ECB FORUM ON CENTRAL BANKING

27–29 June 2022

Challenges for monetary policy in a rapidly changing world

Global supply chain pressures, international trade and inflation

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EUROSYSTEM

Global Supply Chain Pressures, International Trade, and Inflation

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June 2022

Abstract

We study the effects of the Covid-19 pandemic on the Euro Area inflation in comparison to other countries such as the United States over the two-year period 2020-21. Our model-based calibration exercises deliver four key results: 1) Compositional effects – the switch from services to goods consumption – are amplified through global input-output linkages, affecting both trade and inflation. 2) International trade did not respond to changes in GDP as strongly as it did during the 2008-09 crisis despite strong demand for goods. These lower trade elasticities in part reflect supply chain bottlenecks. 3) Inflation can be higher under sector-specific labor shortages relative to a scenario with no such supply shocks. 4) Foreign shocks and global supply chain bottlenecks played an outsized role relative to domestic aggregate demand shocks in explaining Euro Area inflation over 2020-21. These four results imply that policies aimed at stimulating aggregate demand would not have produced as high an inflation as the one observed in the data without the negative sectoral supply shocks.

1 Introduction

Covid-19 has been a historically unique shock to the global economy. While all countries have been impacted by the virus, the effects have not been synchronized across borders like the 2008-09 global financial crisis. The Covid-19 shock impacted economies' sectors heterogeneously and countries were affected repeatedly at different points in time. These uneven impacts were due to several factors. First, responses to the pandemic by governments differed, both in terms of health policies and the degree of monetary and fiscal stimuli. Second, Covid-19 encompassed a combination of sectoral demand and supply shocks, which propagated within and across countries via input-output networks, creating demand-

[1] The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Banks of New York or any other person affiliated with the Federal Reserve System.

supply imbalances on a global scale, resulting in the so-called supply chain bottleneck problem.[2]

This pandemic cycle has captured the world economy since early 2020. In the initial "lockdown phase" of the pandemic, there was a large decline in global economic activity with trade and GDP both collapsing. The roll-out of vaccines combined with unprecedented monetary and fiscal stimulus since late 2020/early 2021 has led to a fast but asymmetric recovery across countries and sectors.[3] The service sectors that rely on face-to-face interactions were slow to recover given the repeated nature of the health shock from different variants, on-and-off lockdowns and labor shortages, whereas manufacturing sectors, especially durables, rebounded quickly as consumption was tilted to these sectors, driving the quick recovery in global trade – for example, global demand skewed towards stationary-bicycles when gyms remained closed.

The goal of this paper is to quantify how the distinct aspects of the Covid-19 shock have impacted both the flow of goods across countries as well as their prices. To accomplish this goal, we provide three distinct model-based quantitative exercises. First, we examine how observed cross-country and cross-sectoral consumption changes spilled-over across countries via the global production network, thereby rationalizing observed trade flows. We follow the methodology of Bems, Johnson, and Yi (2010) and examine how the Covid-19 crisis differed from the 2008-09 global financial crisis. Second, we build on the theoretical work of Baqaee and Farhi (2022) and Guerrieri, Lorenzoni, Straub, and Werning (2022) to quantify the effects of the pandemic on inflation over the period spanning both the collapse and recovery phases of the economy. This framework not only allows us to examine the cumulative impact of the pandemic from 2019Q4 to 2021Q4 on inflation, but also decomposes the contribution of demand- and supply-side factors underlying the observed inflation. Third, we extend the Baqaee and Farhi (2022) approach to a multi-country framework to capture the importance of international spillovers in generating inflation.

The evolution of the pandemic and inflation. The early phase of the pandemic witnessed a negative supply shock, creating the initial supply chain disruption, and combined with uncertainty, created insufficient demand, ending with a large collapse in GDP. During the recovery phase, pent-up demand created further pressure on supply chains, leading to inflation. The initial phase of the pandemic can be thought of as a series of sectoral negative supply and demand shocks in lockdown/contact

[2] The ECB Blog by Philip Lane, February 2022. Accessed: https://www.ecb.europa.eu/press/blog/date/2022/html/ecb.blog220210~1590dd90d6.en.html

[3] See Gourinchas, Kalemli-Özcan, Penciakova, and Sander (2021) on uneven recovery across emerging markets and advanced economies due to co-existence of demand and supply constrained sectors and inequality in fiscal space across countries. See Çakmaklı, Demiralp, Kalemli-Ozcan, Yesiltas, and Yildirim (2022), who developed a model that predicts country-sector asymmetry in recovery under unequal global vaccinations as sectoral supply shocks in unvaccinated countries travel through global production network, affecting vaccinated countries.

intensive sectors and a positive sectoral demand shock in others (e.g., online deliveries versus restaurants). The recovery phase involved a positive aggregate demand shock, in part due to stimulative government policies. Note that during this recovery phase, the negative sectoral supply shocks were still in place as the pandemic were in place globally.[4] The compositional shifts in demand between services and goods sectors, combined with supply-constrained sectors are important to consider in our analysis.

In our framework, like Baqaee and Farhi (2022), domestic inflation is driven by aggregate demand shocks as well as sectoral demand and labor supply shocks. We therefore argue that while some economists warned of looming inflation (e.g., Summers, February 2021), few anticipated the prolonged and drastic shift in spending from services to goods and the effects of such a shift under an economy where labor shortages in certain sectors led economy-wide supply constraints that are persistent. Indeed, in the US, as early as March 2021, the FOMC expected CPI inflation to be 2.4 percent in 2021. While in Europe, as of the October 2021 meeting of the ECB, inflation was not even a concern.[5]

Our analysis lays bare how the inherent pandemic-driven labor dislocations were bound to show up as inflation when combined with aggregate demand stimulus. While the increase in consumer spending barely brought economies back to pre-pandemic levels, this rebound in economic activity coincided with supply chains problems that were slow to dissipate. This mismatch in demand and supply led to inflation being less transitory, broad based and higher than 2021Q1 expectations. For example, Figure 1 below shows that even though employment and real GDP surged during recovery, employment (purple dashed line) was still substantially below its pre-pandemic level in 2020Q4 when inflation (pink solid line) started increasing in the US and Europe, indicating the potential importance of supply constraints and limits to the production capacity of the economy in driving inflation.[6]

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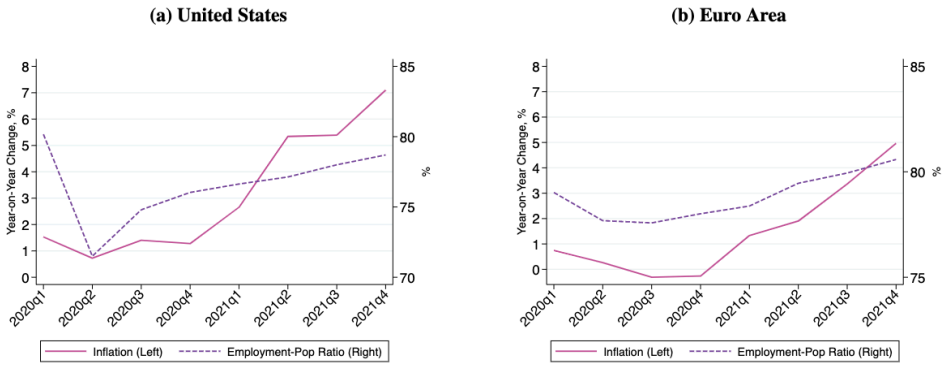
[4] We do not formally model the differential impact of health-related labor shortages, the so-called "Great Resignation," and a slow return of labor given search-match frictions, but instead treat them all as sectoral labor supply shortages/constraints. Çakmaklı, Demiralp, Kalemli-Ozcan, Yesiltas, and Yildirim (2022) open economy global model justifies never disappearing negative sectoral supply shocks under the pandemic.

[5] https://www.federalreserve.gov/newsevents/pressreleases/monetary20210317a.htm
https://www.ecb.europa.eu/press/pr/date/2021/html/ecb.mp211028~85474438a4.en.html

[6] See Krugman's New York Times article for a similar argument (https://www.nytimes.com/2022/01/21/opinion/inflation-us-economy-biden.html). Relatedly, Gourinchas, Kalemli-Ozcan, Penciakova, Sander (2021) estimate a low fiscal (output) multiplier, but a higher `employment' multiplier of fiscal transfers under supply constraints.

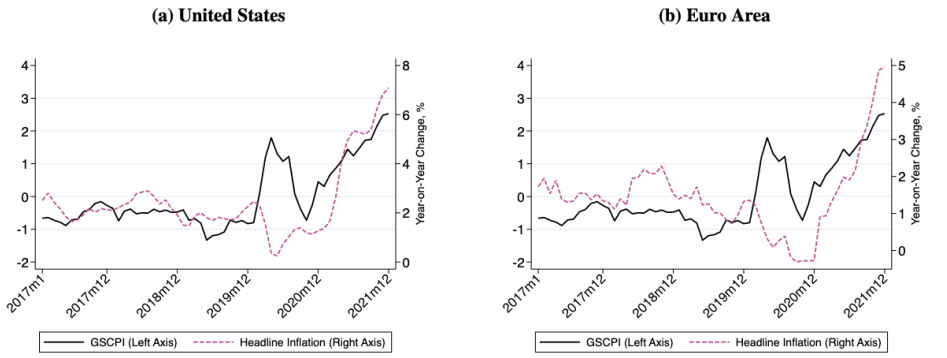
Figure 1. Inflation and Unemployment in the United States and Euro Area



Notes: Both panels plot headline inflation (year-on-year change) on the left axis and employment to population rate (25-54 years, total) on the right axis. All data comes from the Federal Reserve Economic Data (FRED) maintained by the Federal Reserve of St. Louis.

