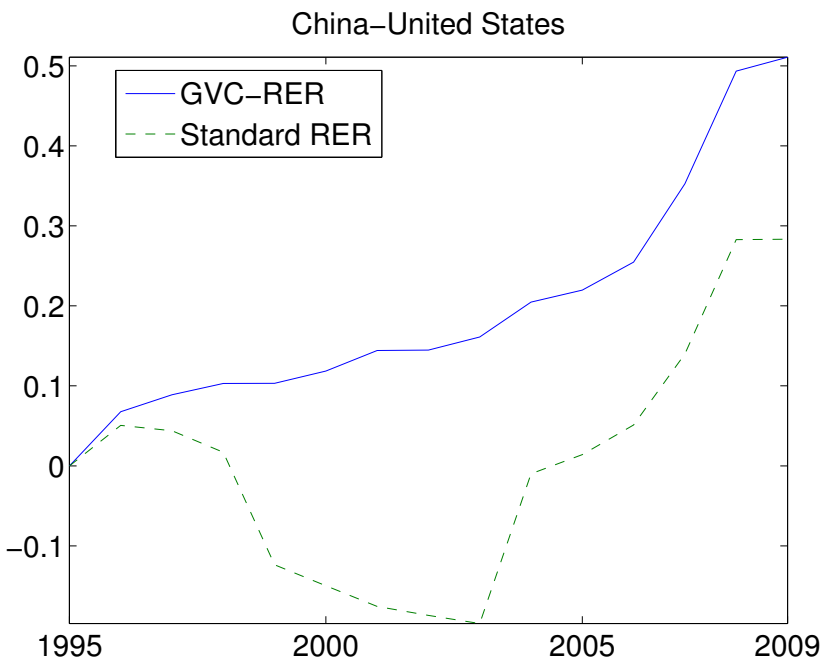
GVC-RER: A New Way to Construct Bilateral Real Exchange Rates

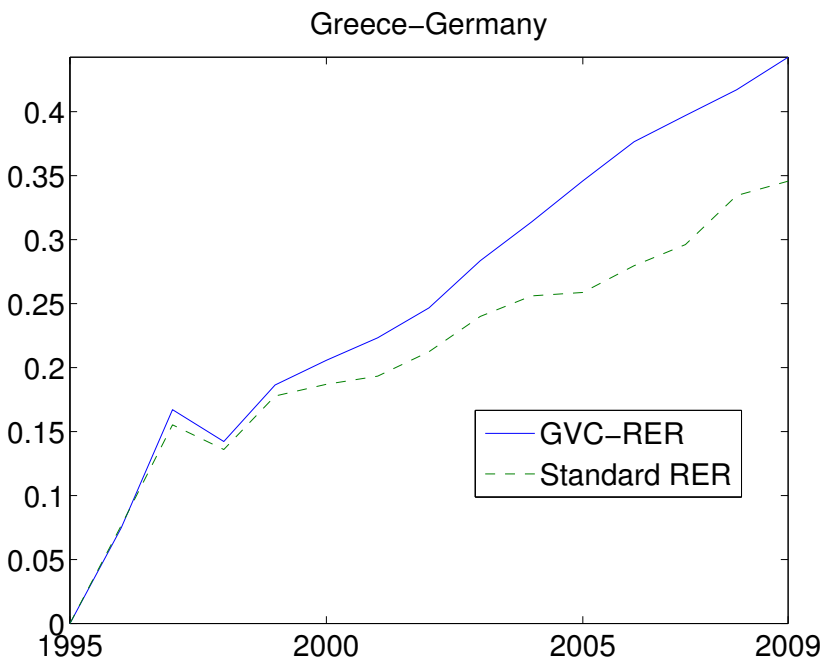
We define bilateral RER using GVC-REER weights as follows:

$$GVC - RER^{hf} = \sum_{i=1}^{m} \frac{p_i^{vh} V_i^h}{\sum_{j=1}^{m} p_j^{vh} V_j^h} \left[\sum_{j=1}^{m} w_{ij}^{hh} \hat{p}_j^{vh} + \sum_{j=1}^{m} w_{ij}^{hf} \hat{p}_j^{vf} \right]$$

- $\hat{GVC - RER}^{US-CH} = -0.01 \implies$ fall in US competitiveness

Comparison of GVC-RER and standard RER bilateral exchange rates





Conclusion

Main contribution: REER measure improving upon existing measures along four dimensions

- Trade in intermediates and distinction between gross and value added flows
- REER indices at the country-sector level
- CES production functions and preferences with elasticities estimated from the data
- Use of sector level prices instead of aggregate country level prices

Other contributions

- Inter country model of sector level trade: can be extended to a DSGE setting.
- First distinct estimates of production and consumption elasticities

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Thank You!

