Quantitative Easing, the Repo Market, and the Term Structure of Interest Rates

RUGGERO JAPPELLI[*], LORIANA PELIZZON[†], MARTI G. SUBRAHMANYAM[‡]

Preliminary and Incomplete.

ABSTRACT

We develop a quantity-driven general equilibrium model that integrates the term structure of interest rates with the repurchase agreements (repo) market to shed light on the combined effects of quantitative easing (QE) on the bond and money markets. We characterize in closed form the endogenous dynamic interaction between bond prices and repo rates, and show (i) that repo specialness dampens the impact of any given quantity of asset purchases due to QE on the slope of the term structure and (ii) that bond scarcity resulting from QE increases repo specialness, thus strengthening the local supply channel of QE.

JEL Codes: E43, E52, G12.

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[*]Leibniz Institute for Financial Research SAFE, Goethe University Frankfurt, Theodor-W.-Adorno-Platz 3, 60323, Frankfurt am Main, Germany, jappelli@safe-frankfurt.de.

[†]Leibniz Institute for Financial Research SAFE, Goethe University Frankfurt, Theodor-W.-Adorno-Platz 3, 60323, Frankfurt am Main, Germany, Ca' Foscari University of Venice, Dorsoduro, 3246, 30123 Venezia, Italy, and CEPR, pelizzon@safe.uni-frankfurt.de.

[‡]New York University, Leonard N. Stern School of Business and NYU Shanghai, Kaufman Management Center, 44 West Fourth Street, 9-68, 10012, New York, msubrahm@stern.nyu.edu.

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1 Introduction

The recent experience with quantitative easing (QE) programs around the world has starkly demonstrated the importance of demand factors in fixed-income markets and thus on the term structure of interest rates, highlighting two aspects that are worthy of analytical examination. First, the price of nearly riskless securities delivering known streams of payments rises persistently with large purchases by central banks (Bernanke, 2020). This calls into question the assumption of perfectly inelastic supply underlying both the standard bond valuation models in the financial economics literature and the Ricardian equivalence theories in the macroeconomic literature. Hence, a large demand shock affects the yield curve. Second, these asset purchases induce a scarcity of high-quality collateral and exert downward pressure on the rates at which the targeted securities trade in the repurchase agreements (repo) market, the main secured money market.[1] Thus, a large demand shock also affects the secured money market. Given the essential role of both money markets and the term structure of interest rates in the financial and economic system, these important stylized facts require an understanding of fixed-income markets in which durable assets such as bonds not only serve as investment vehicles but also as collateral for loans, in the spirit of Kiyotaki and Moore (1997).

Does the term structure of interest rates interact with secured money markets, where investors use bonds to collateralize loans? Intuitively, it should. For instance, in the traditional models of the term structure, bond prices reflect the current realizations of money market rates and a premium attached to the risk that these rates might change in the future. Generally, the money market is summarized by the stochastic behavior of a unique, exogenous interest rate. However, this partial equilibrium approach does not allow for shifts in the bond market itself to affect borrowing and lending rates, since the latter are presumed to be exogenous. Such a restriction is at odds with the growing recognition that demand and supply forces, particularly QE, affect *both* the prices in the bond market (D'Amico and King, 2013; Greenwood and Vayanos, 2010, 2014; Vayanos and Vila, 2021) and the repo rates associated with bonds in the secured money market in the US and EU, among others (D'Amico et al., 2018; Arrata et al., 2020; Corradin and Maddaloni, 2020; Pelizzon et al., 2022). These two robustly documented empirical facts have generally been considered in isolation, even though the bond scarcity generated by QE is the common driving force behind both of them. Our paper is the first attempt to offer a

[1] The repo, the main secured money market instrument, is a secured short-term loan that serves the dual role of providing collateral and obtaining cash. A repo contract achieves collateralized financing and consists of the spot sale of a cash bond combined with a forward agreement to repurchase the bond on a specified future trading day. The counterparty enters the reverse side of the trade (reverse repo) by buying the collateral on the spot market and stipulating a forward contract to sell the security.

comprehensive, quantity-driven model of the term structure of interest rates that integrates the two effects and endogenizes the secured money market with the bond market. In particular, we show that the impact that QE exerts on secured money markets dampens its effect on the compression of the term premium.

The importance of the repo market and its close connection to the bond market underscores the relevance of the quantity-driven framework in which we model both markets in our two-markets general equilibrium exchange economy. The repo market is the lifeblood of the financial system since it provides liquidity to holders of financial assets while providing an avenue to engage in short-term securities lending for those with cash. Repo contracts are the primary financial instrument for money market transactions, where institutional investors routinely obtain collateralized financing, and the size the repo market is simply enormous – much larger than the bond market itself. The average daily volume of outstanding repo transactions is about $12 trillion, roughly 14% of the world's GDP, of which Treasury repo transactions constitute about $8 trillion. By contrast, the daily volume in the US Treasury bond market averages around $0.6 trillion.[2] Moreover, it is well known that the repo market is segmented (see, e.g., Buraschi and Menini, 2002) and elastic to demand (D'Amico et al., 2018), frictions that we leverage in our model. Any repo contract is a short-term loan collateralized by a bond. A particular government bond ("special collateral or SC repo") or any bond from a predefined basket ("general collateral or GC repo") can be used as collateral. GC repo agreements are often called "cash-driven" transactions, because their primary purpose is to achieve collateralized financing that provides liquidity. In these transactions, each bond in a certain basket can be delivered as collateral. On the other hand, repo transactions can be motivated by the demand for a particular bond; in that case, they are "security-driven." An issue of securities that is subject to excess demand compared with others with very similar cash flows is said to be "on special." Competition to buy or borrow a special issue, perhaps to cover short selling commitments, causes buyers in the repo market to accept a lower interest rate in exchange for cash in these SC repo transactions. By lowering the attainable financing rate, special bonds yield a "repo dividend" (Duffie, 1996) that varies with the tenor and type of the collateral (D'Amico and Pancost, 2022) and the demand for that *particular* bond.

Importantly, as Duffie (1996) shows, the price of a bond is connected by an arbitrage relation to its special repo rate, which describes its value as collateral. However, since Duffie's foundational contribution, most of the literature on money market rates has largely abstracted from term structure considerations and most research on term structure issues largely assumes that

[2]Sources: Bank for International Settlements (BIS), www.bis.org/publ/cgfs59.htm; US Department of the Treasury, home.treasury.gov/system/files/136/IAWG-Treasury-Report.pdf.

the money market rates are exogenous. Our model helps fill this gap between the bond and repo markets and shows that, dynamically, bond scarcity, repo specialness, and the term structure feature nontrivial and previously undocumented interactions with one another.

Our model delivers two key results. First, we show that repo specialness strongly influences the term premium along with the entire yield curve for both GC and SC bonds. High levels of repo specialness are evidence of the significant costs of carry trades and hedging strategies, both of which are limits to arbitrage that attenuate the response of the yield curve to demand forces such as QE. Intuitively, repo specialness reduces the attractiveness of carry trade strategies, which short-sell bonds and invest the proceeds in the sequence of overnight rates, thus increasing the duration of the portfolio of term structure arbitrageurs and hampering the duration extraction channel of QE. That is, repo specialness dampens the impact of any given quantity of asset purchases on the term premium of the term structure of interest rates. By inducing frictions in money markets, asset purchases become less effective in achieving their main purpose of reducing the term premium. We show that QE becomes less effective in compressing the term premium as the repo specialness QE generates in the SC segment of the secured money market becomes larger.

Second, bond scarcity increases repo specialness, strengthening the local supply channel of QE. As documented by D'Amico and King (2013), QE often brings about local supply effects, defined as relative-price anomalies of closely related assets induced by demand. Such effects are typically absent in equilibrium term structure models (TSMs), where bonds must be priced consistently with one another by arbitrage. The SC repo market structure offers a natural solution to this puzzle. When a bond is subject to exceptional demand pressure in the market, it becomes overpriced relative to instruments with equivalent cash flows. The lure of price deviations from economic fundamentals induces term structure arbitrageurs like hedge funds to borrow the overpriced bond and sell it short. Arbitrageurs must then deliver that specific security at the end of the contract. Their behavior gradually raises the demand for high-quality collateral in the repo market, exerting endogenous downward pressure on special repo rates and thus eliminating arbitrage. Bond scarcity generates strongly localized supply effects like kinks in the term structure. Therefore, the apparent anomaly of "overpriced bonds" disappears once the term structure is integrated with the repo market.[3]

[3]For instance, yield curve fitting errors of Treasury securities are widely used by academics, policymakers, and practitioners. In an influential paper, Hu et al. (2013) use the dispersion in Treasury yield curve fitting errors as a measure of pricing noise, which proxies for the shortage of arbitrage capital in the economy. One caveat for considering the Treasury market in isolation from the repo market is that bond mispricing might not be executable if the borrowing cost of a position in the repo market is large. Thanks to endogenizing specialness, our model is able to explain the yield curve fitting errors in a manner that is consistent with the absence of arbitrage.

To derive our results, we build on the Vayanos and Vila (2021) (VV) TSM of the bond market. Unlike VV, we focus on the preferences of investors for specific characteristics. For example, in the US Treasury bond market, securities with the same cash flows can be *on-the-run* or *off-the-run*. Traders prefer the former and bid up their prices.[4] A more recent phenomenon involves QE, in which central banks purchased large quantities of several bonds, making many of them special (see Arrata et al., 2020; Ballensiefen et al., 2023). We designate bonds subject to excess demand as "special." In doing so, we introduce a new dimension to TSMs: bonds that share the same tenor might differ in their exposure to demand forces. The notion of demand forces inducing bond specialness puts us in a comfortable position to model QE. To ensure that equilibrium demand-driven price differences between instruments with equivalent cash flows are consistent with the classical notion of arbitrage, we must account for the *different* borrowing cost of the bond in the SC repo market, where investors borrow a specific bond and lend cash. Asset purchases exert direct price pressure on special bonds. On the opposite side of the bilateral purchases of preferred-habitat investors (e.g., the central bank), aggressive market participants that we refer to as arbitrageurs sell the special bonds short and reinvest the proceeds until maturity in the money market. As a group, arbitrageurs thus borrow long-term bonds and invest cash at the series of overnight short rates in the money market, replicating the securities for which asset purchases generate excess (QE) demand through their carry strategy. This endogenous response of the private sector induces three effects.

1. Arbitrageurs intensify their search for collateral on the repo market to borrow special bonds in the face of increasing scarcity. Since the supply of any special bond is finite, its repo specialness increases along with the bond price due to the arbitrage between general and special bonds presented by Duffie (1996). GC bonds, which can be exactly replicated through interest rate derivatives, are not directly affected and will only be affected indirectly through risk adjustment effects. This mechanism leads to the presence of *local supply effects* when comparing bonds across different time-to-maturity buckets.

[4] For instance, it is common for traders to roll over their positions into each successive *on-the-run* issue, perhaps because of their exceptional liquidity and because they are often the cheapest among the basket of deliverable bonds for the settlement of futures contracts (Merrick Jr et al., 2005). This pattern has been well documented empirically. Cornell and Shapiro (1989) were among the first to show the existence of mispricing between bonds with equivalent cash flows. Among others, Barclay et al. (2006) show, using clearing records, that both the trading volume and the market share of electronic intermediaries decline by about 90% when Treasury securities go *off-the-run*. The terms *on-the-run* and *off-the-run* do not carry as much significance in other markets – including most of Europe, Japan, and India – because sovereign bonds in those markets are often issued "on tap." Thus, in principle, all bonds can be reissued and their specialness cannot be ascribed to recency of issuance. What explains specialness in other markets? In general, securities go on special when they attract a significant degree of excess demand, which sometimes arises when a bond becomes the *cheapest to deliver* in the futures market, when a bond is used as a hedge, or when the issue is labelled Green or Islamic.

2. Asset purchases increase the exposure of arbitrageurs to their carry trade strategy, compressing the term premium, as pointed out by VV. This channel is often referred to as the *duration extraction channel* and affects both GC and SC bonds.

3. In general equilibrium, the above channels interact with each other. Bond scarcity in money markets induces a reduction in SC rates, which results in lower yields for the corresponding special bonds on the bond market (Duffie, 1996). A distinct but related effect of bond scarcity is reducing the willingness of arbitrageurs to carry their trades across the curve to meet the exceptional demand prompted by preferred-habitat investors. Gradually, special bonds become more costly to borrow in the SC repo market, and their scarcity brings about the concrete risk of short squeezes. Arbitrageurs continue to borrow long-term bonds in the repo market but reduce their exposures and thus the compression effect of asset purchases on the term premium for both GC and SC bonds. Intuitively, higher levels of specialness in the SC repo market partially unwind the QE effect on the term premium. Frictions in the money markets thus impair the transmission of asset purchases to the term premium, which is reduced by less than in the benchmark case, which is absent specialness.

In our closed-form solutions, the equilibrium price of bonds targeted by exceptional demand exceeds the price of otherwise equivalent bonds by the risk-adjusted present value of their stream of repo dividends. Repo specialness is stochastic, dynamic, and affected by excess demand in the bond market. As a result, the bond and repo markets feature non-trivial interactions with each other, over and above the known arbitrage connection studied by Duffie, warranting interest in a general equilibrium approach. Moreover, the expected return-risk ratio on the bond market is consistent with the absence of arbitrage only when the short riskless rate varies at the instrument level, requiring empirical studies of the bond market to consider a broader picture that includes the rates at which securities are financed (see Cherian et al. (2004) and Chen et al. (2022)).

A calibration of our theory using realistic parameters quantitatively illustrates our main findings. For comparability with previous studies, we use the US Treasury bond data from Gürkaynak et al. (2007). From our setup, two distinct yield curves of general and special bonds are obtained by rolling over GC and SC repo contracts, consistent with the price premium commanded by near-money assets (Krishnamurthy and Vissing-Jorgensen, 2012; Nagel, 2016; Van Binsbergen et al., 2022). Subsequently, we demonstrate the diverse impacts of QE on the term premium through calibration. First, we examine the scenario where the repo market exhibits inelasticity to quantity, allowing us to capture the duration effect of QE on the term premium as modeled by VV. Second, we investigate the case where the supply of bonds on the repo market displays elasticity to quantity, in line with the documented empirical evidence. We establish that elasticity within the repo market impairs the effect of QE on the term premium.

Overall, our framework proposes a paradigm shift from a focus on "conceptual" arbitrage, at the core of finance, to one on "executable" arbitrage, in the spirit of a recent strand in the literature (Gabaix et al., 2007; Du et al., 2018; Fleckenstein and Longstaff, 2020; Jermann, 2020; Pelizzon et al., 2022). Our theory is distinguished by the fact that price differences are not attributable to specialized or constrained marginal investors but rather stem from the holding cost of arbitrage – that is, the cost of repeatedly borrowing a position to sell it short – as documented by Fontaine and Garcia (2012).