The evolution of the pandemic and international trade. As supply chains are global, the compositional change in consumption is important in understanding the developments of not only inflation but also of international trade. The co-existence of a rapid recovery in trade together with slow-to-dissipate supply chain disruptions and resulting inflation in many countries can in part be explained by such compositional effects. The tilting of consumption from services to goods exasperated the impact of supply-driven constraints on global supply chains, so while trade flows appeared to have recovered quickly, the excessive demand for manufactured goods eventually fed through to prices.

Figure 2. Supply Chain Pressure Index and Headline Inflation



Notes: This figure plots the FRBNY global supply chain index (GSCPI) on the right-hand axis of panels (a) and (b), along with US inflation in panel (a) and Euro Area inflation in panel (b). GSCPI comes from the Federal Reserve Bank of New York and inflation numbers comes from the Federal Reserve Economic Data (FRED) maintained by the Federal Reserve of St. Louis.

Figure 2 shows that the Federal Reserve Bank of New York's global supply chain pressure index (Benigno, di Giovanni, Groen, and Noble, 2022), which measures a common factor of several cross-country and global indicators of supply chain pressures (e.g., delays in shipments and delivery times and shipping costs after purging these from demand measured by new orders), moves together with inflation both in the US and in the Euro Area, where both started their ascent in early 2021.[7]

The time series depicted in Figure 2 reveals an issue with focusing solely on trade data as a metric to gauge global supply chain problems as both the supply pressure index and inflation rose with trade flows in the recent period. Trade is an equilibrium outcome, whereas supply chain disruptions are about mismatch between demand and supply indicating disequilibrium. However, examining trade flows can still be informative on how shocks, such as sectoral demand changes, are transmitted across borders via global supply chains. One key channel through which these shocks are transmitted arise from "demand spillovers," whereby changes in domestic demand for final goods lead to changes in exports and imports of both foreign intermediate and final goods. In fact, the Great Trade Collapse (GTC) of 2008-09 led economists to study the role of such spillovers via intermediate goods trade and cross-border supply chains to help explain why during the GTC trade fell so much more than GDP worldwide.

Model-based exercises. Our first quantitative exercise contrasts the response of global trade to the changes in domestic demand in the GTC and Covid-19 crisis. We base our analysis on the framework of Bems, Johnson, and Yi (2010), who show that the elasticity of world trade to world GDP can be much larger than one, and as high as three, given the amplification of demand shocks via cross-border input-output linkages. Using a simple model of global input-output linkages to map the observed changes in sectoral consumption demand across countries to production and trade flows, they show that the observed collapse in demand could account for 70 percent of the observed collapse in trade during the 2008-09 crisis. While the collapse in demand during the GTC was biased towards the consumption of goods relative to services (reverse is true for Covid-19 though in the early phase demand for goods and services both collapsed), GDP did not fall as much relative to trade given that services make up the majority of most countries' GDP. However, the global trade network still played an important amplification role as the initial change in demand in real value terms (as GDP is measured) was multiplied in gross output terms (as trade is measured) along the global production of

[7] Note that the increase of the index in early 2020 is due to the initial Chinese lockdown. The fall is in the second half of 2020 is then in part explained by re-opening of China and Europe, particularly in Europe during the summer 2020.

the final consumption good. For example, the final consumption of an automobile in the domestic economy requires parts sourced – both directly and indirectly – from countries around the world, which generates production and trade flows at different stages of the production process.

We apply the Bems, Johnson, and Yi (2010) framework to the recent episode to show how Covid-19's specific compositional demand changes, that were much starker and unique relative to any other crisis episode, spilled-over through the global production network. We find lower trade elasticities relative to the 2008-09 episode; that is given the decline in GDP during the early phase of Covid-19, trade declined less relative to the 2008-09 crisis given the same fall in GDP. Perhaps more surprisingly, when GDP rebounded during the pandemic recovery phase, trade also recovered but by much less relative to GDP compared to the response of trade to the collapse and recovery in GDP during the 2008-09 episode. Our interpretation of these results hinges on the importance of supply bottlenecks that arose due to the unique nature of Covid-19, and which led to the shutting down of the service sector.

How do these global patterns relate to inflation in each country? In standard models, an increase in demand for certain sectors' output or a decline in the production capacity in other sectors will be smoothed out through relative price adjustments. Sectors with higher demand will attract factors from other sectors via higher prices. Sectors with limited production capacity due to negative supply shocks will also face higher wages when attracting workers. Relative price adjustments ensure the necessary factor reallocation to solve these issues, rendering a response from monetary policy to cool down these sectoral price pressures unnecessary. In a world where such factor mobility is limited, factors are complements in production, and where some of those factors are imported, the drivers of a country's inflation can be imported and broad-based due to both domestic and global sector specific factor shortages.[8]

Limited factor mobility and complementarities in production arose in the short run due to the pandemic. Since everyone was exposed to the same health-related shock at a global level, it was difficult for firms to reallocate labor between sectors and/or switch and substitute suppliers in the short run – either domestically or internationally. The importance of cross-border production linkages amplified the impact of the inability for firms to substitute between different factors on a global scale,[9] leading to supply chain bottlenecks and a rise in prices that ultimately became persistent. We want to be careful with the use of the word "persistent." Our calibration exercise is based cumulative two-year window, 2019Q4

[8] See, for example, **https://libertystreeteconomics.newyorkfed.org/2021/11/high-import-prices-along-the-global-supply-chain-feed-through-to-u-s-domestic-prices/**

[9] This narrative of complementarities in trade and production is consistent with the evidence, see Atalay (2017), Boehm, Flaaen, and Pandalai-Nayar (2019) and Boehm, Levchenko, and Pandalai-Nayar (2020) who all estimate such degree of high complementarity with elasticity less than one in the short run.

to 2021Q4, where we do not model dynamics. We are thus limited from doing out-of-sample forecasts. Rather, our exercise is meant to provide a decomposition of what demand and supply shocks drove observed inflation, where we highlight the potentially important role of sectoral labor shortages in driving aggregate inflation given higher aggregate demand and compositional imbalances in consumption.

Our closed-economy exercise shows that aggregate demand and sectoral labor shortages contributed to inflation in the Euro Area and in the US. In terms of relative importance, sectoral labor shortages (supply-chain "bottlenecks") explain around half of observed inflation in the Euro Area, while these shocks explain only around one third of inflation in the US. The remaining part of inflation is explained by the demand side, with aggregate demand playing a larger role than sectoral demand shifts. The model structure also allows us to decompose the factor price sources of inflation. The decomposition shows that nominal wage increases contribute more than capital price changes in explaining aggregate inflation. Given that the model ignores other potential sources of price pressures, such as changes in firms' mark-ups, we take this result along with the importance of sectoral labor supply shocks as evidence of the overall importance of "cost push shocks" in driving inflation in the Euro Area. Finally, when we extend the model to the multi-country setting, we find that Euro Area-only shocks can only explain roughly one half of observed inflation. This result confirms the importance of international spillovers in driving the observed 2019Q4-2021Q4 inflation episode and in particular the role of foreign cost shocks in driving Euro Area inflation.

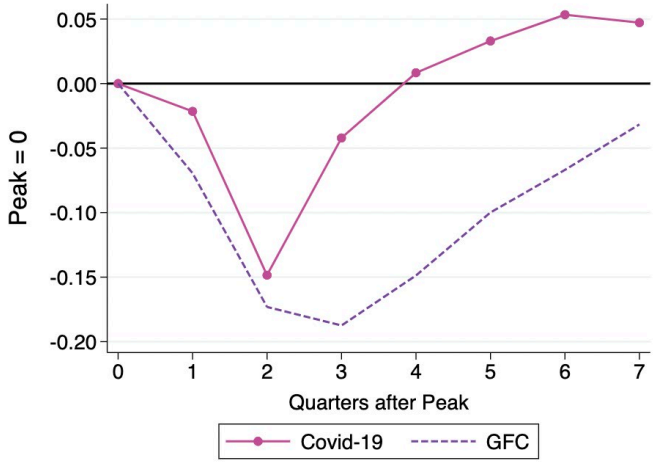
Outline of the paper. Section 2 summarizes descriptive patterns in trade, consumption, output, and prices for several countries since early 2020 and compares these patterns to those of the 2008-09 financial crisis. Section 3 uses a multi-sector trade model with input-output linkages to calculate trade flows given the observed cross-country quarterly sectoral consumption patterns over 2020-21 and compares the implied trade elasticities to those of the GTC 2008-09 episode. Section 4 undertakes a calibration exercise based on the multi-sector input-output network macro model of Baqaee and Farhi (2022) to decompose the drivers of inflation in the US vs Euro Area into demand and supply factors in closed economy setting with a single monetary policy authority. Section 5 revisits these results considering the international linkages of the Euro Area to the rest of the world. Section 6 concludes.

2 Descriptive Data Patterns

The early phase of the pandemic witnessed a steep disruption in world trade. Figure 3 plots this drop in world real imports (which is equivalent to world real exports) together with the same drop observed during the Global Financial Crisis (GFC) of 2008 – the so-called Great Trade

Collapse (GTC). As one can observe, the original drop in trade was worse during the GTC but trade rebounded much more quickly during the Covid-19 period than during the trade rebound after the GFC.