Appendix

Extra Slides

General Model: Production

$$Q^{c}_{\cdot l} = \left[(w^{vc}_{l})^{1/\sigma^{3}(c,l)} (V^{c}_{l}) \frac{\sigma^{3}(c,l)-1}{\sigma^{3}(c,l)} + (w^{Xc}_{l})^{1/\sigma^{3}(c,l)} (X^{c}_{l}) \frac{\sigma^{3}(c,l)-1}{\sigma^{3}(c,l)} \right] \frac{\sigma^{3}(c,l)}{\sigma^{3}(c,l)-1}$$

$$X^{c}_{l} = \left[\sum_{s=1}^{m} (w^{c}_{sl})^{1/\sigma^{2}(c,l)} (X^{c}_{sl}) \frac{\sigma^{2}(c,l)-1}{\sigma^{2}(c,l)} \right] \frac{\sigma^{2}(c,l)}{\sigma^{2}(c,l)-1}$$

$$X^{c}_{sl} = \left[(w^{cc}_{sl})^{1/\sigma^{1h}_{s}(c,l)} (X^{cc}_{sl}) \frac{\sigma^{1h}_{s}(c,l)-1}{\sigma^{1h}_{s}(c,l)} + (w(f)^{c}_{sl})^{1/\sigma^{1h}_{s}(c,l)} (X(f)^{c}_{sl}) \frac{\sigma^{1h}_{s}(c,l)-1}{\sigma^{1h}_{s}(c,l)} \right] \frac{\sigma^{1h}_{s}(c,l)}{\sigma^{1h}_{s}(c,l)-1}$$

$$X(f)^{c}_{sl} = \left[\sum_{i=1, i\neq c}^{n} (w^{ic}_{sl})^{1/\sigma^{1}_{s}(c,l)} (X^{ic}_{sl}) \frac{\sigma^{1}_{s}(c,l)-1}{\sigma^{1}_{s}(c,l)} \right] \frac{\sigma^{1}_{s}(c,l)}{\sigma^{1}_{s}(c,l)-1}$$

back

Genral Model: Final Demand

$$F^c = \left[\sum_{s=1}^{m} (\kappa_s^c)^{1/\rho^2(c)} (F_s^c)^{\frac{\theta^2(c)-1}{\theta^2(c)}} \right]^{\frac{\theta^2(c)}{\theta^2(c)-1}}$$

$$F_s{}^c = \left[(\kappa_s^{cc})^{1/\theta_s^{1h}(c)} (F_s^{cc})^{\frac{\theta_s^1(c)-1}{\theta_s^1(c)}} + (\kappa(f)_s^c)^{1/\theta_s^{1h}(c)} (F(f)_s^c)^{\frac{\theta_s^{1h}(c)-1}{\theta_s^{1h}(c)}} \right]^{\frac{\theta_s^{1h}(c)}{\theta_s^{1h}(c)-1}}$$

$$F_s^c(f) = \left[\sum_{i=1, i\neq c}^{n} (\kappa_s^{ic})^{1/\theta_s^1(c)} (F_s^{ic})^{\frac{\theta_s^1(c)-1}{\theta_s^1(c)}} \right]^{\frac{\theta_s^1(c)}{\theta_s^1(c)-1}}$$

back

Elasticity Estimation Details 1 of 3

- Start with a general armington aggregator:

$$D_t = \left[\sum_{k \in K} (w_k)^{1/\eta} (D_{kt})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

- Demand function:

$$\triangle^r ln(s_{kt}) = -(\eta - 1) \triangle^r ln(p_{kt}) + \epsilon^r_{kt}$$

(where $s_{kt} = \frac{p_{kt} D_{kt}}{\sum_{k \in K} p_{kt} D_{kt}}$)

- Specify supply function exogenously

$$\triangle^r ln(p_{kt}) = \left(\frac{\rho}{1+\rho} \right) \triangle^r ln(s_{kt}) + \delta^r_{kt}$$

back

Elasticity Estimation Details 2 of 3

- Combine the two to get the final estimation equation:

$$Y_{kt} = \theta_1 Z_{1kt} + \theta_2 Z_{2kt} + u_{kt}$$

- Moment condition: $E(u_{kt}) = 0$
  - consistency relies on $T \to \infty$

$$Y_{kt} = (\triangle^r ln(p_{kt}))^2 \ , Z_{1kt} = (\triangle^r ln(s_{kt}))^2$$
$$Z_{2kt} = (\triangle^r ln(p_{kt}))(\triangle^r ln(s_{kt})), \text{and } u_{kt} = \frac{\epsilon^r_{kt}\delta^r_{kt}}{1-\phi}$$
$$\theta_1 = \frac{\phi}{(\eta-1)^2(1-\phi)} \theta_2 = \frac{2\phi-1}{(\eta-1)(1-\phi)}$$

back

Thank You

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