The remainder of the paper is organized as follows. Section 2 surveys the related literature. Section 3 presents a simple theory of the term structure of interest rates integrating capital and money markets. Section 4 presents the main theoretical predictions of the model and a calibration with market data. Section 5 offers concluding remarks. All proofs are available in the Appendix.

2 Literature Review

The Term Structure of Interest Rates. There is a vast literature on modeling the term structure – the relation between time to maturity and bond yield – at a general level. However, we confine ourselves to a discussion of more recent research focusing on the impact of unconventional monetary policies, such as QE, on the term structure. Unconventional monetary policies have renewed efforts by researchers to explain the effects of demand pressure on fixed-income securities in general and sovereign bonds in particular (see, e.g., D'Amico and King, 2013 and Greenwood and Vayanos, 2014). Relatedly, Du et al. (2022) document that the term premium is endogenous to the portfolio holdings of intermediaries. The canonical framework for this recent literature is found in Vayanos and Vila (2021); it provides the analytical structure to harmonize recent empirical findings with the received preferred-habitat theory (pioneered by Culbertson, 1957 and Modigliani and Sutch, 1966), which accounts for the differences in investment horizons across investors.

Our point of departure from the recent literature on habitat preferences in fixed-income markets is concentrating on investors' preferences for special bonds within maturity buckets – rather than on maturities – which could arise from mutual fund investment mandates and liquidity considerations (Pasquariello and Vega, 2009).[5] Moreover, we explicitly consider the two segments of the money market, the GC and the SC repo markets, and endogenize special repo rates by allowing arbitrageurs to finance their positions in the repo market. Furthermore, we allow arbitrageurs to be immune with respect to interest rate risk in the classical sense; namely, by buying two bonds of the same tenor when demand forces induce relative price differences, a strategy commonly referred to as a *convergence trade*. The comparatively higher price of the sought-after special bond is reflected in its appropriate special repo rate by the endogenous search for the collateral necessary to sell the security short. This approach paves the way to the assessment of the effects of bond scarcity due to QE on the money market and to the quantitative evaluation of new policy tools such as securities lending facilities (SLFs).[6]

The Repo Market. Repo contracts are similar to collateralized loans. In a foundational paper, Duffie (1996) shows that bond prices and the rate on the loans they collateralize are connected by an arbitrage restriction and develops a model connecting the two in a static sense (empirically validated by Jordan and Jordan, 1997), where special repo rates – that is, those significantly below prevailing riskless rates – decrease as arbitrageurs intensify the search for collateral to sell a bond short on the secondary market. Unlike Duffie's paper, we explore the repo specialness in a dynamic sense in both the time series and the cross-section of bonds, explaining it as the result of the interaction between demand forces and costly arbitrage.[7] To our knowledge, our paper is the first general equilibrium model formalizing these ideas in a term structure framework where repo specialness arises endogenously due to (i) preferred-habitat investors' demand that generates bond scarcity and (ii) repo market elasticity to quantities.

[5]Naturally, bonds differ in many dimensions other than maturity. For example, Chen et al. (2022) use the constraints on Islamic financial institutions for their investments to comply with Shariah law to identify clientele effects on bond prices and repo rates, and D'Amico et al. (2022) focus on Green premia, the yield differences between maturity-matched conventional and Green bonds.

[6]Recently, the framework proposed by VV has been extended to the foreign exchange market in Greenwood et al. (2023) and Gourinchas et al. (2022), to the credit risk market in Costain et al. (2022), and to the interest rate swaps market in Hanson et al. (2022), by using arbitrage restrictions. However, none of these papers focuses on the effects of demand pressure on the repo market.

[7]Other contributions in this area include Buraschi and Menini (2002), Fisher (2002), and Krishnamurthy (2002). Cherian et al. (2004) document the joint cyclicality of special repo rates and bond specialness over the auction cycle and present a no-arbitrage model where *on-the-run* bonds are discounted at an exogenously modeled special repo rate. We derive such phenomena endogenously by building on recent advances in the literature on heterogeneity in asset demand across investors. Other papers on price differences between securities with equivalend cash flows include Vayanos and Weill (2008) and Gârleanu et al. (2021). We complement their stationary search-based contribution from a term structure perspective, which has the advantage of allowing for time series analyses. Copeland et al. (2014) and Mancini et al. (2016) present extensive descriptions of the institutional aspects of the US and European markets for repurchase agreements. Consistent with our model, Graveline and McBrady (2011) and Maddaloni and Roh (2021) show that inelastic investors participate in the repo market substantially less than in the secondary market, increasing the scarcity of collateral.

He et al. (2022) propose a preferred-habitat model that explains the behavior of Treasury convenience yields in times of crisis, where dealers subject to regulatory constraints provide GC repo financing to leveraged investors. Our paper differs from theirs because we focus on endogenous SC rates and provide a unified framework to price-specific and generic securities, giving rise to equilibrium price differences between bonds with identical cash flows. In models where the short rate is constrained in the cross-section of bonds, such price differentials would normally result in arbitrage opportunities. Instead, in our framework, the equilibrium creates the specialness of the specific bond and satisfies a generalized notion of the Sharpe ratio that allows the short financing rate to depend on the characteristics of the collateral.

Collectively, the consensus view in the literature on the effects of QE in fixed-income markets has highlighted stable empirical patterns that have proven robust across countries and over time.

Stylized Fact 1: QE significantly affects the term structure. To put this into perspective, Christensen and Rudebusch (2012) use data around policy announcements in the US Treasury market to estimate a reduction of the term premium on the order of 29 basis points (bps) (see also Gagnon et al., 2018). Moreover, QE generates local supply effects. D'Amico and King (2013) quantify this effect during the first large-scale asset purchase program in the US at around 30 bps (for evidence in EU markets, see Altavilla et al., 2021; Koijen et al., 2021).[8]

Stylized Fact 2: QE significantly affects repo specialness. In the context of US markets, D'Amico et al. (2018) document an effect of - 1.8 bps per billion dollars purchased by the Fed, with a stronger impact at the short end of the curve. Today, the Fed holds around $5 trillion in US Treasury securities. In the EU, Arrata et al. (2020) document that large-scale asset purchases affect repo specialness through the collateral scarcity channel and estimate that purchasing 1% of the outstanding bond increases its specialness by 0.78 bps. The ECB held 39% of German government bond (Bundesanleihen or bunds) at the end of 2017.[9] The estimates in Corradin and Maddaloni (2020) are even larger.

[8]The *Financial Times* suggests that long rates may have become artificially too low as a result of Fed's bond buying programs – and the curve therefore no longer sends a reliable signal about future economic conditions. The article, titled "Is the yield curve lying?" and dated June 2023, then argues that low rates are not driven by fundamentals but instead by the demand for long-term, risk-free securities for use as collateral.

[9]According to the BIS, the share of special trades in the German repo market increased from around 5% before the introduction of the Public Sector Purchase Programme in 2015 to more than 50% in 2016, peaking at the staggering level of 550 bps (https://www.bis.org/publ/mktc11.pdf, IV.13.)

3 The Model

3.1 Setup

In this section, we develop a model in continuous time $t \in (0, \ldots, \infty)$ that features a market for default-free zero coupon bonds (ZCB). Bonds are indexed by their tenor τ and by their status i = {g, s}; that is, as general as opposed to special bonds. General and special bonds of the same tenor have equivalent cash flows, but their prices can differ because of the demand effects detailed below. At time t, a bond with tenor τ has a price $P^{\tau}_{i,t}$ expressed in dollars per unit of notional principal. The continuously compounded yield to maturity is

$$y^{\tau}_{i,t} = -\frac{1}{\tau} \log P^{\tau}_{i,t}. \tag{1}$$

The short rate r_t follows a Vasicek process whose parameters have the usual interpretation.[10]

$$dr_t = \kappa_r(\overline{r} - r_t)dt + \sigma_r dv^r_t. \tag{2}$$

Bonds can be used as collateral to obtain overnight secured financing in the repo market.[11] As is standard in modeling repurchase agreements, we abstract from collateral rehypothecation and credit risk and assume that the repo market clears once a day (see, e.g., Duffie, 1996).[12] Therefore, the GC repo rate must coincide with the short rate r_t to prevent arbitrage opportunities. In our model, the short-rate process in Equation (2) can thus be interpreted as describing the GC repo rate dynamics (e.g., the SOFR in the US Treasury market). As noted above, the repo market is segmented. Arbitrageurs with overnight cash on their hands have two distinct riskless options to lend money against either SC or GC bonds at their respective market rates, namely:

1. Reverse any of a basket of generic bonds ($i = g$) in the GC market by entering an overnight agreement that earns the GC repo rate r_t.

2. Reverse the position in the SC market ($i = s$), which is elastic in supply, and earn the lower overnight SC repo rate r^{τ}_t, to be determined in equilibrium.

[10]The choice of a Gaussian model is standard. For a discussion of non-Gaussian models, see Berardi et al. (2021).

[11]We focus on overnight repo transactions for simplicity because the modeling of term repos would require an additional index. Empirically, the overnight segment of the repo market attracts by far the dominant proportion of volume. The Fed, e.g., reports that overnight repos are about 80% of the volume in the US triparty market.

[12]We consider unlimited overnight borrowing without default risk. The results hold under re-use of collateral as long as the passthrough of the rehypothecated collateral is less than one, as is well understood empirically.

While the GC secures higher interest rates, arbitrageurs might want to forgo loan returns to borrow the special bonds needed to meet any pending short selling commitments. Specialness premia $l(\tau, t) = r_t - r_t^\tau$ do not result in any arbitrage opportunities, as we demonstrate below. However, the supply of special bonds is *elastic* to quantities, as are SC repo rates. In fact, the amount of outstanding special bonds is fixed, and the repo activity of buy-and-hold investors such as pension funds and insurance companies is limited, (as documented by Maddaloni and Roh, 2021). Thus, incremental quantities of special bonds grant financing at progressively lower repo rates. As an illustration, Appendix Figure OA.1 shows the volume-weighted monthly trailing average of the daily rates on repo transactions collateralized by German treasury bonds ranging from 2012 to 2018, using data from Mercato Telematico dei titoli di Stato (MTS) with millisecond precision. We distinguish between GC and SC transactions and plot the latter for benchmark time-to-maturity buckets; SC rates are generally lower than GC rates. Further, it is clear that SC rates can vary stochastically across tenors and over time. We endogenize the difference between the GC and the SC repo rates as a result of the demand effect on the bond market, which induces the search for collateral on the repo market.

3.1.1 Preferred-Habitat Investors

As a group, preferred-habitat investors such as central banks have a demand for the *special* bond of a certain tenor. These investors have habitat preferences, which we allow to be a function of tenor, toward bonds with specific characteristics.[13] Preferred-habitat investors are not active on the repo market, or at least they are less so than arbitrageurs.[14] We define as special those bonds that are targeted by preferred-habitat investors and index them through $i = s$; for clarification, think of securities eligible for QE as obvious candidates for specialness.[15] Conversely, we refer to bonds of all maturities for which the excess demand is permanently zero as general and index them through their status $i = g$; one example is *far-off-the-run* bonds. The demand of preferred-habitat investors is expressed net of the size of the issue supplied by the government, which is normalized to zero, without loss of generality. Borrowing the specification from VV,

[13] We wish to emphasize that our focus on preferences for specific bond characteristics that are clearly observable in market data is not vulnerable to the criticism of the preferred-habitat view of the term structure based on the argument that interest rate derivatives allow for hedging maturity habitats.

[14] Bond market mutual funds often target bellwether indices composed of *on-the-run* bonds of selected maturities and have mandates preventing them from achieving leverage through the repo market because of the risk involved (Krishnamurthy, 2002). Fleckenstein and Longstaff (2021) document that Treasury convenience premia have discontinuities at specific annual maturities induced by clientele effects unrelated to fundamentals.

[15] The set of securities targeted by excess demand includes but is not limited to bonds that are targeted by the purchases of central banks to achieve local effects, *on-the-run*, Green, and Islamic.

we define the excess demand $Z_{i,t}^{\tau}$ for bonds with tenor τ by

$$Z_{i,t}^{\tau} = \begin{cases} -\alpha_{\tau} \log P_{i,t}^{\tau} - \beta_{t}^{\tau} & i = s, \\ 0 & i = g. \end{cases} \tag{3}$$

In the above, $\beta_t^{\tau} = \theta_{\tau} + \omega_{\tau} q_t$ is composed of a term θ_{τ} that is constant over time but can depend on maturity and a loading ω_{τ} on the stochastic demand factor q_t, which evolves as

$$dq_t = \kappa_q(\overline{q} - q_t)dt + \sigma_q dv_t^q. \tag{4}$$

Equation (3) is a definition of segmented markets according to which exceptional demand risk factors affect only special bonds. The process for demand risk in Equation (4) is autoregressive and mean-reverting. The parameters κ_q, $\overline{q}$, and σ_q have the usual interpretation of speed of mean reversion, long-run mean, and standard deviation of a process that has normal innovations. To express the model in full generality, we allow for demand shocks and GC rate innovations to be correlated with coefficient ϱ. Under normal market conditions, Equation (3) describes preferences for liquidity and those arising from coordination equilibria among investors. In the context of QE, this formulation captures the purchases by central banks of targeted bonds relative to non-targeted bonds.