Figure 3. World Imports for the GFC and Covid-19: WTO Quantity Index

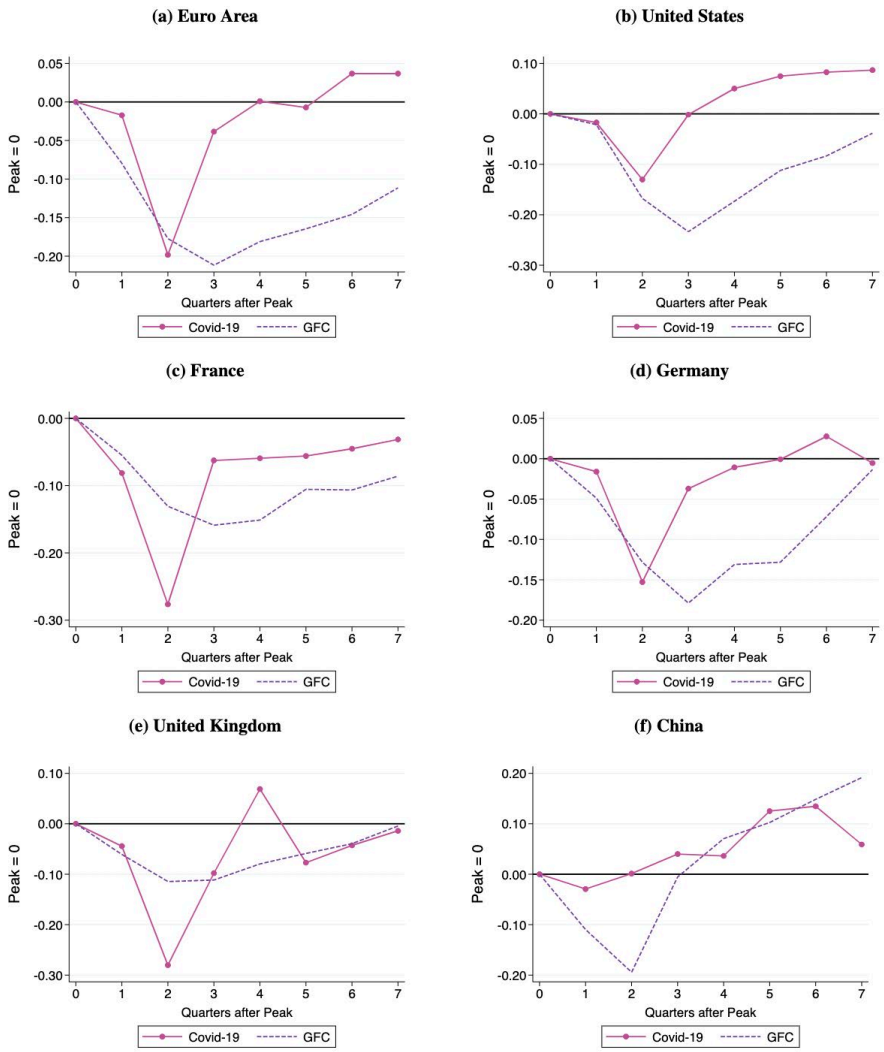


Notes: This figure compare the behavior of world imports quantity during the great financial crisis (GFC) to the Covid-19 pandemic. We compute log- deviations (percentage points) from the pre-crisis peaks that correspond to 2008Q3 for GFC and 2019Q4 for Covid-19. Data refers to merchandise trade (goods trade) and comes from the World Trade Organization (WTO) Trade database. Series are seasonally adjusted.

This aggregate narrative hides a large degree of country heterogeneity as shown in Figure 4. While Covid-19 led to a steeper drop in real imports for Euro Area countries and the U.K. relative to the GFC, the opposite held for the US and China. Figure 5 plots real exports and shows that the Covid-19 period is worse than the GFC for all countries except for China. Arguably, these differences reflect the consumption composition change that begin early during the pandemic, particularly in the US, where consumers' consumption of goods remained robust during the initial lockdown period as they substituted away from services. This consumption was in turn in part driven by imports from China.

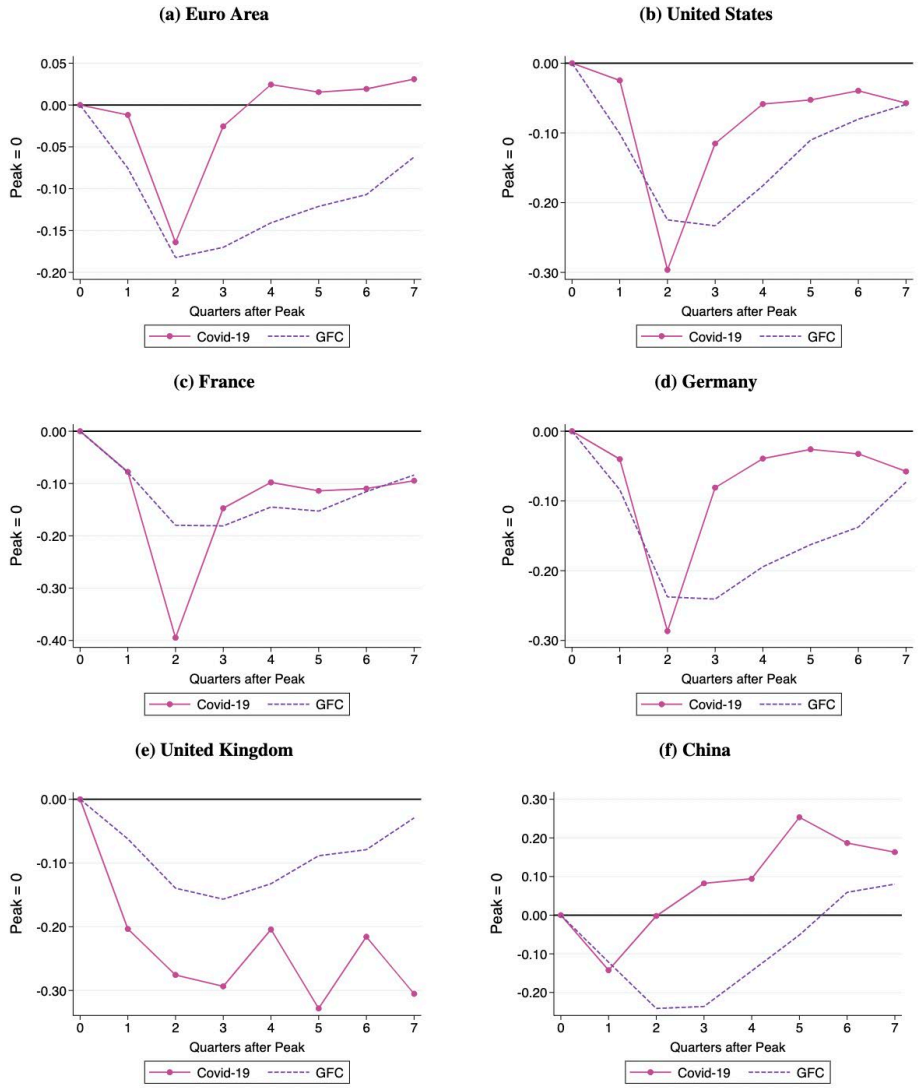
It is also interesting how exports are much slower to recover during Covid-19 for European countries, both relative to imports during Covid-19 and relative to the GFC in several countries, indicating possibly both lower demand for these goods (e.g., capital goods from Germany) from the rest of the world and limits to production capacity.

Figure 4. Import Quantity in Selected Countries during the GFC and Covid-19



Notes: This figure provides a cross-country comparison of the behavior of imports quantities for a set of countries. We compute log-deviations (percentage points) from the pre-crisis peaks in the import quantities series. We pick the same pre-crisis dates for all countries in both crises. We set 2008Q3 for the pre-crisis date in the GFC and 2019Q4 for the pre-crisis date in the Covid-19 crisis. Data refer to merchandise trade (goods trade) and comes from the World Trade Organization (WTO) for all countries except China data that comes from the CPB World Trade Monitor. All series are seasonally adjusted.

Figure 5. Export Quantity in Selected Countries during the GFC and Covid-19



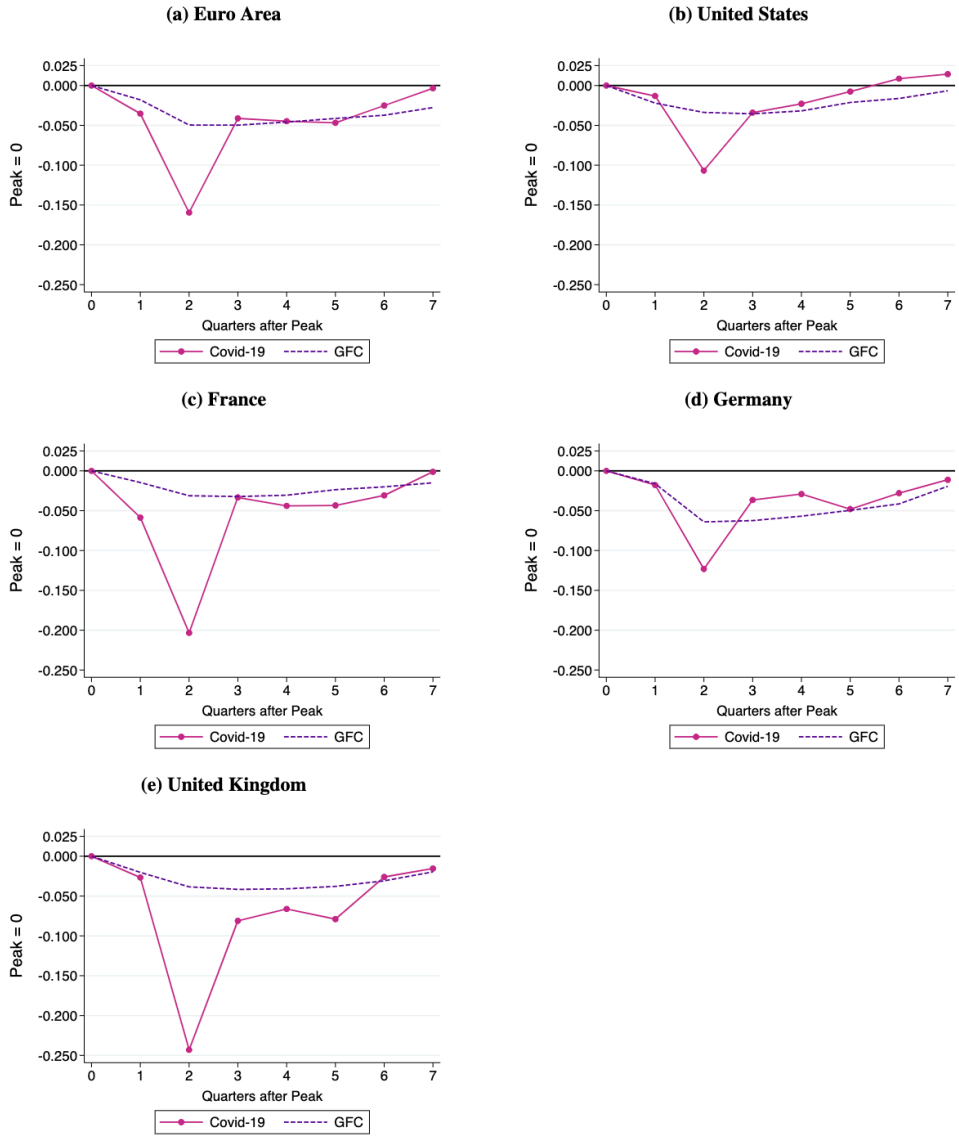
Notes: This figure provides a cross-country comparison of the behavior of exports quantities for a set of countries. We compute log-deviations (percentage points) from the pre-crisis peaks in the export quantities series. We pick the same pre-crisis dates for all countries in both crises. We set 2008Q3 for the pre-crisis date in the GFC and 2019Q4 for the pre-crisis date in the Covid-19 crisis. Data refer to merchandise trade (goods trade) and comes from the World Trade Organization (WTO) for all countries except China data that comes from the CPB World Trade Monitor. All series are seasonally adjusted.

Figures 6 and 7 show that the drops in real GDP and real private consumption were both much larger during Covid-19 relative to the GFC in the US and Europe, making the early phase of Covid-19 the worst recession since the Great Depression. What is unique about Covid-19 is that during the early phase, even though total consumption expenditures dropped, there were large compositional shifts from services to goods.

As can be seen in Figure 8, which plots consumption growth, durable consumption started rising as early as 2020Q2 with the speed being much faster in the US, while services only started picking up in 2021Q2. Figure 9 shows the same growth patterns for real consumption, with similar

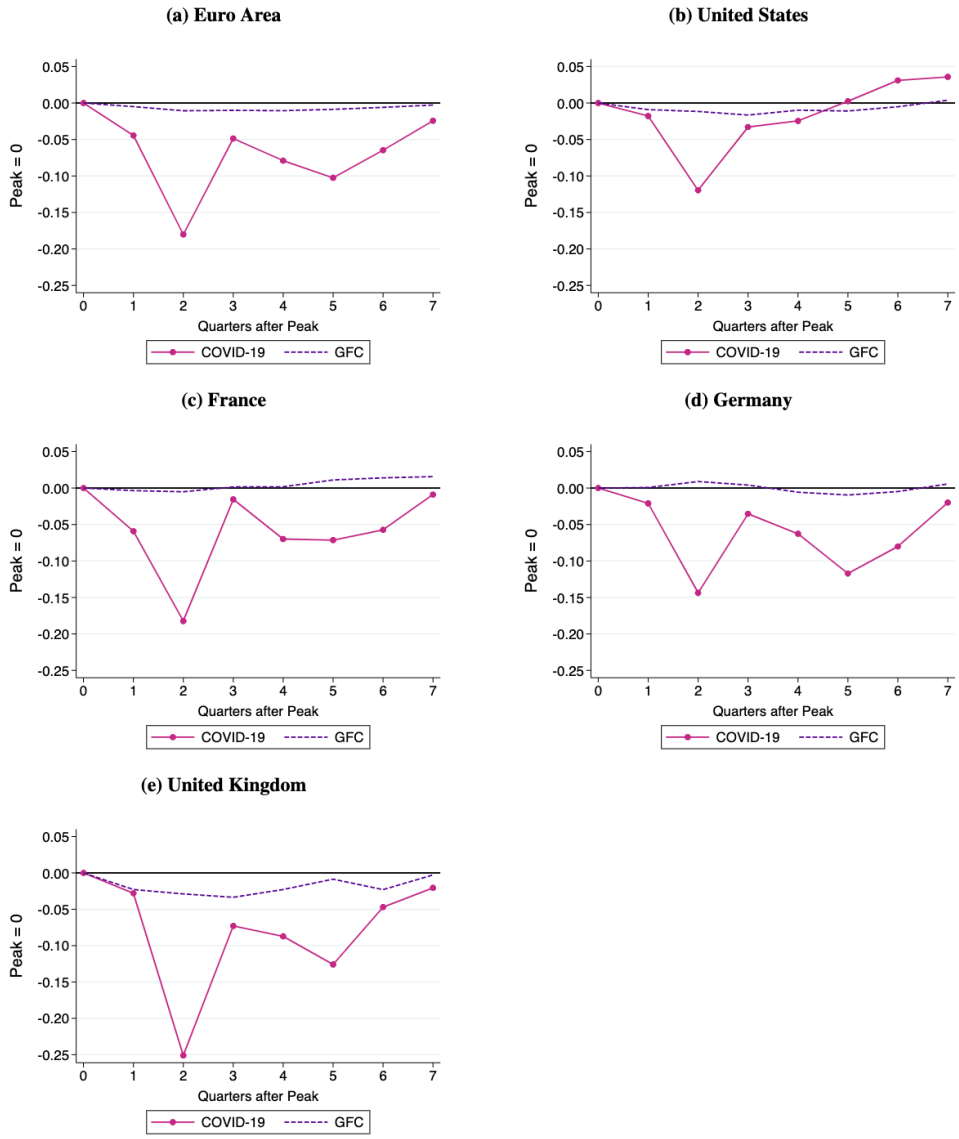
timing differences across durables and services and very large increase in the US for durables.

Figure 6. Real GDP in Selected Countries during GFC and Covid-19



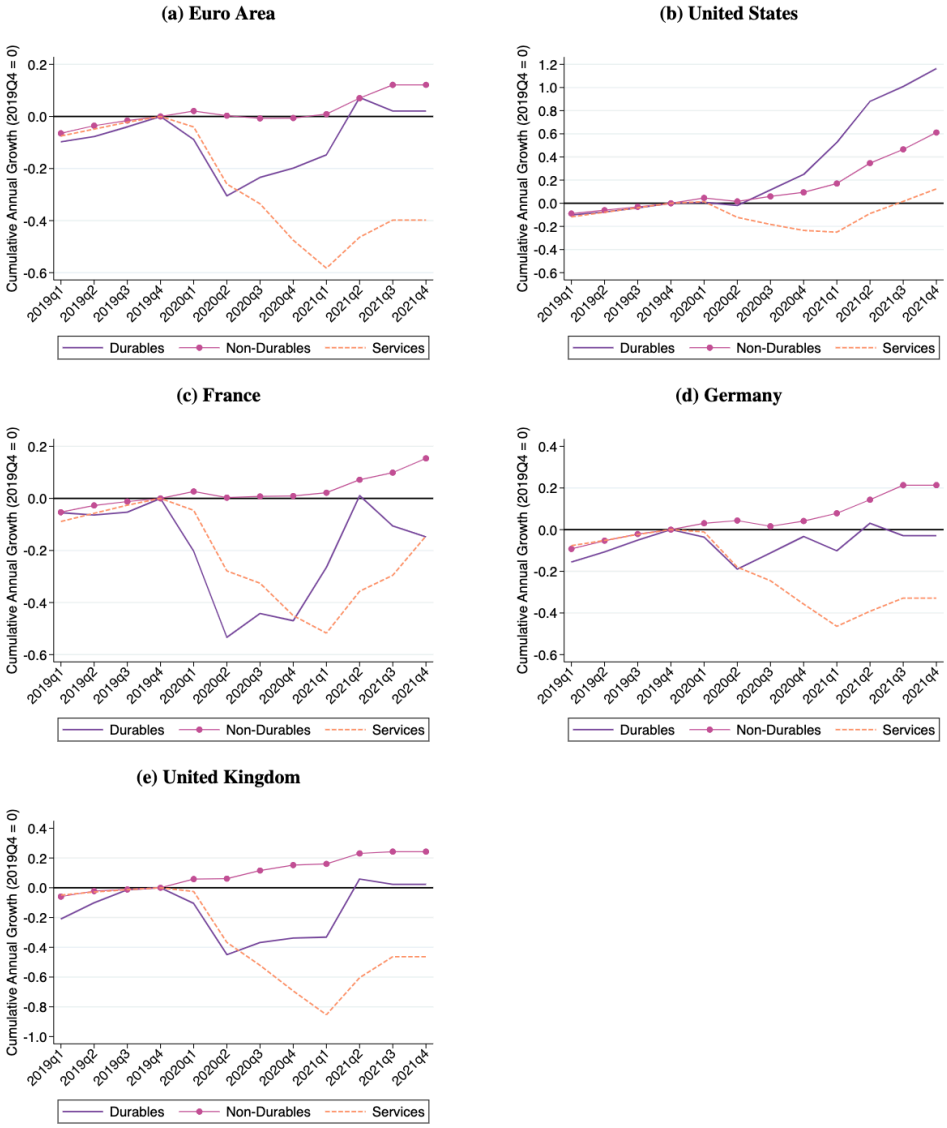
Notes: This figure provides a cross-country comparison of the behavior of real gross domestic product (GDP) for a set of countries. We compute log-deviations (percentage points) from the pre-crisis peaks in the real GDP series. We pick the same pre-crisis dates for all countries in both crises. We set 2008Q3 for the pre-crisis date in the GFC and 2019Q4 for the pre-crisis date in the Covid-19 crisis. Data comes from the International Financial Statistics (IFS) of the International Monetary Fund (IMF). All series are seasonally adjusted.