3.1.2 Arbitrageurs

Arbitrageurs resort to short-term repo financing and engage in term structure trades to smooth out price differences that would otherwise arise in a segmented equilibrium. For example, arbitrageurs such as hedge funds would short-sell a bond that is overpriced as a result of demand pressure. To this end, they would reverse their position in the bond earning the repo rate and simultaneously sell outright the collateral exerting downward pressure on the bond price. The reverse repo contract would then be rolled over until the bond matures or the position is closed. The portfolio holdings of arbitrageurs are denoted through $X_{i,t}^{\tau}$. The bond market clearing condition is

$$Z_{i,t}^{\tau} + X_{i,t}^{\tau} = 0. \tag{5}$$

Due to market clearing, and since the demand for general bonds does not exceed their supply from the government, arbitrageurs are only active in special bonds in equilibrium. Of course, nothing prevents arbitrageurs from trading general bonds as well, so that in equilibrium these securities would be as profitable as special bonds from their perspective. General bonds are inherently financed at the overnight GC rate, since there is no excess demand for these securities. Effectively, arbitrageurs issue synthetic τ-maturity special bonds by accepting the rollover risk associated with short sales financed through SC repurchase agreements. Thus, higher activity from preferred-habitat investors increases repo specialness by locking up the bond and symmetrically increasing the search for collateral to short the bond by arbitrageurs.[16] The next expression is the dynamics of arbitrageurs' wealth W_t.

$$dW_t = r_t W_t dt + \underbrace{\int_0^\infty X_{g,t}^\tau \left(\frac{dP_{g,t}^\tau}{P_{g,t}^\tau} - r_t \right) d\tau}_{\text{General bonds}} + \underbrace{\int_0^\infty X_{s,t}^\tau \left(\frac{dP_{s,t}^\tau}{P_{s,t}^\tau} - r_t^\tau \right) d\tau}_{\text{Special bonds}} . \tag{6}$$

Equation (6) is *not* a standard law of motion of wealth, even if the restriction $r_t^\tau = r_t \ \forall \tau$ will be shown to correspond to the VV case in which the short rate is constant in the cross-section of bonds. Notably, our approach departs from the textbook portfolio allocation problem between a riskless money market account and a set of risky assets. Here, the holdings of leveraged arbitrageurs are financed on the repo market for collateralized lending.[17] The first integral on the right side of the equation captures cash investments. Invested wealth W_t achieves the remuneration r_t offered by the GC rate, the highest among short rates. Similarly, cash shortages are inherently financed at the GC rate in the absence of SC bonds. The second integral is the marked-to-market value of the portfolio of special bonds net of their financing costs, each represented by the respective SC repo rate r_t^τ. Arbitrageurs establish a long (short) position by buying (selling) the bond outright in the spot market and finance that purchase (sale) by using the bond as collateral to enter an overnight (reverse) repo agreement. The next trading day, arbitrageurs must either close the outright position or roll over the short-term collateralized financing. Unlike an opportunity cost interpretation, r_t^τ thus denotes the cost of the collateralized loan (which repos the bond) to finance the position, in the spirit of Tuckman and Vila (1992).[18]

Why are repo rates more interesting than a simple exogenous process for the short rate? Market considerations aside, the hallmark of special repo rates is the exposure to demand forces (Duffie, 1996). From a theoretical asset pricing perspective, there is simply no room for demand pressure to impact the exogenously specified short rate process in Equation (2). In the

[16] Our approach is consistent with Banerjee and Graveline (2013), who decompose the *on-the-run* premium of Treasury bonds into higher prices encountered by long investors and larger borrowing costs for short sellers.

[17] Without any loss of generality, we assume that arbitrageurs use the repo market to finance their bond portfolios since it is optimal to do so.

[18] For details on how institutional investors finance Treasury trades, see Fisher (2002). A similar insight on their budget constraint can be found in He et al. (2022), where the GC rate results from regulatory frictions. We complement their approach by focusing on SC rates that vary across bonds, induced by exceptional demand.

model we propose, the demand forces that affect bond prices contribute to the endogenous determination of special repo rates r_t^τ. Special repo rates are important from a quantitative viewpoint. For example, using data from the New York Fed, Copeland et al. (2014) estimate SC repo transactions to be about 60% of the daily volume in the US market, with the remaining 40% constituted by GC transactions. The SC daily volume share of the EU repo market is even larger; for instance, Arrata et al. (2020) report an average value of 87%. Thus, the TSMs that exogenously specify the process for the short rate are suitable for describing the GC repo market but leave the larger SC segment of the market unmodeled.

3.1.3 General Bonds, Special Bonds

Two issues of the same tenor may differ in terms of collateral value: for instance, bonds with the same time to maturity might be SC as *on-the-run* securities or GC as *far-off-the-run* ones. While both are exposed to the same duration risk, only the former is targeted by preferred-habitat investors and thus affected by demand pressure. To highlight this distinction in our model, we define as special those bonds that are exposed to two risk factors and as general those bonds exposed to one risk factor. Formally, let us conjecture that the price process is exponentially affine in the short rate and, conditionally on the bond status, in demand forces.

$$-\log P_{i,t}^\tau = \begin{cases} A_\tau r_t + B_\tau X_{s,t}^\tau + C_\tau & i = s, \\ A_\tau r_t + C_\tau & i = g. \end{cases} \tag{7}$$

Specific to our framework, bonds with identical cash flows can trade at different prices because of demand pressure. This feature adds a layer of realism to the TSMs and arises because the exposure of GC bonds to demand risk is restricted to zero (by construction), so that the price of these bonds reflects only the risk of changes in the short rate r_t. Equation (7) reflects a market segmentation, as the compensation for (GC) interest rate risk r_t is common to GC and SC bonds, while demand forces only exert direct pressure on the price of bonds targeted by preferred-habitat investors. In Treasury markets, we regularly observe that bonds on special are overpriced with respect to general bonds with identical cash flows. Let $B_{i,\tau} = B_\tau \mathbb{1}_{[i=s]}$, where $\mathbb{1}_{[i=s]} = 1$ if $i = s$ and 0 otherwise, and write Equation (7) more compactly as

$$-\log P_{i,t}^\tau = a_{i,\tau} r_t + b_{i,\tau} \beta_t^\tau + c_{i,\tau}. \tag{8}$$

In the above, we have used the bond market clearing condition $X_{i,t}^\tau = \alpha_\tau \log P_{i,t}^\tau + \beta_\tau$ and we have defined $a_{i,\tau} = A_\tau (1 + \alpha_\tau B_{i,\tau})^{-1}$, $b_{i,\tau} = B_{i,\tau}(1 + \alpha_\tau B_{i,\tau})^{-1}$, and $c_{i,\tau} = C_\tau (1 + \alpha_\tau B_{i,\tau})^{-1}$.

To develop intuition, we solve the model without demand risk ($\omega_\tau = 0$). In Appendix [TBA], we solve the model with demand risk.

3.2 *Equilibrium in the Bond Market*

The equilibrium in the bond market is a set of bond prices such that the market clears and arbitrageurs behave optimally given the demand of preferred-habitat investors. The next few steps follow the structure in VV, generalizing that model to multiple instantaneous rates r_t^τ in the cross-section of bonds. Our contributions become clear thereafter. Replace Equations (4), (5), and (2) into Equation (8) to derive bond prices dynamics.

$$\frac{dP_{i,t}^\tau}{P_{i,t}^\tau} = \mu_{i,t}^\tau dt - a_{i,\tau}\sigma_r dv_t^r, \tag{9}$$

$$\mu_{i,t}^\tau \equiv \dot{a}_{i,\tau} r_t + a_{i,\tau}\kappa_r (r_t - \overline{r}) + \frac{1}{2} a_{i,\tau}^2 \sigma_r^2 + \dot{b}_{i,\tau}\theta_\tau + b_{i,\tau}\dot{\theta}_\tau + \dot{c}_{i,\tau}.$$

As usual, $\mu_{i,t}^\tau$ is the expected return from a bond. The volatility of bond returns depends on the innovations in the short rate dv_t^r, whose effects vary across maturities. We note that Equation (9) describes the returns of both general and special bonds, since the coefficients $(a_{i,\tau}, b_{i,\tau}, c_{i,\tau})$ depend on the bond status. In equilibrium, through market clearing, arbitrageurs' net exposures at the close of the business day are only short positions in special bonds.[19] Arbitrageurs set their portfolio holdings as the solution of

$$\max_{\{X_{i,t}^\tau\}} \quad \frac{\mathbb{E}_t\left[dW_t\right]}{dt} - \frac{\gamma}{2}\frac{\mathbb{V}_t\left[dW_t\right]}{dt}. \tag{10}$$

Substituting Equation (9) into the arbitrageurs' wealth dynamics in Equation (6), we obtain

$$dW_t = \left[W_t r_t + \int_0^\infty X_{g,t}^\tau \big(\mu_{g,t}^\tau - r_t\big) + X_{s,t}^\tau \big(\mu_{s,t}^\tau - r_t^\tau\big) d\tau\right] dt - \left[\int_0^\infty a_{g,\tau} X_{g,t}^\tau + a_{s,\tau} X_{s,t}^\tau d\tau\right] \sigma_r dv_t^r.$$

Replacing the above expression into Equation (10), we obtain

$$\max_{\{X_{i,t}^\tau\}} W_t r_t + \int_0^\infty X_{g,t}^\tau \big(\mu_{g,t}^\tau - r_t\big) + X_{s,t}^\tau \big(\mu_{s,t}^\tau - r_t^\tau\big) d\tau - \frac{\gamma}{2}\left[\sigma_r^2 \int_0^\infty a_{g,\tau} X_{g,t}^\tau + a_{s,\tau} X_{s,t}^\tau d\tau\right]^2.$$

[19] Empirically, D'Amico et al. (2018) use the repo volume spread, calculated as the volume of reverse repo versus repo contracts, to measure excess demand for bonds and proxy for the number of short positions. Their estimates show that the repo volume spread is 10 times larger for *on-the-run* than *off-the-run* Treasury bonds.

The first-order condition with respect to the position in the bond with tenor τ and status i is

$$\mu_{i,t}^{\tau} - r_t^{\tau} = -a_{i,\tau}\lambda_{r,t}, \tag{11}$$

where

$$\lambda_{r,t} = -\gamma\sigma_r^2 \int_0^{\infty} a_{g,\tau}X_{g,t}^{\tau} + a_{s,\tau}X_{s,t}^{\tau} d\tau. \tag{12}$$

Imposing the market clearing condition, we conclude that the market price of risk is given by

$$\lambda_{r,t} = -\gamma\sigma_r^2 \int_0^{\infty} 0 + a_{s,\tau}\big[\theta_\tau - \alpha_\tau\big(a_{s,\tau}r_t + b_{s,\tau}\theta_\tau + c_{s,\tau}\big)\big] d\tau. \tag{13}$$

Equation (11) is an equilibrium term structure equation where the drift of the bond price $\mu_{i,t}^{\tau}$ is compared against the rate at which arbitrageurs can exchange cash for the special bond r_t^{τ}. This result is key to determining the linkage between the term structure of bond prices and the equilibrium rates in the repo market. Unlike the classical formulation in which the borrowing rate is the short rate, a long (short) position in the special bond must be financed (remunerated) at its own SC repo rate. Intuitively, this result suggests that in equilibrium the deterministic change in the risk-adjusted price of the bond must equal the repo rate against which the market allows arbitrageurs to finance their positions. From Equation (12), we note that the market price of interest rate risk depends on the entire bond portfolio of arbitrageurs, delivering the *transmission* of the duration extraction channel of QE from special bonds to the general term structure of bonds ineligible for asset purchases. We stress that arbitrageurs ensure that prices are consistent both between bonds of different tenor and between bonds of different status.

Equation (11) closely resembles the familiar TSM arbitrage equation, with one difference that is our first important contribution: the riskless rate r_t is replaced by the cross-section of overnight special repo rates, r_t^{τ}. Since the foundational paper by Vasicek (1977), the characterization of TSMs by the absence of arbitrage is routinely based on the restriction $r_t^{\tau} = r_t$ $\forall\tau$. In practice, however, financing costs differ across bonds since they can be used for collateralized borrowing at a variety of special rates. Hence, we relax this assumption and propose a generalized equilibrium condition that allows the short rate to vary with the collateral value the bond grants to its holder. Canonical TSMs are based on the standard arbitrage restriction: Since a portfolio consisting of the appropriate combination of bond exposures achieves perfect immunization against interest rate risk, such a portfolio should realize the same return as an investment remunerated at the spot rate. Therefore, one should observe a constant ratio between mean return and standard deviation across all traded instruments.

Building on the idea of a constant excess return to risk (Sharpe) ratio, we note that in practice borrowing is often collateralized. Hence, it is necessary to employ our equilibrium concept that different bonds give rise to different costs of financing for market participants to fund their positions. Thus, we must adjust the Sharpe ratio, since the risk-free rate is not constant in the cross-section of bonds. That is natural once we recognize that special bonds are simply bonds with an additional stream of repo dividends.[20] We propose a paradigm shift from a focus on arbitrage to one on *executable* arbitrage. The TSM of VV reflects a portfolio allocation decision à la Merton between a riskless spot rate and risky bonds. In our interpretation, however, the equilibrium results from the choices of leveraged investors that use their positions as collateral to borrow cash. For market participants, differences in the collateral value between bonds are crucial determinants of portfolio choices. Special repo spreads are sometimes referred to as the "*repo dividends*" of a bond. The payoffs of the securities must be redefined on account of their holding costs, which our model determines endogenously as a result of market demand segmentation. We next provide a general solution of the model that endogenizes repo specialness l_t^τ, which is defined as the difference between GC and SC rates conditional on time to maturity:

$$l_t^\tau = r_t - r_t^\tau. \tag{14}$$

3.3 Equilibrium in the Repo Market

The equilibrium in the repo market is a set of repo rates such that the market clears given the demand of collateral by arbitrageurs and the supply of collateral by preferred-habitat investors.[21] In the GC segment of the repo market, there is no excess demand and the supply is elastic with slope $\mathcal{E}_g = 0$, since bonds can be substituted among each other. The SC segment of the repo market differs both in demand and supply of collateral. Arbitrageurs have entered the commitment to deliver the specific bond, and their demand for collateral $-X_{i,t}^\tau = Z_{i,t}^\tau$ is inelastic to its price. With rightward shifts in the demand curve for SC bonds in the repo market, equilibrium specialness increases because collateral holders require greater compensation to pledge additional units of the special security (Duffie, 1996). We assume that the supply curve of SC is linear in specialness, with slope $\mathcal{E}_s$. Preferred-habitat investors may or may not supply their bonds in the repo market. In the context of QE, central banks may either lend the bonds purchased through a Securities Lending Facility or may not lend their bonds against cash in the money market. We thus study the effects of QE with and without the SLF.

[20]The equilibrium concept extends to equities by replacing special repo rates with securities lending rebate rates.

[21]Repo rates are determined at the inception of the contract and involve no risk. Conversely, bonds are forward-looking expectations of the relevant future repo rates, whether general or special. Thus, bond prices include risk compensation because the notional principal is discounted at the entire stream of stochastic future repo rates.

Equilibrium with Securities Lending Facility: $l_t^\tau = \mathcal{E}_i(X_{i,t}^\tau + Z_{i,t}^\tau) = 0$.
In the presence of the SLF, the central bank is willing to meet the excess demand for special bonds by arbitrageurs, and QE does not create repo specialness.

Equilibrium without Securities Lending Facility: $l_t^\tau = \mathcal{E}_i Z_{i,t}^\tau$.
In the absence of the SLF, the central bank generates the demand for special bonds by arbitrageurs, who borrow the bond in the money market by paying a specialness premium which increases in their demand.

In sum, repo specialness equals zero for GC bonds that are in inelastic supply. Figure 1 illustrates the equilibrium in the SC segment of the repo market. The figure is a general representation of the SC segment of the repo market that holds independently of the bond tenor.[22]

3.4 *Bond Price and Repo Rates*

Bond prices can be also represented by the conditional Laplace transform

$$P_{i,t}^\tau = \exp\Big(- A_\tau r_t - B_\tau X_{i,t}^\tau - C_\tau \Big) = \mathbb{E}_t^{\mathbb{Q}}\Big[\exp\Big(- \int_0^\tau r_{t+u}^{\tau-u} du \Big) \Big], \tag{15}$$

provided a parametrization is admissible (Duffie and Kan, 1996).[23] The notional principal at maturity is priced using the appropriate bond-specific discount factor (Buraschi and Menini, 2002), with factors that are more persistent exerting a stronger impact on long-term yields.

Thus far, we have derived the equilibrium by using the absence of arbitrage in the time series of bond prices and interest rates. An important difference arises when we turn to their cross-section. While term structure carry trade portfolios require the risky rollover of short-term financing, the cross-sectional static arbitrage between GC and SC bonds is riskless, since both their prices and repo rates are known.[24] Hence, the demand pressure in the cash market has its mirror image in the repo specialness generated by arbitrageurs' search for collateral in the money market.

[22] The first order relation between specialness and demand risk that captures a linear SC supply curve results from the affine specification and can be generalized to higher orders. For example, a second-degree polynomial would result from a quadratic TSM, and so on for higher-order specifications.

[23] Grasselli and Tebaldi (2008) establish conditions for closed-form bond prices in admissible TSMs. We note that the coefficients A_τ, B_τ, and C_τ project the current value of the risk factors on the risk-adjusted rational expectations forecast of their future conditional realizations to impound their information into market quotes.

[24] We abstract from search costs in over-the-counter markets (see Duffie et al., 2005; Jankowitsch et al., 2011).

Lemma 1. *In equilibrium,*

$$\exp\left(B_\tau X_{s,t}^\tau\right) = \frac{\mathbb{E}_t^{\mathbb{Q}}\Big[\exp\Big(-\int_0^\tau r_{t+u}du\Big)\Big]}{\mathbb{E}_t^{\mathbb{Q}}\Big[\exp\Big(-\int_0^\tau r_{t+u}du\Big)\exp\Big(-\mathcal{E}_s\int_0^\tau X_{t+u}^{\tau-u}du\Big)\Big]}.$$

Proof. See Appendix A. The result follows from the price ratio of general to special bonds.

Both general and special bonds promise the payment of equivalent cash flows at maturity. Therefore, their relative price (on the left side of the expression above) in equilibrium must be equal to the ratio of the holding cost of replicating the two bonds through a series of overnight repo contracts, in expected risk-adjusted terms (on the right side of the equation).

Demand pressure induces different valuations between bonds with equivalent cash flows. Such price differences equal the risk-adjusted present discounted value (PDV) of repo specialness from the pricing date until the bond matures. When $\mathcal{E}_s = 0$, there is no repo specialness and $B_\tau = 0$. For given levels of demand pressure, repo specialness increases in the elasticity of collateral supply. As a result, the relative price anomaly between special and general bonds on the cash market must also widen. We conclude that B_τ rises in $\mathcal{E}_s$.

Among others, Buraschi and Menini (2002) and Cherian et al. (2004) suggest that repo specialness must be included in the pricing of special bonds. However, these papers are silent on what determines repo specialness.

3.5 Equilibrium in the Bond and in the Money Market

Proposition 1. Equilibrium with Securities Lending Facility. *In the presence of the SLF, the coefficients governing bond returns in Equation (9) satisfy*

$$\begin{aligned} a_{i,\tau} &= \frac{1-e^{-\kappa_r^*\tau}}{\kappa_r^*} \\ b_{i,\tau} &= 0 \\ c_{i,\tau} &= \kappa_r^*\overline{r}^*\int_0^\infty a_{i,u}du - \frac{\sigma_r^2}{2}\int_0^\infty a_{i,u}du \end{aligned}$$

where the scalars $(\kappa_r^*, \overline{r}^*)$ *are defined by*

$$\kappa_r^* = \kappa_r + a_{i,\tau}\gamma\sigma_r^2\int_0^\infty \alpha_\tau a_{s,\tau}^2 d\tau, \qquad \kappa_r^*\overline{r}^* = \kappa_r^* + a_{i,\tau}\gamma\sigma_r^2\int_0^\infty a_{s,\tau}\big[\theta_\tau - \alpha_\tau c_{s,\tau}\big]d\tau.$$

The short rate r_t is unique and there is no repo specialness.

Proof. Special case of Proposition 2.

This equilibrium with money markets corresponds to Vayanos and Vila (2021). However, the interest rate is unique and exogenous. This occurs because preferred-habitat investors lend their bonds on the repo market, eliminating repo specialness. Since there is no repo specialness, there are no price differences between bonds with equivalent cash flows. In the absence of the SLF, repo rates are however endogenous to quantities, leading to price differences between bonds with equivalent cash flows and aggregate effects on the market price of risk formalized in the next result.

Proposition 2. Equilibrium without Securities Lending Facility. *In the absence of the SLF, the coefficients governing bond returns in Equation (9) satisfy*

$$\begin{aligned} a_{i,\tau} &= \frac{1-e^{-\kappa_r^*\tau}}{\kappa_r^*} \\ b_{i,\tau} &= \frac{\mathcal{E}_i(1-g_\tau)(1-e^{-\int \overline{\theta}_\tau d\tau})}{\overline{\theta}_\tau} \\ c_{i,\tau} &= \kappa_r^*\overline{r}^* \int_0^\infty a_{i,u}du - \frac{\sigma_r^2}{2}\int_0^\infty a_{i,u}du \end{aligned}$$

where the scalars $(\kappa_r^*, \overline{r}^*)$ are defined by

$$\kappa_r^* = \kappa_r + \alpha_\tau\mathcal{E}_i + a_{i,\tau}\gamma\sigma_r^2\int_0^\infty \alpha_\tau a_{s,\tau}^2 d\tau,\ \kappa_r^*\overline{r}^* = \kappa_r^* + a_{i,\tau}\gamma\sigma_r^2\int_0^\infty a_{s,\tau}\big[\theta_\tau - \alpha_\tau(b_{s,\tau}\theta_\tau + c_{s,\tau})\big]d\tau.$$

Moreover, $\overline{\theta}_\tau = \dot{\theta}_\tau(1+\alpha_\tau\mathcal{E}_i)/\theta_\tau$ and g_τ is the solution of $g_\tau = e^{-\int \overline{\theta}_\tau d\tau}\big(\int_0^\infty \frac{\alpha_\tau c_{i,\tau} e^{\int \overline{\theta}_\tau d\tau}}{\theta_\tau}\big)d\tau$. There is a cross-section of money market rates $r_t^\tau = r_t - l_t^\tau$, whose repo specialness is given by

$$l_t^\tau = \mathcal{E}_i Z_{i,t}^\tau.$$

Proof. See Appendix A.

Proposition 2 shows that the absence of the SLF introduces price distortions between securities with equivalent cash flows on the bond market, captured by the b_τ coefficients. These price differences on the bond market are reflected by bond special repo rates l_t^τ in the money market, and are thus not executable. Crucially, repo specialness affects the coefficients of special bonds $(a_{s,\tau}, b_{s,\tau}, c_{s,\tau})$ and thus the *entire* yield curve, since the holdings of arbitrageurs influence the market price of interest rate risk $\lambda_{r,t} = -\gamma\sigma_r^2\int_0^\infty a_{s,\tau}\big[\theta_\tau - \alpha_\tau\big(a_{s,\tau}r_t + b_{s,\tau}\theta_\tau + c_{s,\tau}\big)\big]d\tau$, which is common for *both* general and special bonds.

Remark 1. *The $b_{g,\tau}$ coefficients are a sequence of zeros for GC bonds, as their repo supply is inelastic. Conversely, for SC bonds the $b_{s,\tau}$ coefficients are positive, leading to higher bond prices because the securities are in elastic supply on the repo market.*

$$\begin{cases} \mathcal{E}_g = 0 \Longleftrightarrow b_{g,\tau} = 0 \quad \forall \tau & i = g, \\ \mathcal{E}_s > 0 \Longleftrightarrow b_{s,\tau} > 0 \quad \forall \tau & i = s. \end{cases}$$

Remark 1 is intuitive: The b_τ coefficients switch off to zero for GC bonds, which are not subject to demand pressure and symmetrically are in inelastic supply on the repo market. As an immediate consequence, for general bonds we have $a_{g,\tau} = A_\tau$ and $c_{g,\tau} = C_\tau$. Conversely, SC bonds are overpriced relative to those that are not subject to demand pressure. The sign restriction $b_{i,\tau} \geq 0 \quad \forall\ \tau$ maps to the well-known result that repo rates are lower for issues on special that guarantee cheaper cash equivalence. The economic reasoning follows Duffie (1996).[25] In general, we prefer not to rule out the unlikely event of negative specialness that could result from selling pressure. However, unless the demand pressure $Z^\tau_{s,t}$ is negative, SC repo rates are below the GC rate ($r^\tau_t \leq r_t$) and special bond prices are above general bond prices ($P^\tau_{s,t} \geq P^\tau_{g,t}$).

Proposition 2 is novel because securities with identical cash flows would have the same price in earlier TSMs. Instead, this result shows that in equilibrium price differences arise for bonds targeted by demand pressure, *ceteris paribus*. The key insight is that our setup does not restrict the collateral value of all securities to a common exogenous short rate.

The recursion for the $b_{i,\tau}$ coefficients in Proposition 2 is parametrized by $\mathcal{E}_i$, which captures the elasticity of collateral supply. We nest more traditional models as special cases which obtain by setting $\mathcal{E}_i = 0$, a case corresponding to TSMs where there is no pricing of exceptional demand pressure, the lending rate is exogenous, and the collateral is general.

Remark 2. *Arbitrageurs consider general and special bonds equally profitable in equilibrium.*

3.6 Bond Scarcity and Conventional Monetary Policy

Conventional monetary policy affects the yield curve by changes in the level or dynamics of the GC rate r_t. The empirical evidence in Nguyen et al. (2023), who document that bond scarcity

[25] Assume by contradiction $b_\tau < 0$ for some tenor τ that would occur if net demand pressure were to reduce some equilibrium price. Since the GC borrowing rate r_t is not sensitive to quantity, arbitrageurs would want to buy an infinite amount of the relatively underpriced special issue and short sell the general one in order to create a portfolio that achieves a perfect hedge against the financing costs of the position (i.e., its short rate risk) and generates riskless profits when both bonds reach maturity, thus contradicting the concept of equilibrium that requires market clearing, i.e., finite quantities.

impairs the pass through of rate hikes to the money market and hence to the bond market.[26] Our model replicates this feature of the data. Recall that special repo rates are given by

$$r_t^\tau = r_t + \mathcal{E}_i X_{i,t}^\tau = r_t + \mathcal{E}_s\big[\theta_\tau - \alpha_\tau\big(a_{s,\tau} r_t + b_{s,\tau}\theta_\tau + c_{s,\tau}\big)\big].$$

Therefore, while the pass through of monetary policy to the GC rate is complete, SC rates only respond to changes in r_t by a factor of $1 - \alpha_\tau a_{s,\tau}\mathcal{E}_s$. Intuitively, GC rate hikes lower the price of special bonds and thus increase their excess demand from preferred-habitat investors.

On the bond market, the sensitivity of the yields of general bonds to short rate shocks is A_τ, and the sensitivity of yields of special bonds to short rate shocks is $a_{s,\tau} = A_\tau(1 + \alpha_\tau B_{s,\tau})^{-1}$. In our model, the transmission of short rate shocks across the yield curve is stronger for GC bonds than it is for SC bonds, since $A_\tau > A_\tau(1 + \alpha_\tau B_{s,\tau})^{-1}$. Therefore, special bond yields react less to rate hikes.

3.7 Bond Scarcity and Unconventional Monetary Policy

The literature on QE suggests that asset purchases affect the term structure by influencing the term premium and by inducing local supply effects (see, e.g., D'Amico et al., 2012). In Section 3.7.1, we discuss the effects of special repo rates on the term premium. Section 3.7.2 discusses Quantitative Tightening (QT). In Section 4.2, we show by calibration that our TSM generates strongly localized supply effects, a feature that, to our knowledge, is new in the literature.

3.7.1 Bond Scarcity and the Term Premium

The term premium is the yield difference between long- and short-term general bonds, since the central bank is interested in the transmission of monetary policy to the general term structure of interest rates. The yield of general bonds is $y_{g,t}^\tau = \frac{1}{\tau}\big(\frac{1-e^{-\kappa_r^*\tau}}{\kappa_r^*} r_t + c_{g,\tau}\big)$. The term premium of τ bonds is the difference between the short rate and the yield of general bonds, $TP(\tau) = y_{g,t}^\tau - r_t$. QE is an exogenous shock to demand $\Delta\theta_\tau < 0$ that does not affect the short rate r_t (recall that θ_τ is the negative of the intercept of the preferred-habitat demand).

$$\frac{\partial TP(\tau)}{\partial\theta_\tau} = \frac{1}{\tau}\frac{\partial c_{g,\tau}}{\partial\theta_\tau} = \frac{1}{\tau}\gamma A_\tau a_{s,\tau}\sigma_r^2(1 - \alpha_\tau b_{s,\tau})\int_0^\infty A_u du \geq 0. \tag{16}$$

Equation (16) shows the magnitude by which QE ($\Delta\theta_\tau < 0$) compresses the slope of the yield curve. Dysfunctional money markets dampen the effect of QE on the term premium, since:

[26] Eisenschmidt et al. (2024) document that repo rate dispersion impairs the transmission of monetary policy.

- $a_{s,\tau}$is decreasing in κ_r^* and thus in $\mathcal{E}_s$, implying $a_{s,\tau} \leq A_\tau$.
- From Lemma 1, $b_{s,\tau}$ is increasing in $\mathcal{E}_s$.

These two effects reinforce each other, as formalized in the next result.