Figure 7. Real Consumption in Selected Countries during the GFC and Covid-19



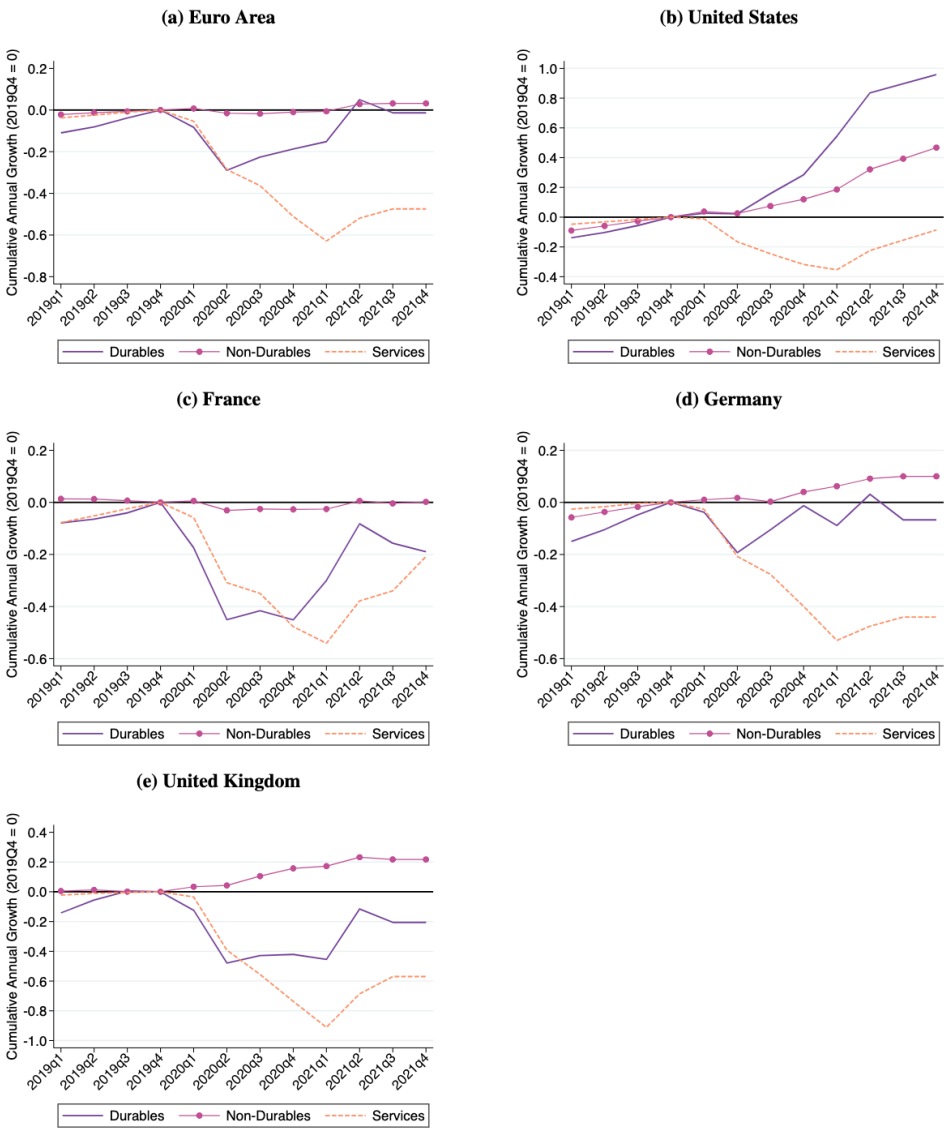
Notes: This figure provides a cross-country comparison of the behavior of real private consumption for a set of countries. We compute log-deviations (percentage points) from the pre-crisis peaks in the real consumption series. We pick the same pre-crisis dates for all countries in both crises. We set 2008Q3 for the pre-crisis date in the GFC and 2019Q4 for the pre-crisis date in the Covid-19 crisis. Data comes from the International Financial Statistics (IFS) of the International Monetary Fund (IMF). All series are seasonally adjusted.

Figure 8. Nominal Consumption Growth in Selected Countries by Sector during Covid-19



Notes: This figure plots nominal consumption growth in each quarter vis-à-vis 2019Q4 and cumulated for three different consumption series: durables, non-durables and services. All series are nominal, de-seasonalized and comes from the OECD Quarterly National Accounts. We construct the Euro Area numbers using data for the following countries that contains information for the three series: Austria, Estonia, Finland, France, Germany, Ireland, Italy, Latvia, Luxembourg and Netherlands.

Figure 9. Real Consumption Growth in Selected Countries by Sector during Covid-19



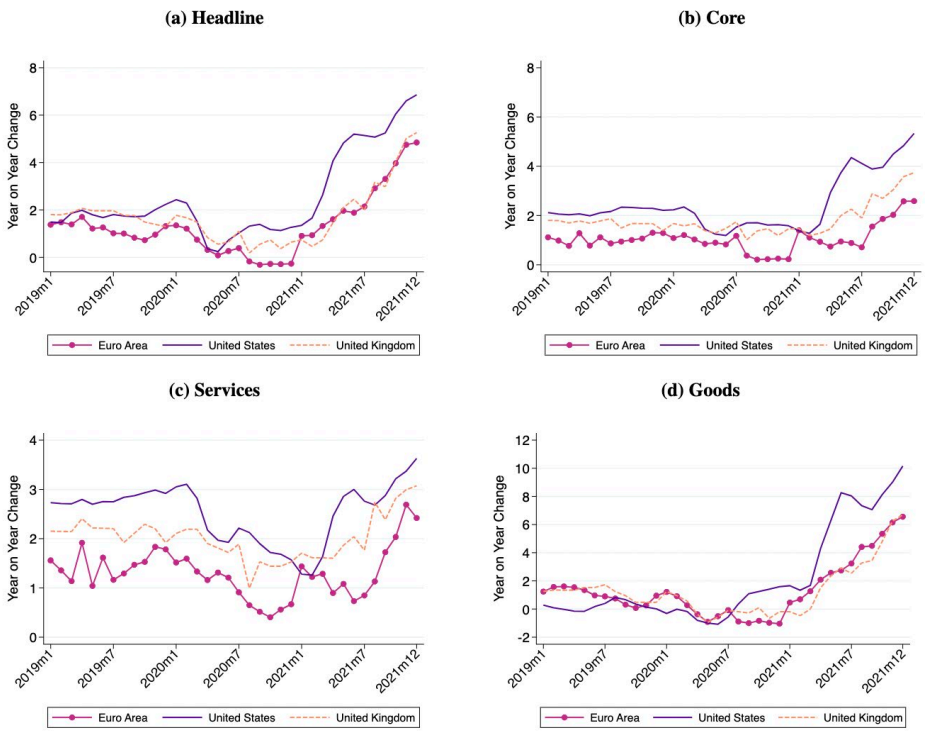
Notes: This figure plots real consumption growth in each quarter vis-à-vis 2019Q4 and cumulated for three different consumption series: durables, non-durables and services. All series are nominal, de-seasonalized and comes from the OECD Quarterly National Accounts. We construct the Euro Area numbers using data for the following countries that contains information for the three series: Austria, Estonia, Finland, France, Germany, Ireland, Italy, Latvia, Luxembourg and Netherlands.

Figure 10 plots the inflation trends that are consistent with the compositional shifts we document above. Both headline and core inflation started increasing at similar times in the US, Euro Area and the UK during early 2021, though the slope is much steeper for the US and began a few months earlier there.

What is important is the differences shown in panels c and d of this figure where services inflation was still less than 3 percent in the Euro Area and UK, and just passed this number in the US at the end of 2021, while

goods inflation was almost 8 percent in the Euro Area and UK and over 10 percent in the US.