Lemma 2. ***Money Markets and the transmission of QE.*** *The term premium compression effect exerted by QE in combination with the SLF is larger than that achieved by QE without the SLF.*

$$\frac{1}{\tau}\gamma A_\tau^2\sigma_r^2 \int_0^\infty A_u du \geq \frac{1}{\tau}\gamma A_\tau\sigma_r^2 a_{s,\tau}(1-\alpha_\tau b_{s,\tau}) \int_0^\infty A_u du.$$

Proof. Follows by comparing the effects of QE ($\Delta\theta_\tau < 0$) in Propositions 1 and 2. *Q.E.D.*

We view this result as one of our key contributions and one that has a natural interpretation. When the short rate is exogenous, QE lowers the term premium by inducing arbitrageurs to increase their short selling activity. However, when the short rate is endogenous, QE generates repo specialness which act in the *opposite* direction, raising term premia. The transmission of QE is weaker when the special collateral is supplied with higher elasticity $\mathcal{E}_s$, raising the difference between the interest rate risk of general and special bonds A_τ and $a_{s,\tau}$ and their relative mispricing $b_{s,\tau}$.

The market price of risk governs the slope of the yield curve and the term premium. Thus, a larger magnitude of arbitrageurs' exposures reduces the slope of the yield curve (Vayanos and Vila, 2021). When we also endogenize the effects of asset purchases in the money market, repo specialness dampens the transmission of QE by reducing the magnitude of arbitrageurs' portfolio through two channels.

The first channel operates through market clearing. Repo specialness increases the cost of bonds in the cash market, reducing the demand of preferred-habitat investors and, symmetrically, the exposure of arbitrageurs and their required compensation for the risk of short-selling long-term bonds and investing the proceeds at the risk-free rate. The second channel operates through the optimality of arbitrageurs. Repo specialness is the cost of carry trade arbitrage positions hedged against interest rate risk. As a limit to arbitrage, repo specialness reduces the incentives to short-sell long-term bonds and invest the proceeds at the series of overnight rates. With higher specialness, arbitrageurs scale down their positions, *ceteris paribus*. On aggregate, repo specialness thus increases the duration of the portfolio of arbitrageurs and dampens the duration extraction channel of QE.

One of the most important takeaways of our undertaking can be illustrated by the optimality of arbitrageurs, which requires that $\mu_{i,t}^\tau - r_t + l_t^\tau = -a_{i,\tau}\lambda_{r,t}$. If repo specialness l_t^τ rises, for given levels of interest rates r_t, only two things can occur: either the expected returns $\mu^{\tau}_{i,t}$ from shorting the specific bond also rise relative to its duration risk $a_{i,\tau}$ (strengthening the local supply effect), or the absolute value of the market price of interest rate risk $-\lambda_{r,t}$ rises, steepening the entire yield curve (dampening the duration extraction channel). Therefore, an increase in the specialness of a bond is either transmitted to a lower yield or distributed to the entire term structure through a change in the market price of risk flattening the yield curve. The FOC of arbitrageurs thus underscores a novel trade-off between local supply effects and duration extraction of QE. Overall, a reduction in specialness – e.g., through the SLF versus cash program – results in stronger impacts of QE on the compression of the yield curve and on less localized supply effects.

3.7.2 Bond Scarcity and Quantitative Tightening

Our model offers implications for QT, an exogenous shock to demand $\Delta\theta_\tau > 0$ that does not affect the short rate r_t (recall that θ_τ is the negative of the intercept of the preferred-habitat demand). On the money market, QT reduces the repo specialness generated by QE. On the bond market, we have highlighted a trade-off between the duration extraction and the local supply channels of QE. The same trade-off is at work with QT, which achieves stronger term premium (local supply) effects on the bond market when repo specialness is lower (higher). On the repo market, while QE (QT) increases (reduces) specialness, the effect of QT is nonlinear since repo specialness has a lower bound at zero.

3.8 Testable Predictions

Perhaps the most interesting testable prediction of our theory is a preference-free asset pricing equation that generalizes the classical term structure equilibrium equation. Based on the notion of arbitrage, we point out that the excess return to risk ratio should be constant in the cross-section of nearly risk-free bond returns, but only after taking into account the convenience yield (that is, the repo specialness) of the asset. Equation (11) is relatively simple to apply to the data. To test its empirical counterpart, we require a panel of nearly riskless bonds that consists of observations of their secondary market and repo quotes. The data should include both generic and special bonds with the same tenor n.

A formal empirical analysis is beyond the scope of this paper, but we can sketch the necessary steps. It is natural to estimate the (Jensen-adjusted) drift term of each bond $\hat{m}^{\tau}_t$ as the period-to-period bond return using market data and to assess the robustness of the estimates to different frequencies. Similarly, a common approach is to use variation in returns to proxy for the standard deviation $\hat{s}^{\tau}$. Finally, the exercise requires a measure for the risk-free rate r_t and one for the tenor-specific overnight special repo rate r_t^{τ}. One can compute both by using volume-weighted averages of GC rates and SC repo market rates, grouping bonds by their tenor, and use time fixed effects to soak up the adjustment in the market price of risk. The repo specialness l_t^{τ} can be inferred from the GC and SC rates. Next, the following simple panel linear regression model could test whether the proposed equilibrium TSM reasonably improves on the canonical specification (Vasicek, 1977; Brennan and Schwartz, 1979) by accounting for bond-specific short rates.

$$\frac{\hat{m}_t^{\tau}}{\hat{s}^{\tau}} = \text{Time FE} + \delta_1 \frac{r_t}{\hat{s}^{\tau}} + \delta_2 \frac{l_t^{\tau}}{\hat{s}^{\tau}} + \text{error term}$$

Our model suggests that δ_2 should be negative to prevent arbitrage opportunities. Intuitively, special bonds should have lower excess returns relative to general bonds, since the former generate additional cash flows on the repo market. We caution the reader that while this preliminary analysis may be useful, a formal test of the above requires more sophisticated specifications to account for the simultaneous determination of bond prices and repo specialness.

Moreover, Proposition 2 can be tested by regressing the term premium on the average specialness $\bar{l}_t$ after controlling for variables in Ξ_t:

$$y_{g,t}^{10} - y_{g,t}^{2} = \beta_0 + \beta_1 \Xi_t + \beta_2 \bar{l}_t + \text{error term}$$

Our model suggests that β_2 should be positive, since high levels of repo specialness reduce the short selling behavior of term structure arbitrageurs and their required compensation for risk. We leave to future research the task of carrying out a formal econometric test of these specifications. Clearly, the term premium should be estimated by dropping highly special securities from the pool of high-quality bonds used to fit the yield curve – a practice currently followed by the Fed but not by the ECB, even though the specialness of German bunds routinely reaches as much as 50 bps.

4 Calibration

4.1 Two Yield Curves

The calibration of our model is tantamount to the combined modeling of the general and special yield curves in the bond market and of the specialness in the repo market. The aim of this calibration is to highlight the effects of counterfactual scenarios determined by conventional monetary policy tools that guide short rate behavior and the use of unconventional instruments through QE, which act through demand pressure on the bond and repo markets.[27] We recast the model in discrete time and refer to well-established contributions in the literature on financial economics. We emphasize that the calibration serves the purpose of illustrating our findings qualitatively.

For comparability with VV, we set the maximum maturity to 30 years, and use publicly available 1985–2020 US Treasury data from Gürkaynak et al. (2007) (GSW). It is worth emphasizing that the latter data set excludes bonds targeted by exceptional demand pressure, thus fitting well with our purpose of calibrating the general yield curve. We express all rates on a per annum basis. We take a standard value for the long-run mean $\overline{r}$ from He and Milbradt (2014) and specify κ_r and σ_r to match the autocorrelation and standard deviation of the one-year yield, respectively. The market price of GC bond risk λ_r in this calibration is considered constant and equal to 0.42, replicating the average 10-year bond yield in the GSW data. To measure $\mathcal{E}_s$, we use the estimated impact of bond purchases on their returns conditional on other characteristics in D'Amico and King (2013). To model demand risk, we use a homogeneous level of excess demand θ for the special bond across tenors which reverts to zero at the speed φ.

We set θ to 26 bps to match the average *on-the-run* repo spread of 19.4 bps documented by D'Amico et al. (2018). This value approximates the GC repo/T-bill spread of 23.65 bps found by Nagel (2016), although it is more conservative, and far lower than the repo spread of around 40 bps observed in the German bund repo market. We tune the persistence parameter φ to the ratio between the average *on-the-run* repo spread to the average repo spread of *second-off-the-run* and older bonds on special of 4.88 bps in D'Amico et al. (2018). Thus, the half-life of θ is six months. To illustrate local supply effects in our model, θ_{10} reproduces the 10-year special bond price residual from the GSW model estimates in D'Amico et al. (2018). We explain these choices in detail in Table I.

As shown in Figure 2, our model features several salient characteristics. First, as the top panel shows, two yield curves – general and special – co-exist simultaneously. For each tenor, the yield to maturity of the special bond exposed to demand pressure is lower (i.e., its price is higher) than that of the general bond. Thus, the yield curve constructed by interpolating the prices of SC zero coupon bonds lies below the yield curve of GC bonds, but their difference shrinks with time to maturity as demand pressure shocks decline over time. That is intuitive,

[27] In affine TSMs, the persistence parameters define the curvature of the yield curve, and the relative importance of shocks is more pronounced at shorter maturities, as current realizations of stationary risk factors are relatively more informative for the near future.

given that the two curves are generated by rolling over GC and SC rate risk and that SC repo rates are generally below GC ones. In fact, the vertical distance between the GC and the SC curve at short residual maturities reflects the elasticity of the repo market supply of SC $\mathcal{E}_s$ and the persistence φ at the longer end of the yield curve. The gradually decreasing pattern of bond specialness recalls the spread between *on-the-run* and GSW-fitted yields documented in Figure 1 in Greenwood et al. (2015). Second, the joint modeling of the GC and SC yield curves in the bond market is only possible in the context of our theory, because we account for differentials in the special repo rates induced by these bonds. In the bottom panel of Figure 2, we show the repo rate for GCs in red, which is assumed to be constant across time to maturity, and for SC transactions in blue. That is, the SC rate captures the average special repo rates across all the transactions of special bonds with that maturity. The SC rate is $\mathcal{E}_s\theta$, defining the specialness as lower than the GC rate, except for the most special 10-year bond, since we use the variable θ_{10} to illustrate local supply effects, as described in subsection 4.2. In subsection 4.3, we then relax the assumption of constant risk aversion of the arbitrageurs in the tradition of Vasicek and instead assume, as in VV, that the market price of risk and hence the term premium depend on the arbitrageurs' holdings.

4.2 Local Supply Effects

In Figure 2, exceptional demand pressure directed toward the 10-year maturity special bond, θ_{10}, is stronger. This targeted demand pressure may capture the structural intervention of central banks through policies such as QE. A central bank can be modeled as a buy-and-hold investor that exerts extraordinary purchasing pressure on the market for nearly riskless sovereign bonds with particular tenors.[28] Targeted net excess demand may also reflect institutional constraints on investors, the reopening of a Treasury auction, or short squeezes, as diverse positions may induce a spike in valuations in otherwise common value settings (Nyborg and Strebulaev, 2003). In the top panel of Figure 2, excess demand induces a proportional kink in the yield curve (as noted, among others, by Gürkaynak et al., 2007, in Figure 4). Thus, from a modeling perspective, the flexibility of our framework allows for nonmonotonicity and bridges the gap between equilibrium models of the term structure of interest rates and econometric interpolation techniques (in the spirit of Nelson and Siegel, 1987). The mirror image of the intervention by the central bank is represented in the bottom panel of Figure 2, where the cross-section of special repo rates reaches a trough for the 10-year tenor SC that is more aggressively targeted, illustrating the endogeneity of repo rates. Simply put, when some investors exert significant demand

[28]For instance, the Fed reports its Treasury portfolio holdings by tenor in its system open market account.

pressure that raises a bond's price and lowers its yield, arbitrageurs borrow the bond in the SC market to respond to the large demand for this bond created by preferred-habitat investors, thus increasing its repo specialness. Since SC cannot be replaced with similar bonds on the repo market, the net supply effects on both prices and special repo rates are strongly localized.

This calibration exercise generates several interesting and important policy implications. To cite just one, consider any two levels of exceptional demand for long- and short-term bonds, respectively, which both have the same effect on special repo rates. Then, the demand pressure at the short end of the yield curve has a larger effect on bond yields. The intuition is straightforward: If the decay of exceptional demand pressure is rapid, the bonds at the two maturities will be exposed to approximately the same repo dividend, although that dividend is of course discounted more heavily at the long end of the yield curve. Perhaps a more subtle remark is that policymakers can fine-tune the persistence of their asset purchases to be impactful for the yield of long-term bonds, while minimizing distortions on the repo market. Simply put, prices have a forward-looking outlook while special repo rates reflect the *existing* stock of collateral. As the rate of decay of exceptional demand pressure diminishes, the bond price immediately increases, thus reflecting expectations that its future specialness will decline. On the other hand, what matters for the degree of collateral specialness is the quantity of bonds available on the repo market at each point in time. Thus, by fixing the overall amount purchased of a bond and the effect of the purchase on its yield, predictable repeated reverse auctions smooth the distortions in the repo market across intervention dates when compared to a one-time operation. This is generally consonant with the practice of the major central banks, including the ECB, the Bank of Japan, and the Fed, over the past decade.[29] As noted, we expect the SLF policies to reduce repo specialness and allow the resulting kinks to be arbitraged away in the yield curve, affecting the localization of supply effects and inducing market forces to smooth them.

4.3 Bond Scarcity and the Term Premium

The second main result of our model is that repo specialness generated by QE in the repo market impairs the effect that QE has on the term premium of the yield curve. To illustrate the effects of QE on the slope of the yield curve, the received wisdom suggests that the market price of risk λ should depend on the asset purchases, as argued by VV. We show the resulting

[29] From the FAQ on the Public Sector Purchase Program available on the ECB website: "The need to preserve smooth market functioning calls for the necessary amount of purchases at yields below the Deposit Facility Rate [special bonds] to be distributed over time, rather than abruptly changing the sectors of the yield curve where asset purchases take place." Thus, the ECB distributes its bond purchases to smooth distortions, as we argue it should. We are not aware of other models of QE and the term structure that generate this striking pattern.

term structure in Figure 3. In order to model QE, it appears reasonable to consider an economy at the zero lower bound (ZLB), as shown in Panel A. We start from a level of the term premium of $y_g^{10} - y_g^2 = 0.39$ in the general term structure of interest rates ($i = g$), delivered by the parameters in Table I when the economy is at the ZLB. When the short interest rate is in the proximity of this zero bound, the central bank may influence the yield curve by purchasing assets. In Panel B, we show the effect of asset purchases on the yield curve in the Vayanos and Vila (2021) framework. To this end, we specify the values of arbitrageurs' risk aversion as $a = 4.5$ and a constant demand elasticity of habitat investors $\alpha = 6.21$, in line with the literature. In this scenario, the central bank reduces the term premium to 0.24, compressing it by around 38%. However, since collateral is infinitely available and $\mathcal{E}_s = 0$, asset purchases do not influence money markets. This lack of an effect on the short-term interest rate appears to be at odds with the consensus view in the empirical literature discussed in the stylized facts above. In our model, we thus incorporate the effects of asset purchases on money markets. In Panel C, we again set $\mathcal{E}_s$ at 0.68, as we have in previous calibrations to match the evidence in D'Amico and King (2013) on US markets, while keeping everything else fixed. Bonds directly purchased by the central bank become special and trade at lower yields. By inducing specialness, asset purchases become less effective in influencing the general term structure of interest rates, inducing a term premium of 0.27, which is lower than the baseline but substantially higher than the case with exogenous money market. *Ceteris paribus*, in counterfactual scenarios where collateral is scarce, asset purchases thus achieve a lower reduction of the term premium. In Panel D, we set $\mathcal{E}_s$ at 0.78 to sketch an illustration of EU markets, where bonds issued on tap reach comparatively higher levels of repo specialness. This calibration may help model the findings in Arrata et al. (2020); those authors estimate that purchasing 1% of a bond outstanding is associated with a decline in its repo rate of 0.78 bps. We kept the same values for other parameters to allow for a better comparison with the previous panels. QE induces even higher levels of specialness, as measured by the vertical distance between the term structure of general and special bonds. In Panel D, the term premium is 0.29 and QE achieves a reduction with respect to the baseline 0.39 value of around 25%. In general, limits to arbitrage arising from money markets may substantially impair the transmission of QE to the term premium, as demonstrated in Proposition 2. For clarity of exposition, we have muted local supply effects.

We draw two main lessons by endogenizing money markets in these calibrations. First, by inducing repo specialness, QE generates a vertical distance between the GC and the SC yield curves. Second, the resulting repo specialness dampens the effect of QE on the term premium, highlighting the dynamic interactions between bond scarcity, repo specialness, and the term structure of interest rates. The key mechanism that drives our results is that SC rates largely reflect bond scarcity, a limit to arbitrage that may substantially prevent fixed-income intermediaries from entering aggressive short positions over the long term. Substantial levels of special repo rates thus induce marginal agents to hold more conservative positions, reducing their exposure to interest rate changes and their need for greater compensation for bearing negative duration risk in the form of a term premium, given their short positions on bonds that are scarce. As an important insight, this counterfactual result highlights that dysfunctional money markets impair the transmission of unconventional monetary policy.

5 Conclusion

Empirical fixed-income market research in the last two decades has documented systematic patterns in the spread between general and special bonds that are difficult to explain in the context of uncertainty in short-rate dynamics. The existing literature lacks a coherent theory to reconcile this evidence with existing models of the term structure of interest rates. In the present study, we have proposed an endogenous explanation for special repo rates based on the short selling behavior of term structure arbitrageurs. We have done so by characterizing the equilibrium relation between bond prices and repo specialness across the entire term structure of interest rates. The preferred-habitat approach that we have used gives rise to equilibrium price differences between bonds with identical cash flows that are reflected in their respective repo spreads. Our derived equilibrium concept accounts for the collateral value of the bonds in the market for repurchase agreements, both general and special.

We draw three main lessons by endogenizing money markets into the modeling of the term structure. First, by inducing repo specialness, quantitative easing generates a vertical distance between the general collateral and special collateral yield curves. Second, by requiring the delivery of specific securities, SC money markets reconcile the quantitative discipline imposed by the absence of arbitrage opportunities with the presence of strongly localized supply effects of QE on the term structure of interest rates. Third, the repo specialness induced by QE also dampens the effect of QE on the term premium, highlighting the dynamic interactions between bond scarcity, repo specialness, and the term structure of interest rates. The key mechanism that drives this result is that SC rates largely reflect bond scarcity, which is a cost for arbitrageurs and therefore a limit to arbitrage that may substantially prevent fixed-income intermediaries from entering aggressive short positions over the long term. Thus, lower levels of special repo rates may induce marginal agents to hold more conservative positions, reducing their exposure to interest rate changes and the compensation they require for bearing duration risk in the form of a term premium.

In the present study, we have relaxed the standard assumption of the uniqueness of the instantaneous interest rate by proposing an endogenous market for the risk-free asset whose supply is elastic to quantities. (We have, however, abstracted from credit risk and market liquidity considerations, which may give rise to additional effects.) For ease of comparison with other techniques in the literature, our model was implemented without taking a particular stance on investor preferences. At the same time, we illustrate how our general formulation nests preference-based approaches as special cases.

The theory that we have presented has two especially attractive features. First, we have provided a unified framework that connects the secondary market for (nearly) risk-free bonds, such as US Treasury bonds, with the repo market for collateralized financing. Policymakers could use our model to assess the combined effects of exceptional demand pressure, such as QE or tapering, on the secondary market for government bonds and on the repo market for collateralized financing. Second, we have developed a generalized term structure equilibrium concept that accounts for the collateral value of bonds. Our framework is attractive for applied researchers, who may exploit exogenous shocks in both the bond and repo markets rather than considering these markets in isolation. Third, we have characterized the many dynamic and multifaceted connections between bond scarcity, repo specialness, and the term structure of interest rates.

We have derived our results in closed form so as to perform comparative statics experiments and derive testable predictions and illustrated them through calibrations on bond and money markets. We have then proposed three simple extensions of our model to consider regular US Treasury auctions that account for cyclicality in specialness, enabling us to derive the equilibrium effects of heterogeneous arbitrageurs through haircuts and borrowing constraints and to examine the equilibrium effects of substitutability between bonds in the demand of preferred-habitat investors.

The present study has discussed the demand pressure for special issues that have the same cash flows as benchmark securities; applications could focus on Green or Islamic bond premia. The structure we derive suggests that by estimating the yield curve on both general and special bonds together, a common practice, may result in a distorted fit that no longer sends reliable signals about impending economic conditions. Future research could generalize the method that we have proposed to multifactor or quadratic term structure models from the theory side and test its predictions empirically. Overall, the paper shows that dysfunctional money markets, in the context of the large expansion in the role of their collateral-driven segment, can substantially impair the transmission of unconventional monetary policy across the yield curve. Hence, these effects on both the bond and repo markets should be considered jointly in the conduct of monetary policy.

A Proof of Lemma 1

By substituting Equation (7) into the affine representation in Equation (15), we obtain the price of general and special bonds, since $X^{\tau}_{g,t} = 0$ for general bonds whose status is $i = g$:

$$P^{\tau}_{g,t} = \mathbb{E}^{\mathbb{Q}}_t\big[e^{-\int_0^{\tau} r_{t+u}du}\big] = e^{(-A_\tau r_t - C_\tau)},$$
$$P^{\tau}_{s,t} = \mathbb{E}^{\mathbb{Q}}_t\big[e^{-\int_0^{\tau} r^{\tau-u}_{t+u}du}\big] = e^{(-A_\tau r_t - B_\tau X^{\tau}_{s,t} - C_\tau)}.$$

The result follows by taking the price ratio of the general bond $P^{\tau}_{g,t}$ to the special bond $P^{\tau}_{s,t}$, since the repo market equilibrium requires $r^{\tau}_t = r_t + \mathcal{E}_i X^{\tau}_{i,t}$. *Q.E.D.*

B Proof of Proposition 1

The optimality condition for arbitrageurs on the bond market is

$$\mu^{\tau}_{i,t} - r^{\tau}_t = a_{i,\tau}\gamma\sigma_r^2 \int_0^{\infty} a_{g,\tau} X^{\tau}_{g,t} + a_{s,\tau} X^{\tau}_{s,t} d\tau. \tag{17}$$

Substitute $\mu^{\tau}_{i,t}$ and r^{τ}_t to derive the following relationship.

$$\begin{aligned}&\dot{a}_{i,\tau} r_t + a_{i,\tau}\kappa_r(r_t - \overline{r}) + \frac{1}{2}a^2_{i,\tau}\sigma^2_r + \dot{b}_{i,\tau}\theta_\tau + b_{i,\tau}\dot{\theta}_\tau + \dot{c}_{i,\tau} - r_t + l^{\tau}_t \\ &= a_{i,\tau}\gamma\sigma_r^2 \int_0^{\infty} a_{g,\tau} X^{\tau}_{g,t} + a_{s,\tau} X^{\tau}_{s,t} d\tau.\end{aligned}$$

From the repo market equilibrium without the SLF, we have $l^{\tau}_t = \mathcal{E}_s Z^{\tau}_{s,t}$. From the bond market equilibrium, we have $X^{\tau}_{g,t} = 0$ and $X^{\tau}_{s,t} = -Z^{\tau}_{s,t} = \theta_\tau - \alpha_\tau\big(a_\tau r_t + b_\tau\theta_\tau + c_\tau\big)$. Therefore,

$$\begin{aligned}&\dot{a}_{i,\tau} r_t + a_{i,\tau}\kappa_r(r_t - \overline{r}) + \frac{1}{2}a^2_{i,\tau}\sigma^2_r + \dot{b}_{i,\tau}\theta_\tau + b_{i,\tau}\dot{\theta}_\tau + \dot{c}_{i,\tau} - r_t - \mathcal{E}_i\big[\theta_\tau - \alpha_\tau\big(a_{i,\tau} r_t + b_{i,\tau}\theta_\tau + c_{i,\tau}\big)\big] \\ &= a_{i,\tau}\gamma\sigma_r^2 \int_0^{\infty} a_\tau\big[\theta_\tau - \alpha_\tau\big(a_{s,\tau} r_t + b_{s,\tau}\theta_\tau + c_{s,\tau}\big)\big] d\tau.\end{aligned} \tag{18}$$

Equation (18) must hold for all values of r_t. By matching the coefficients in the risk factors, we derive the following ordinary differential equation (ODE).

$$\dot{a}_{i,\tau} + a_{i,\tau}\big(\kappa_r + \alpha_\tau \mathcal{E}_i\big) - 1 = -a_{i,\tau}\gamma\sigma_r^2 \int_0^\infty \alpha_\tau a_{s,\tau}^2 d\tau. \tag{19}$$

Moreover, the terms that are independent of the risk factors must add up to zero, implying

$$\begin{aligned}\dot{c}_{i,\tau} &= a_{i,\tau}\kappa_r\overline{r} - \frac{1}{2}a_{i,\tau}^2\sigma_r^2 - \dot{b}_{i,\tau}\theta_\tau - b_{i,\tau}\dot{\theta}_\tau + \mathcal{E}_i(\theta_\tau - \alpha_\tau(b_{i,\tau} + c_{i,\tau})) \\ &\quad + a_{i,\tau}\gamma\sigma_r^2 \int_0^\infty a_{s,\tau}\big[\theta_\tau - \alpha_\tau(b_{s,\tau}\theta_\tau + c_{s,\tau})\big] d\tau.\end{aligned}$$

We further require that the optimality holds for both general and special bonds. Lemma 1 implies $b_{g,\tau} = 0$. We obtain

$$\begin{aligned}\dot{b}_{s,\tau} + b_{s,\tau}\overline{\theta}_\tau &= \mathcal{E}_s\left(1 - \frac{\alpha_\tau c_\tau}{\theta_\tau}\right) \\ \dot{c}_{i,\tau} &= a_{i,\tau}\kappa_r\overline{r} - \frac{1}{2}a_{i,\tau}^2\sigma_r^2 + a_{i,\tau}\gamma\sigma_r^2 \int_0^\infty a_{s,\tau}\big[\theta_\tau - \alpha_\tau(b_{s,\tau}\theta_\tau + c_{s,\tau})\big] d\tau,\end{aligned}$$

where we have defined $\overline{\theta}_\tau = \dot{\theta}_\tau(1 + \alpha_\tau\mathcal{E}_s)/\theta_\tau$. We thus have a system of three ODEs. Since at maturity a bond repays the principal notional, we have $A_0 = B_0 = C_0 = 0$, implying that the system must be solved with initial conditions $a_{i,0} = b_{i,0} = c_{i,0} = 0$. We follow Vayanos and Vila (2021) and proceed in two steps. First, we take the integrals as given and solve the above equations as linear ODEs. Second, we require that the solution is consistent with the value of the integrals. We obtain

$$\begin{aligned}a_{i,\tau} &= \frac{1 - e^{-\kappa_r^*\tau}}{\kappa_r^*} \\ b_{i,\tau} &= \frac{\mathcal{E}_i(1 - g_\tau)(1 - e^{-\int \overline{\theta}_\tau d\tau})}{\overline{\theta}_\tau} \\ c_{i,\tau} &= \kappa_r^*\overline{r}^* \int_0^\infty a_{i,u} du - \frac{\sigma_r^2}{2} \int_0^\infty a_{i,u} du\end{aligned}$$

where the scalars $(\kappa_r^*, \overline{r}^*)$ are defined by

$$\kappa_r^* = \kappa_r + \alpha_\tau \mathcal{E}_i + a_{i,\tau}\gamma\sigma_r^2 \int_0^\infty \alpha_\tau a_{s,\tau}^2 d\tau$$
$$\kappa_r^* \overline{r}^* = \kappa_r^* + a_{i,\tau}\gamma\sigma_r^2 \int_0^\infty a_{s,\tau}\big[\theta_\tau - \alpha_\tau(b_{s,\tau}\theta_\tau + c_{s,\tau})\big] d\tau$$

and the function g_τ is the solution of

$$g_\tau = e^{-\int \overline{\theta}_\tau d\tau} \left(\int_0^\infty \frac{\alpha_\tau c_{i,\tau} e^{\int \overline{\theta}_\tau d\tau}}{\theta_\tau} \right) d\tau$$

Proposition 1 achieves as a particular case when repo specialness $l_t^\tau = 0$. *Q.E.D.*

References

Altavilla, C., G. Carboni, and R. Motto. 2021. Asset purchase programmes and financial markets: lessons from the euro area. *International Journal of Central Banking* 17:1–48.

Arrata, W., B. Nguyen, I. Rahmouni-Rousseau, and M. Vari. 2020. The scarcity effect of QE on repo rates: Evidence from the euro area. *Journal of Financial Economics* 137:837–856.

Ballensiefen, B., A. Ranaldo, and H. Winterberg. 2023. Money Market Disconnect. *The Review of Financial Studies* forthcoming.

Banerjee, S., and J. J. Graveline. 2013. The cost of short-selling liquid securities. *The Journal of Finance* 68:637–664.

Barclay, M. J., T. Hendershott, and K. Kotz. 2006. Automation versus intermediation: Evidence from Treasuries going off the run. *The Journal of Finance* 61:2395–2414.