Figure 10. Inflation by Sector in the Euro Area, US and UK, Covid-19



Notes: This figure plots headline, core (headline minus food and energy), services and goods annual inflation for the Euro Area, US and the UK. Data comes from the Federal Reserve Economic Data (FRED) maintained by the Federal Reserve of St. Louis.

3 The Composition of Demand and International Trade

3.1 Analytical Framework

The previous section highlighted several stylized facts on the cross-country and cross-sector differences in the economic collapse and recovery during the Covid-19 pandemic. Further, consumption, output and international trade also behaved differently during the Covid-19 period relative to the GFC. In particular, given the rebound in world trade (see Figure 3) in the recent period compared to the GFC, some economists had inferred that the problems with the global supply chains should also smooth out quickly. We argue that the recovery in trade may not be a good metric on its own to understand supply chain issues and

bottlenecks and support this point below by using a simple decomposition accounting framework.

We compare the two periods to point out why the patterns we observe in the data are not surprising given the nature of the Covid-19 health shocks vs. what we observed during the GFC. Our analysis follows the work of Bems, Johnson, and Yi (2010), who provide a partial equilibrium global input-output framework that links changes in domestic sector-level consumer demand to foreign countries' output across sectors. Given their model setup and assumptions, a change in a country's demand for a given sector's goods will spillover across countries due to (i) imports of final goods in that sector, and (ii) intermediate trade arising from the production of the sector's goods along the global value chain.

We do not derive the whole quantitative framework. Instead, we lay out the key equations which we use to conduct our accounting exercise and refer the interested reader to Bems, Johnson, and Yi (2010) for the full derivation of the framework and their fascinating analysis of the role of the global production network in generating international spillovers during the GFC, explaining the Great Trade Collapse of 2008-09.

The framework allows for C countries, N sectors, each with constant returns to scale production that combines local factor inputs along with domestic and foreign intermediate goods. Denote the quantity of final goods produced in a given country m, sector j by y_{jm}; the quantity of sector-country good jm used as intermediates for production in sector-country kc by $x_{kc,jm}$, and final demand for sector-country good jm by country c by $x_{0c,jm}$,[10] then market-clearing for the good jm implies that:

$$y_{jm} = \sum_{c}\sum_{k} x_{kc,jm} + \sum_{c} x_{0c,jm}, \tag{3.1}$$

where the double summation on the right-hand side of equation (1) measures total intermediate demand for good jm across sector-country pairs kc, and the second term captures final demand for the good across all countries c.

Taking Equation (3.1) and applying a set of model assumptions, Bems, Johnson, and Yi (2010) show that the percentage change of a sector-country output over two points in time can be related to output changes across all sectors in all countries in the world and final demand for the goods across countries by the following equation:

[10] To interchangeably use the sector indices in consumption, we define sector 0 to be the "consumption" sector.

$$\hat{y}_{jm} = \sum_{c}\sum_{k}\left(\frac{p_{jm}x_{kc,jm}}{p_{jm}y_{jm}}\right)\hat{y}_{kc} + \sum_{c}\left(\frac{p_{jm}x_{0c,jm}}{p_{jm}y_{jm}}\right)\hat{x}_{0c,j}, \qquad (3.2)$$

where the hat notation refers to percentage changes. Let's define $M_{kc,jm} \equiv p_{jm}x_{kc,jm}$ as the value of sector-country kc's use of sector-country jm good as an intermediate, and $D_{c,jm} \equiv p_{jm}x_{0c,jm}$ as the value of sector-country jm good demanded by country c, $Y_{jm} \equiv p_{jm}y_{jm}$ as the value of output in sector-country jm. We assume $\hat{x}_{0c,j} \equiv \hat{x}_{0c,jm}$; i.e., the demand change of final good j by country c is identical across potential source countries m. With these definitions, we can rewrite Equation (3.3) as:

$$\hat{y}_{jm} = \sum_{c}\sum_{k}\left(\frac{M_{kc,jm}}{Y_{jm}}\right)\hat{y}_{kc} + \sum_{c}\left(\frac{D_{c,jm}}{Y_{jm}}\right)\hat{x}_{0c,j}\,. \qquad (3.3)$$

Equation (3.3) shows that a sector j's output change in country m is equal to a weighted sum of output changes of other countries and demand changes of final goods. The first term on the right-hand side is a weighted sum of a sector-country outputs, where the weights measure the share of good jm used by sector-country kc as an intermediate relative to total output of good jm. This term captures the importance of the global production network, whereby output changes spillover across country-sectors due to intermediate usage. The share is deflated by jm's total output to reflect how important the use of the good by kc relative to jm's total output. The second term on the right-hand side of Equation (3.3) captures the importance of demand changes for jm's goods across countries, again scaled by jm's total output.

What is key to note in Equation (3.3) is that the first term that captures intermediate goods demand can be brought over to the left-hand side of the equation and after stacking this equation across all pairs, we can invert the system (a matrix) in order to express the vector of sector-country output changes, $\hat{y}_{jm}$, as a linear function of sector-country final demand changes, $\hat{x}_{0c,j}$. In particular, the output and demand changes are related by a matrix $\mathbf{\Omega}$ that captures global input-output linkages (intermediate goods linkages) and the importance of a sector-country good in countries' final demand. This formulation, therefore, allows to use observed intermediate and final goods shares from global input-output tables to construct $\mathbf{\Omega}$, and then feed in observed final demand changes across pairs to calculate the corresponding sector-country output changes implied by this global input-output framework.

Thus far, this framework feeds in observed consumption data and produces output at the sector-country level. We can next use these series and aggregate up to calculate country level measures of total output, GDP, exports, and imports using the following equations:

$$\hat{Q}_m = \sum_i \frac{Y_{jm}}{Y_m} \hat{y}_{jm}\,, \tag{3.4}$$

$$\widehat{GDP}_m = \sum_i \frac{VA_{jm}}{VA_m} \hat{y}_{jm}\,, \tag{3.5}$$

$$\widehat{EX}_m = \sum_{m \neq c} \sum_j \left[\sum_k \left(\frac{M_{kc,jm}}{EX_m} \right) \hat{y}_{kc} \; + \left(\frac{D_{c,jm}}{EX_m} \right) \hat{x}_{0c,j} \right], \tag{3.6}$$

$$\widehat{IM}_m = \sum_{m \neq c} \sum_j \left[\sum_k \left(\frac{M_{jm,kc}}{IM_m} \right) \hat{y}_{jm} \; + \left(\frac{D_{m,jk}}{IM_m} \right) \hat{x}_{0m,j} \right]. \tag{3.7}$$

Equation (3.4) calculates the country m's output change using the countries sectoral shares of total output. Equation (3.5) calculates country m's GDP change by weighting sectoral output growth by value added shares.[11]

Note that the assumption in this aggregation is that sector-country value added grows at the same rate as total output. Equations (3.6) and (3.7) calculate country-level export and import growth by essentially aggregating up over Equation (3.2) to the country-level, while removing domestic demand. Specifically, the first term on the right-hand side of Equation (3.6) measures how much of country m's exports are driven by intermediate demand for its goods from abroad, while the second term captures the contribution from final goods exports. Meanwhile, the first term in Equation (3.7) measures the importance of imported intermediate inputs to country m's total import growth, while the second term measures the contribution of imported final consumption goods to aggregate import growth.

3.2 Data

The accounting framework requires several pieces of data. First, we require data on both intermediate and final goods trade as well as demands shares of final goods at the country-sector level to construct the matrix $\mathbf{\Omega}$, which allows us to map observed consumer demand changes to final output changes using the matrix version of Equation (3.2). These data are available at the annual level in global input-output tables. We source these data from the OECD ICIO tables for 2007 and 2018 for the GFC and Covid-19 period exercises, respectively. We use data at the onset of the shock on purpose to keep the global linkages at their pre-shock level, assuming during the shock in the short run these supplier relations cannot change. These data are available for 67 countries and 45 sectors.

[11] We create these shares by aggregating domestic sectors valued added in each country, as observed in the input-output table, to calculate GDP.

Second, we need to feed in data series for changes in real domestic demand at the country-sector level, $\hat{x}_{0m,j}$. We source direct measures of real household consumption growth at the quarterly level for OECD countries for three aggregate sectors: durable goods, non-durable goods, and services. These three sectors are the same that Bems, Johnson, and Yi (2010) examined in their analysis of the Great Trade Collapse, but they did not have direct data available for these series. Instead, they estimated sectoral consumption demand from measures of total domestic demand using various data sources and assumptions.[12] While we follow this methodology to fill in missing non-OECD countries and sectors not covered by the OECD for some countries in the sample, our baseline measure is the observed real household consumption growth from the OECD, which is particularly relevant to consider given our analysis focuses on Euro Area, the US, and other industrial countries.

Finally, note that since we only have time series data for the consumption series for three sectors, we are forced to aggregate the more detailed OECD input-output data to match these three sectors. We aggregate up the OECD input-output tables from 45 sectors to the three sectors by using the same concordance as Bems, Johnson, and Yi (2010). We do this when calculating input-output coefficients as well as domestic and foreign demand shares. Appendix C presents further details on the input-output aggregation and the data series we use for $\hat{x}_{0m,j}$.