Berardi, A., R. Brown, and S. Schaefer. 2021. Bond risk premia: The information in (really) long term rates. Working Paper.

Bernanke, B. 2020. The new tools of monetary policy. *American Economic Review* 110:943–83.

Brennan, M., and E. Schwartz. 1979. A continuous time approach to the pricing of bonds. *Journal of Banking & Finance* 3:133–155.

Buraschi, A., and D. Menini. 2002. Liquidity risk and specialness. *Journal of Financial Economics* 64:243–284.

Chen, M., J. Cherian, Z. Li, Y. Shao, and M. G. Subrahmanyam. 2022. Clientele effect in sovereign bonds: Evidence from Islamic Sukuk bonds in Malaysia. *Critical Finance Review* 11:677–745.

Cherian, J., E. Jacquier, and R. Jarrow. 2004. A model of the convenience yields in on-the-run treasuries. *Review of Derivatives Research* 7:79–97.

Christensen, J. H., and G. D. Rudebusch. 2012. The response of interest rates to US and UK quantitative easing. *The Economic Journal* 122:F385–F414.

Copeland, A., A. Martin, and M. Walker. 2014. Repo runs: Evidence from the tri-party repo market. *The Journal of Finance* 69:2343–2380.

Cornell, B., and A. Shapiro. 1989. The mispricing of US Treasury bonds: A case study. *The Review of Financial Studies* 2:297–310.

Corradin, S., and A. Maddaloni. 2020. The importance of being special: Repo markets during the crisis. *Journal of Financial Economics* 137:392–429.

Costain, J., G. Nuño, and C. Thomas. 2022. The term structure of interest rates in a heterogeneous monetary union. Working Paper.

Culbertson, J. 1957. The term structure of interest rates. *The Quarterly Journal of Economics* 71:485–517.

D'Amico, S., W. English, D. López-Salido, and E. Nelson. 2012. The Federal Reserve's large-scale asset purchase programmes: Rationale and effects. *The Economic Journal* 122:F415–F446.

D'Amico, S., R. Fan, and Y. Kitsul. 2018. The scarcity value of treasury collateral: Repo-market effects of security-specific supply and demand factors. *Journal of Financial and Quantitative Analysis* 53:2103–2129.

D'Amico, S., and T. King. 2013. Flow and stock effects of large-scale treasury purchases: Evidence on the importance of local supply. *Journal of Financial Economics* 108:425–448.

D'Amico, S., J. Klausmann, and N. A. Pancost. 2022. The benchmark greenium. Working Paper.

D'Amico, S., and N. Pancost. 2022. Special repo rates and the cross-section of bond prices: The role of the special collateral risk premium. *Review of Finance* 26:117–162.

Du, W., B. M. Hébert, and W. Li. 2022. Intermediary balance sheets and the treasury yield curve. Working Paper.

Du, W., A. Tepper, and A. Verdelhan. 2018. Deviations from covered interest rate parity. *The Journal of Finance* 73:915–957.

Duffie, D. 1996. Special repo rates. *The Journal of Finance* 51:493–526.

Duffie, D., N. Gârleanu, and L. Pedersen. 2005. Over-the-counter markets. *Econometrica* 73:1815–1847.

Duffie, D., and R. Kan. 1996. A yield-factor model of interest rates. *Mathematical Finance* 6:379–406.

Eisenschmidt, J., Y. Ma, and A. L. Zhang. 2024. Monetary policy transmission in segmented markets. *Journal of Financial Economics* 151:103738.

Fisher, M. 2002. Special repo rates: An introduction. *Economic Review-Federal Reserve Bank of Atlanta* 87:27–44.

Fleckenstein, M., and F. Longstaff. 2020. Renting balance sheet space: Intermediary balance sheet rental costs and the valuation of derivatives. *The Review of Financial Studies* 33:5051–5091.

Fleckenstein, M., and F. A. Longstaff. 2021. Treasury richness. Working Paper.

Fontaine, J.-S., and R. Garcia. 2012. Bond liquidity premia. *The Review of Financial Studies* 25:1207–1254.

Gabaix, X., A. Krishnamurthy, and O. Vigneron. 2007. Limits of arbitrage: Theory and evidence from the mortgage-backed securities market. *The Journal of Finance* 62:557–595.

Gagnon, J., M. Raskin, J. Remache, and B. Sack. 2018. The financial market effects of the Federal Reserve's large-scale asset purchases. *International Journal of Central Banking* 7:45–52.

Gârleanu, N., S. Panageas, and G. Zheng. 2021. A long and a short leg make for a wobbly equilibrium. Working Paper.

Gourinchas, P.-O., W. Ray, and D. Vayanos. 2022. A preferred-habitat model of term premia, exchange rates, and monetary policy spillovers. Working Paper.

Grasselli, M., and C. Tebaldi. 2008. Solvable affine term structure models. *Mathematical Finance: An International Journal of Mathematics, Statistics and Financial Economics* 18:135–153.

Graveline, J., and M. McBrady. 2011. Who makes on-the-run Treasuries special? *Journal of Financial Intermediation* 20:620–632.

Greenwood, R., S. Hanson, and J. Stein. 2015. A comparative-advantage approach to government debt maturity. *The Journal of Finance* 70:1683–1722.

Greenwood, R., S. Hanson, J. Stein, and A. Sunderam. 2023. A quantity-driven theory of term premia and exchange rates. *Quarterly Journal of Economics* forthcoming.

Greenwood, R., and D. Vayanos. 2010. Price pressure in the government bond market. *American Economic Review* 100:585–90.

Greenwood, R., and D. Vayanos. 2014. Bond supply and excess bond returns. *The Review of Financial Studies* 27:663–713.

Gürkaynak, R., B. Sack, and J. Wright. 2007. The US Treasury yield curve: 1961 to the present. *Journal of Monetary Economics* 54:2291–2304.

Hanson, S., A. Malkhozov, and G. Venter. 2022. Demand–supply imbalance risk and long-term swap spreads. Working Paper.

He, Z., and K. Milbradt. 2014. Endogenous liquidity and defaultable bonds. *Econometrica* 82:1443–1508.

He, Z., S. Nagel, and Z. Song. 2022. Treasury inconvenience yields during the covid-19 crisis. *Journal of Financial Economics* 143:57–79.

Hu, G., J. Pan, and J. Wang. 2013. Noise as information for illiquidity. *The Journal of Finance* 68:2341–2382.

Jankowitsch, R., A. Nashikkar, and M. Subrahmanyam. 2011. Price dispersion in OTC markets: A new measure of liquidity. *Journal of Banking & Finance* 35:343–357.

Jermann, U. 2020. Negative swap spreads and limited arbitrage. *The Review of Financial Studies* 33:212–238.

Jordan, B., and S. Jordan. 1997. Special repo rates: An empirical analysis. *The Journal of Finance* 52:2051–2072.

Kiyotaki, N., and J. Moore. 1997. Credit cycles. *Journal of Political Economy* 105:211–248.

Koijen, R. S., F. Koulischer, B. Nguyen, and M. Yogo. 2021. Inspecting the mechanism of quantitative easing in the euro area. *Journal of Financial Economics* 140:1–20.

Krishnamurthy, A. 2002. The bond/old-bond spread. *Journal of Financial Economics* 66:463–506.

Krishnamurthy, A., and A. Vissing-Jorgensen. 2012. The aggregate demand for treasury debt. *Journal of Political Economy* 120:233–267.

Maddaloni, A., and H. Roh. 2021. Not equally special: Collateralised trading of non-banks. Working Paper.

Mancini, L., A. Ranaldo, and J. Wrampelmeyer. 2016. The euro interbank repo market. *The Review of Financial Studies* 29:1747–1779.

Merrick Jr, J., N. Naik, and P. Yadav. 2005. Strategic trading behavior and price distortion in a manipulated market: anatomy of a squeeze. *Journal of Financial Economics* 77:171–218.

Modigliani, F., and R. Sutch. 1966. Innovations in interest rate policy. *The American Economic Review* 56:178–197.

Nagel, S. 2016. The liquidity premium of near-money assets. *The Quarterly Journal of Economics* 131:1927–1971.

Nelson, C., and A. Siegel. 1987. Parsimonious modeling of yield curves. *Journal of Business* 60:473–489.

Nguyen, B., D. Tomio, and M. Vari. 2023. Safe Asset Scarcity and Monetary Policy Transmission. Working Paper.

Nyborg, K. G., and I. A. Strebulaev. 2003. Multiple unit auctions and short squeezes. *The Review of Financial Studies* 17:545–580.

Pasquariello, P., and C. Vega. 2009. The on-the-run liquidity phenomenon. *Journal of Financial Economics* 92:1–24.

Pelizzon, L., M. Subrahmanyam, and D. Tomio. 2022. Central bank-driven mispricing. Working Paper.

Tuckman, B., and J.-L. Vila. 1992. Arbitrage with holding costs: A utility-based approach. *The Journal of Finance* 47:1283–1302.

Van Binsbergen, J., W. Diamond, and M. Grotteria. 2022. Risk-free interest rates. *Journal of Financial Economics* 143:1–29.

Vasicek, O. 1977. An equilibrium characterization of the term structure. *Journal of Financial Economics* 5:177–188.

Vayanos, D., and J.-L. Vila. 2021. A preferred-habitat model of the term structure of interest rates. *Econometrica* 89:77–112.

Vayanos, D., and P.-O. Weill. 2008. A search-based theory of the on-the-run phenomenon. *The Journal of Finance* 63:1361–1398.

TABLE I: **Calibration**

Yield Curve Calibration on 1985–2020 Data			
Parameter	Value	Source	Data and Moment
$\overline{r}$ Long-run mean of r_t	0.0200	He and Milbradt (2014)	Table I Risk-free rate, long-run mean
κ_r Persistence of r_t	0.9	Gürkaynak et al. (2007) data	Autocorrelation of 1-year yields Equal to 0.9
σ_r Standard deviation of r_t	0.0115	Gürkaynak et al. (2007) data	Volatility of 1-year yields Equal to 2.63
λ_r Market price of GC risk	0.42	Gürkaynak et al. (2007) data	Average of 10-year yields Equal to 0.0517
Demand Pressure			
Parameter	Value	Source	Data and Moment
$\mathcal{E}_s$ Slope of special collateral supply	0.68	D'Amico and King (2013)	Table VII Purchases conditional impact on returns
θ Level of excess demand for the Special bonds	0.0026	D'Amico et al. (2018)	Table I Average general/special Repo spread equal to 19.4 bps
θ_{10} Level of excess demand for the 10-years tenor special bond	0.0100	D'Amico et al. (2018)	Table I Average price residual of 10-year Special bonds equal to 53 bps of par
φ Persistence of demand	0.25	D'Amico et al. (2018)	Table I Average new to old special bonds Repo spread ratio equal to 0.25

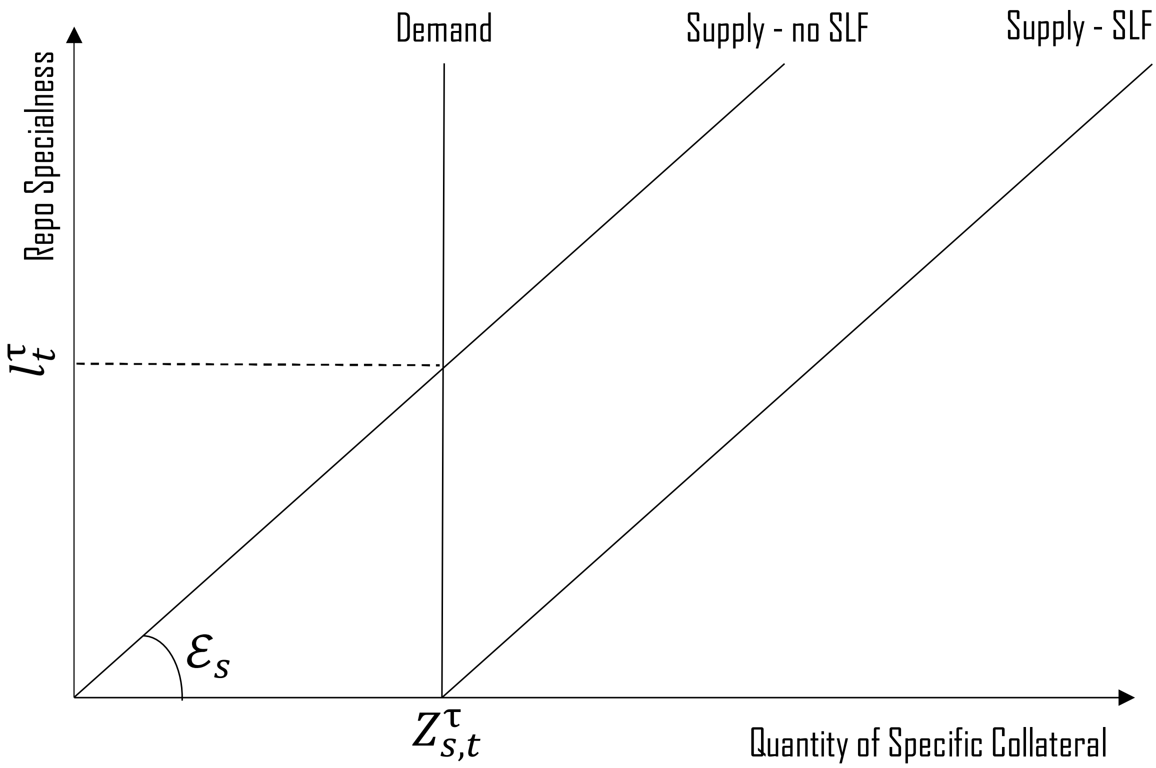

FIGURE 1: **Demand and supply of special collateral.** This figure illustrates the functioning of the market for repurchase agreements collateralized by special bonds. The horizontal axis shows demand pressure on the cash market, and the vertical axis shows repo specialness. The demand curve is inelastic because of the commitment of short sellers to deliver the specific issue. The supply, by contrast, is elastic, as holders of special collateral bonds require greater compensation to pledge additional units of the security on the repo market. The securities lending facility corresponds to a rightward shift of the supply of collateral.

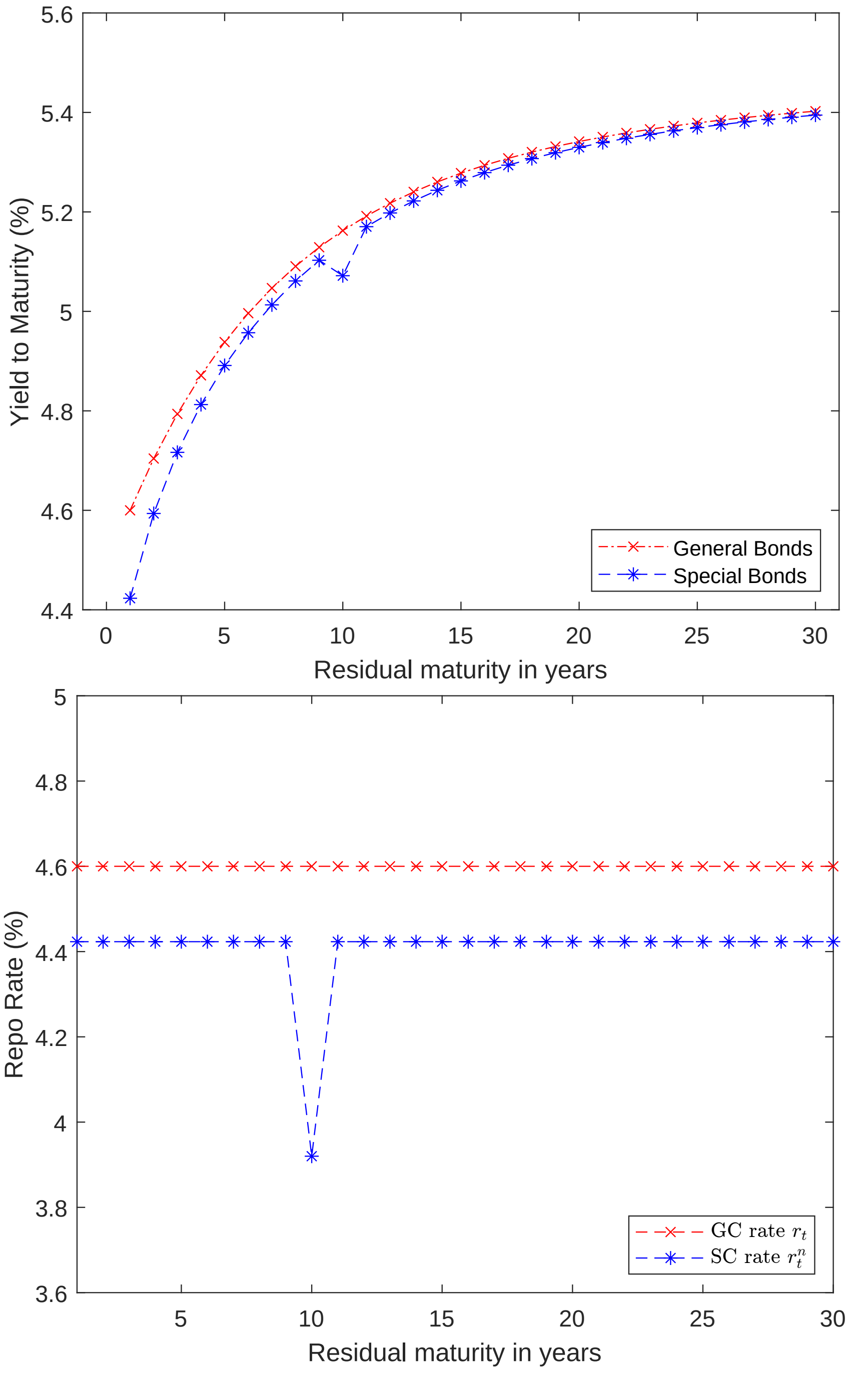

FIGURE 2: **Yield curves and repo rates.** The top panel shows the term structure of interest rates. The bottom panel shows general and special overnight repo rates plotted against collateral tenor. Rates are expressed on a per annum basis. The curves in red show the general bonds, which are not exposed to demand pressure. The curves in blue show special bonds, which are targeted by exceptional demand pressure. Table I presents the calibration.

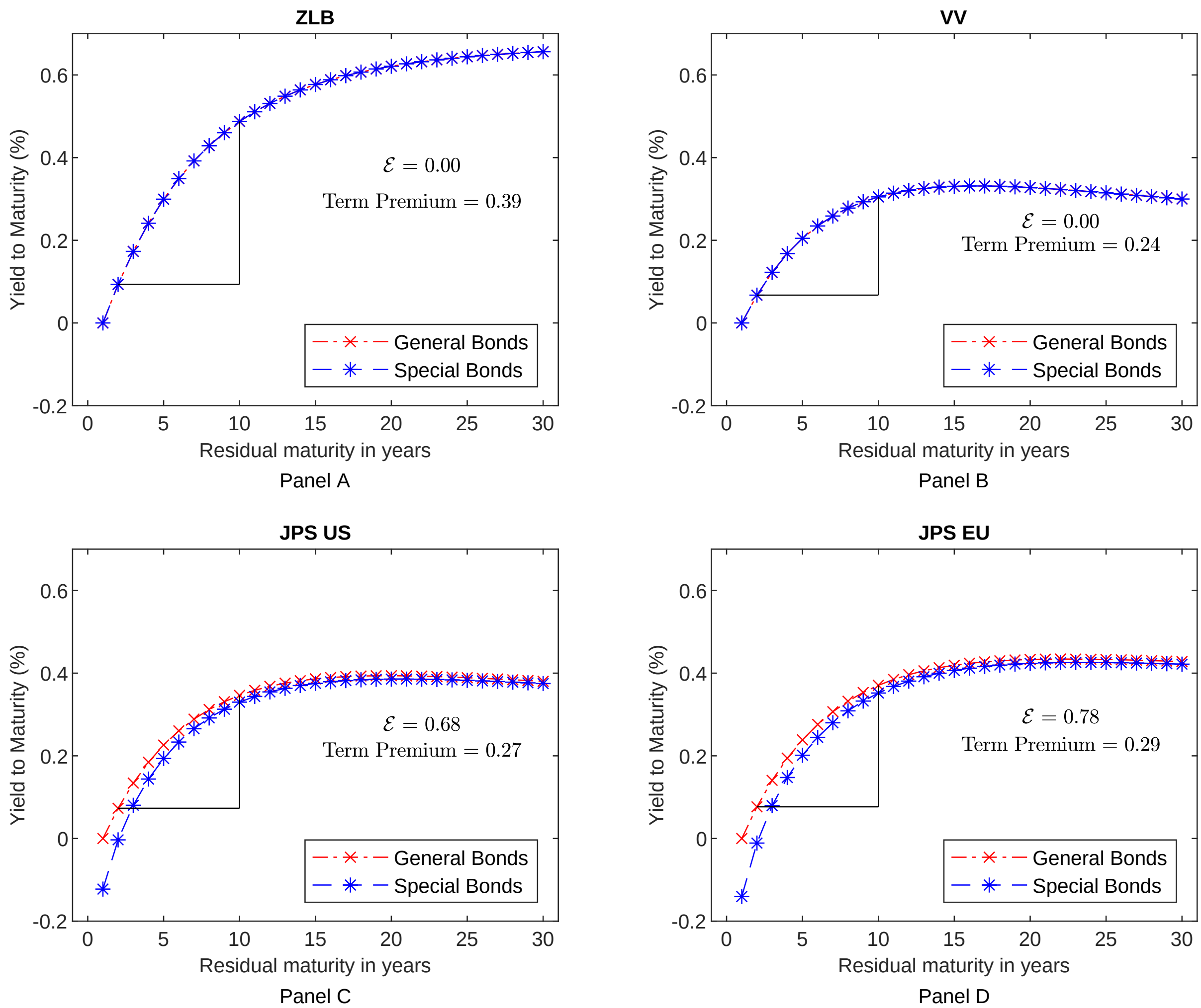

FIGURE 3: **Bond Scarcity and the Term Premium.** Panel A of the figure shows the term structure of interest rates at the zero lower bound (ZLB). Panel B shows the effect of quantitative easing (QE) in the Vayanos and Vila (VV) model. Panels C and D present our calibrations (JPS), where we relax the assumption that the collateral is in infinite supply, *ceteris paribus*. Panel C presents the calibration of our model to the US market, while Panel D shows its calibration to the EU market. Rates are expressed on a per annum basis. The curves in red show the general bonds, which are not exposed to demand pressure. The curves in blue show the special bonds differently targeted by exceptional demand pressure. Section 4.3 presents the calibration.

Internet Appendix

“Quantitative Easing, the Repo Market, and the Term Structure of Interest Rates”

Ruggero Jappelli, Loriana Pelizzon, and Marti G. Subrahmanyam.

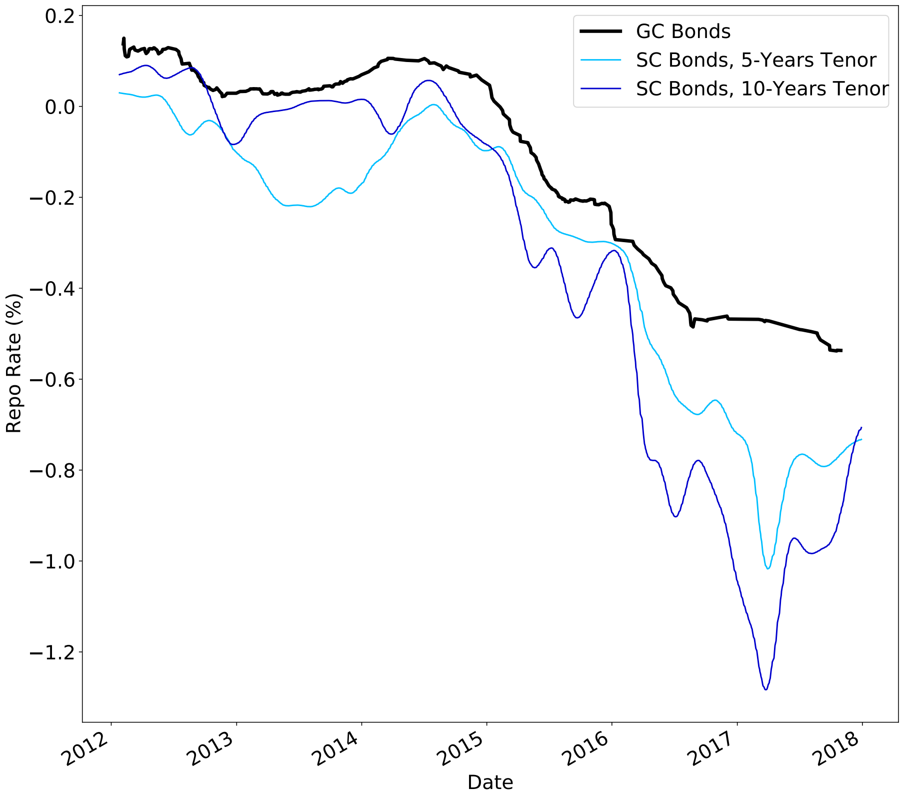

FIGURE OA.1: **General and Special Repo Rates for German Treasury Bonds**. This figure shows the volume-weighted monthly trailing average of the daily rates on tick-by-tick repo transactions collateralized by German treasury bonds, as recorded by MTS markets from 2012 to 2018. Daily rates are the volume-weighted average of intraday rates. Each trading day, repo transactions for 22 trading days are averaged. We distinguish between general collateral (GC) and special collateral (SC) transactions; the latter are shown for benchmark time-to-maturity buckets.

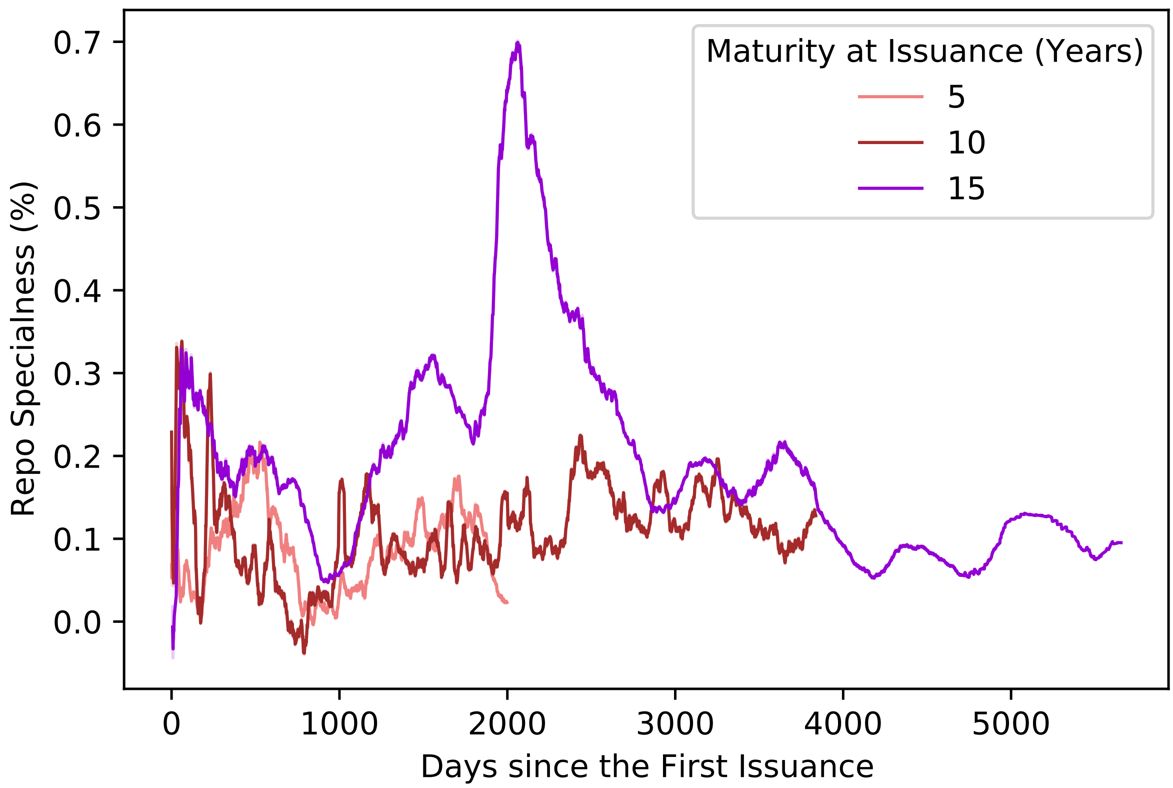

FIGURE OA.2: **Repo Specialness of Italian Treasury Bonds**. This figure shows the volume-weighted six-month trailing average of the daily rates on tick-by-tick repo transactions collateralized by Italian treasury bonds (most notably Buoni del Tesoro Poliennali, Buoni Ordinari del Tesoro, and Certificati di Credito del Tesoro), as recorded by MTS markets from 2012 to 2018. Daily rates are the volume-weighted average of intraday rates. Each day, repo transactions for 365 days are averaged. We distinguish between three benchmark bond maturities at issuance.