3.3 Results

Table 1 begins by presenting summary statistics for the durables, non-durables, and services sectors based on the 2018 Input-Output table sourced from the OECD ICIO database. We present each sector's share as a share of gross output, value added, domestic final demand, imports, and exports for the United States (US), Euro Area (EA), United Kingdom (UK), and the World, where shares for country groups are weighted averages based on the country sample.

Two key facts emerge from looking across the countries and sectors. First, the services sectoral share dominates a country's gross output, value added (i.e., GDP), and domestic final demand. Further, while not as important as for production and final demand, services also represent the largest share of both imports and exports.[13] Second, when zooming in on the goods' sectors (durables vs. non-durables), we see that non-durables tend to be a larger share for an economy's production as well as imports

[12] See Appendix C of their paper.

[13] Note that the services sector share of trade is larger than what is reported by countries' customs and that would be calculated using product-level data. This reflects the fact that we include the Wholesale & Retail sector as part of the services sector and domestic wholesalers often act as middlemen in international trade Bernard, Jensen, Redding, and Schott (2010).

and exports, but domestic total demand is approximately the same across durables and non-durables.

Table 1. Sectoral Shares Based on 2018 Input-Output Tables

	Output	VA	Final Demand	Imports	Exports
United States					
Durables	0.06	0.05	0.08	0.31	0.22
Non-Durables	0.13	0.08	0.08	0.29	0.25
Services	0.81	0.87	0.83	0.40	0.52
Euro Area					
Durables	0.11	0.07	0.12	0.20	0.22
Non-Durables	0.16	0.10	0.10	0.32	0.35
Services	0.73	0.83	0.78	0.48	0.43
United Kingdom					
Durables	0.06	0.04	0.07	0.20	0.16
Non-Durables	0.10	0.07	0.09	0.24	0.20
Services	0.84	0.89	0.85	0.56	0.64
World					
Durables	0.09	0.06	0.10	0.21	0.21
Non-Durables	0.20	0.14	0.12	0.35	0.44
Services	0.71	0.79	0.78	0.45	0.35

Notes: This table presents the sectoral shares of output, value added, final demand, imports and exports for the three sectors in the economy: (i) Durables, (ii) Non-Durables, and (iii) Services. All calculations are based on the 2018 OECD ICIO table, which has information for 71 countries and 43 sectors, which we aggregate to countries or country-groups and three aggregate sectors. The disaggregate sectoral data are assigned to (i), (ii), or (iii) following Bems, Johnson, and Yi (2010).

Table 2 presents country-level elasticities of real imports and real exports viz. real GDP for the US, Euro Area, UK and the World. Panel I, columns (1)-(4), construct the elasticities based on observed data, while Panel II, columns (5)-(8), construct the elasticities based on the quantitative exercise and equations (3.4)-(3.6). Panel A results are based on data from the Great Financial Crisis and uses year-on-year growth rates between 2008Q2-2009Q2 for the Collapse and 2009Q2-2010Q2 for the Recovery. Panel B results are based on data from the Covid-19 Pandemic and use year-on-year growth rates between 2019Q2-2020Q2 for the Collapse and 2020Q2-2021Q2 for the Recovery.

with the GFC's world elasticity is minor. Trade responded more to the changes in GDP during GFC relative to Covid-19. The difference in elasticities between the two periods is notable and perhaps not surprising given that the shocks hitting economies in the two periods are very different, i.e., the financial shock of the GFC vs. the Covid-19 health shock.

The second fact is that the import and export elasticities are always larger for the US relative to the Euro Area, the UK and the world as a whole. This is true both for imports and exports as well as during periods of collapse and recovery.

To better understand how the composition of demand played a role in the difference in elasticities, we begin by calculating the country-level trade elasticities using the quantitative framework outlined above in Panels IIA and IIB. The structure of these model-based results is identical to what we just described for the elasticities calculated for using realized trade and GDP data, except now we have fed in observed country-sector consumption growth rates and compute the implied growth rates of imports, exports, and GDP given the model setup.

In comparing the model-implied elasticities of columns (5)-(8) to their data counterparts in columns (1)-(4), it is notable that the model-implied elasticities are smaller. This is not surprising given that model framework is partial equilibrium and has several assumptions built into that only approximate reality. However, the quantitative results still match up reasonably well to what we calculate using realized trade and GDP data: the model-implied elasticity is roughly one-half of the actual across all observations in Panel A for the GFC period, and three-quarters of the actual across all observations in Panel B for the Covid-19 period.

Importantly, the results that the trade elasticities are larger during the GFC than the Covid-19 pandemic period hold up for most of the observations when looking at the model results. While we utilize different vintages of input-output tables (2007 and 2018) when implementing the quantitative framework to the two crisis periods, these data do not differ dramatically in the cross-section, providing further support that supply chain relations do not change easily (i.e., a 2007 version of Table 1 looks very similar to the 2018 version presented). Rather, differences in how consumption changed across sectors in the two periods is key for understanding the smaller elasticities during the Covid-19 period relative to the GFC.

Table 2. Trade Elasticities with respect to GDP

	Panel I. Data				**Panel II.** Model			
	Panel A. Great Financial Crisis							
	Collapse		Recovery		Collapse		Recovery	
	Imports (1)	Exports (2)	Imports (3)	Exports (4)	Imports (5)	Exports (6)	Imports (7)	Exports (8)
United States	4.35	3.31	5.90	4.99	2.65	1.74	1.67	2.09
Euro Area	2.74	3.11	5.39	5.65	1.34	2.05	0.86	2.39
United Kingdom	1.99	2.02	3.72	3.04	1.28	0.39	0.87	0.49
World	1.29	1.29	4.15	4.15	1.63	1.63	1.34	1.34
	Panel B. Covid-19 Pandemic							
	Collapse		Recovery		Collapse		Recovery	
	Imports (1)	Exports (2)	Imports (3)	Exports (4)	Imports (5)	Exports (6)	Imports (7)	Exports (8)
United States	2.43	2.63	2.50	1.52	0.60	1.09	1.31	1.20
Euro Area	1.42	1.45	1.49	1.82	0.87	0.74	1.04	1.16
United Kingdom	1.31	0.64	0.92	0.25	0.89	0.29	1.01	0.63
World	1.48	1.48	2.03	2.03	0.89	0.89	1.06	1.06

Notes: This table presents country-level elasticities of real imports and real exports viz. real GDP. Panel I, columns (1)-(4), construct the elasticities based on observed data, while Panel II, columns (5)-(8), construct the elasticities based on the quantitative exercise and equations (3.5)-(3.7). Panel A results are based on data from the Great Financial Crisis and uses year-on-year growth rates between 2008Q2-2009Q2 for the Collapse and 2009Q2-2010Q2 for the Recovery. Panel B results are based on data from the Covid-19 Pandemic and use year-on-year growth rates between 2019Q2-2020Q2 for the Collapse and 2020Q2-2021Q2 for the Recovery.

Table 3 next decomposes the model-based elasticities into responses driven by the intermediate and final goods' components of trade, where the intermediates' contributions are calculated in the first terms of equations (3.6) and (3.7) and the final goods' trade values are in the second terms of the equations. The table's structure follows that of Table 2, but we now report the imports and exports elasticities split into the 'Inter.' and 'Final' terms. For example, the total imports elasticity for the US reported in column (1) of Table 2, Panel A, is a weighted average of the terms in columns (1) and (2) of Panel A in Table 3, where the weights are the intermediate and final goods' share of total imports, respectively.

Table 3. Trade Elasticities Decomposition

Panel A. Great Financial Crisis

	Collapse				Recovery			
	Imports		Exports		Imports		Exports	
	Inter. (1)	Final (2)	Inter. (3)	Final (4)	Inter. (5)	Final (6)	Inter. (7)	Final (8)
United States	1.88	3.53	1.53	2.00	1.43	1.95	1.87	2.36
Euro Area	1.31	1.45	1.64	2.58	1.19	0.52	2.27	2.53
United Kingdom	1.04	1.51	0.36	0.43	0.91	0.83	0.52	0.45
World	1.36	1.98	1.36	1.98	1.27	1.39	1.27	1.39

Panel B. Covid-19 Pandemic

	Collapse				Recovery			
	Imports		Exports		Imports		Exports	
	Inter. (1)	Final (2)	Inter. (3)	Final (4)	Inter. (5)	Final (6)	Inter. (7)	Final (8)
United States	0.80	0.39	1.06	1.14	1.16	1.48	1.19	1.29
Euro Area	0.86	0.89	0.77	0.72	1.05	1.03	1.14	1.18
United Kingdom	0.90	0.88	0.54	0.50	0.95	1.04	0.65	0.64
World	0.92	0.84	0.92	0.84	1.03	1.09	1.03	1.09

Notes: This table presents the decomposition of the model-based country-level elasticities of real imports and real exports viz. real GDP, reported in Table 2, into the contribution of intermediates goods' trade ('Inter.') and final goods trade ('Final'). Panel A results are based on data from the Great Financial Crisis and uses year-on-year growth rates between 2008Q2-2009Q2 for the Collapse and 2009Q2-2010Q2 for the Recovery. Panel B results are based on data from the Covid-19 Pandemic and use year-on-year growth rates between 2019Q2-2020Q2 for the Collapse and 2020Q2-2021Q2 for the Recovery.

Panel A's results show that most the trade's collapse during the GFC was driven by final goods' trade, particular for US imports and Euro Area's exports, confirming results reported in Bems, Johnson, and Yi (2010). A similar pattern holds for the first year of the recovery period. Looking at the world, we see that final goods trade played a somewhat larger role in explaining the total elasticity of both imports and exports in both the collapse and recovery periods.

intermediates trade can still explain roughly half of world trade and more important during Covid-19 episode than the GFC.

To summarize, this section highlights the key role that the composition of demand plays in driving the observed trade patterns during the Covid-19 pandemic. We show that the change in trade relative to GDP was more muted during Covid-19 than the GFC, and that this result follows naturally given the health shock, which resulted in a shutdown of the service sector in the latest crisis, a shortage of labor and the related supply chain issues. We also show that the effects of such lockdowns spilled over across countries given the service sector's reliance on intermediate trade goods.

While it has been argued that trade recovered quickly during Covid-19 relative to the GFC, it should be emphasized that this change in trade was muted compared to the change in domestic output (Table 2). This difference reflects the contrast in shocks in the two periods (financial vs. health), which impacted production as well as the composition of demand. It would therefore be misleading to say that the rebound in trade observed in the Covid-19 recovery period reflected well-functioning supply chains. Indeed, trade flows are an equilibrium outcome, which capture demand and supply pressures. Therefore, focusing only on quantities may be misleading. To address this point, we next turn to a stylized general equilibrium macro model, which allows us to decompose observed inflation into a set of demand and supply shocks to better understand how production bottlenecks contributed to the current inflationary environment.

4 Inflation under Supply Chain Bottlenecks: Closed-Economy Model

4.1 A Simple Framework

In this section, we briefly sketch a theoretical framework borrowed from Baqaee and Farhi (2022). Their model allows us to perform a calibration exercise to obtain broad based inflation under sectoral labor shortages, relative demand shifts and aggregate demand shocks. It also allows us to assess the relative forces of demand and supply driving inflation during the pandemic period. We outline the key components of the model here which are sufficient to make our point regarding inflation and refer the reader to the above paper for further details.

The model is a two-period multi-sector closed economy with perfectly competitive factors and good markets. The intertemporal block assumes

households have perfect foresight and therefore there is no uncertainty about the future.

We follow Baqaee and Farhi (2022) with the following specifications: (i) we assume a unitary elasticity of substitution between present and future consumption, which we denote as $\rho = 1$; (ii) we assume a representative consumer with the ability to borrow, i.e., no hand to mouth consumers; (iii) we assume factors are sector specific, hence, immobile across sectors. Assumptions (i) and (ii) imply that the consumers have log utility with consumption smoothing under no uncertainty on next period's income. Assumption (iii) implies that relative goods' demand changes in different sectors and will lead to relative factor price movements, but no factors moving between sectors. Hence, sectoral demand changes can potentially cause unemployment.

We describe the economy and present key equations below and refer the reader to Appendix 7.2 for the full model derivation.

Production. We assume that each good is produced by a single sector, so sectors and goods can be used interchangeably in what follows. Let N be the number of sectors and F the number of factors. We index sectors by i and factors by f.

To save on notation, we outline the model below with only one sector-specific factor that we call sectoral labor. Hence, the number of factors and goods in this economy coincides $N = F$. Then, when going to the data, and in line with the quantitative exercise in Baqaee and Farhi (2022), we add another sector-specific factor that we call sectoral capital. This additional sector-specific factor serves two broad purposes. First, it allows us to capture better the structure of national accounts where sectoral value-added is decomposed into labor compensation and gross operating surplus (plus taxes) from the supply side. Second, this also allows the quantitative exercise to say something about other factor price changes that are not necessarily wage changes such as rental rates, or, although not strictly a factor price, profits.[14] Note, however, that the model abstracts from other potential drivers of firms' prices such as changes in mark-ups given the competitive market structure.

There is a representative firm in each sector that produces using constant returns to scale production technology, which combines factors and intermediates goods. Unlike Section 3, we assume that there is a single country, m. Hence, to simplify the notation used in Section 3, we define sectors' use of inputs as:

[14] As highlighted in Baqaee and Farhi (2022), this follows from an argument in McKenzie (1959), where we can always express a decreasing return to scale production function, which features profits, as a constant returns to scale production function where an additional "entrepreneurial" factor receives the payments that were labelled as profits before.

$$x_{ik} \equiv x_{im,kc} \text{ and } x_{0i} \equiv x_{0m,mi}.$$

In words, these equations imply that sector k provides inputs to be used in sector i. We use sector index of 0 to denote the consumption to have a unified representation of sectoral linkages. Denote the CRS production function by $\mathrm{G}(\cdot)$, then gross output $y_{i>0}$ can be written as

$$y_i = A_i \mathrm{G}(L_i, \mathbf{X}_i),$$

where $\mathbf{X}_i = (x_{i1}, x_{i2}, \dots, x_{iN})$ is an $N \times 1$ vector of intermediate goods. Each element of this vector records the quantity of good $j = 1, \dots, N$ purchased by sector i to produce its output. L_i is the quantity of factor i used in sector i. A_i is a Hicks-neutral exogenous technology affecting the productivity of the sector.

Under perfect competition, the firm's cost minimization problem yields the following solution for the growth rate in prices:

$$\mathrm{d}\log p_i = \sum_{k=1}^{N} \Psi_{ik}\, \alpha_k\, \mathrm{d}\log w_k - \sum_{k=1}^{N} \Psi_{ik}\, \mathrm{d}\log A_k\,, \qquad (4.1)$$

where sectoral wages and productivities are denoted by w_k and A_k, respectively. The Ψ_{ik} terms capture sector i's use of all sectors' intermediate goods (see Appendix 7.2 for details).

Equation (4.1) shows that prices in each sector are a function of changes in wages in all sectors even if a sector does not use labor from other sectors directly in production. However, given the use of intermediate goods from other sectors, changes in other sectors' wages will feed into marginal cost changes via the price of intermediate goods. This dependence highlights the importance of the production network and of supply chains more broadly in driving relative price movements across the economy.

Equation (4.1) also highlights another related but subtle point: exogenous changes in demand, both aggregate and relative, only affect good prices through changes in wages across sectors because there is no term in the equation that can be linked to the demand side of the economy. That is, demand changes, either aggregate or relative, only change prices via general equilibrium forces.

Households. We keep the household side as simple as possible and assume that there is a representative household who consumes goods from all sectors according to the following Cobb-Douglas intratemporal utility function:

$$\mathrm{U}(C) = \prod_{i=1}^{N} x_{0i}^{\kappa_i}.$$

Where x_{0i} denotes quantity consumed of good i and κ_i represents the expenditure share on good i with $\sum_{i=1}^{N} \kappa_i = 1$. Implicit in this functional form is the fact that the consumer substitutes across consumption goods in given period with an elasticity equal to unity. We denote this elasticity as $\sigma = 1$.The household also owns all factors in the economy and supply them inelastically at the given wages.

Intertemporal utility function is of the form:

$$(1-\beta)\log \mathrm{U}(C) + \beta \log \mathrm{U}(C^*).$$

The superscript * denotes variables in the future and no asterisk for present variables. Here, we already imposed the unitary intertemporal elasticity of substitution $\rho = 1$, which implies the log-utility across periods. The parameter β, on the other hand, governs the weight the consumer puts on future utility. For example, an increase in β signals a desire of the representative consumer to consume less in the present period and postpone its consumption to the future. This parameter will be a key in what follows since it allows us to model an aggregate demand shock in this economy in a simple way.

The household intertemporal budget constraint satisfies

$$PC + \frac{P^*C^*}{1+i} = \sum_{f=1}^{F} w_f L_f + \frac{I^*}{1+i},$$

where $PC = \sum_{j=1}^{N} p_j x_{0j}$ is consumption in the present, $P^*C^* = \sum_{j=1}^{N} p_j^* x_{0j}^*$ is consumption in the future, i is the nominal interest rate, $\sum_{f=1}^{F} W_f L_f$ are factor payments in the present and I^* is income in the future. Note that P and P^* are the price indices and reflect the cost of the consumption bundle in the present and future periods, respectively. Implicit in the intertemporal budget constraint is the assumption that the representative household can smooth consumption over periods and that it owns all factors in the economy. The household takes i, I^* and P^* as given when solving her maximization problems.

The key aspect of the household's maximization problem is how it allows us to model aggregate demand shocks. Specifically, we can express consumption as a function of an aggregate demand shifter and the price index (see Appendix 7.2 for details):

$$\log C = \log \zeta - \log P \implies \log P + \log C = \log \zeta$$

$$\log \zeta = \log\big((1-\beta)/\beta\big) + \log P^*C^* - \log(1+i)$$

where $\log \zeta = \log P + \log C$ is an aggregate demand shifter that coincides with nominal expenditure in the present period. A decrease in the discount factor (β), the interest rate (i) or an increase in nominal expenditure in the future ($P^* C^*$), generate an aggregate demand shifts in the current period such that, given the price index, consumption today goes up.[15]

Equilibrium conditions and CPI Inflation. CPI inflation is a consumption-weighted average of each price change:

$$d \log \mathrm{CPI} = \sum_{i=1}^{N} \kappa_i \, d \log p_i.$$

We can express CPI inflation as a function productivity shocks and factor price changes using the change in goods' prices that we derived for each sector i in Equation (4.1). Before doing so, we first need to define the equilibrium conditions in each market.

The goods market clearing condition is

$$y_i = x_{0i} + \sum_{j=1}^{N} x_{ji}\,, \tag{4.2}$$

which states that total gross output in sector i, y_i, goes to either final consumption good i, x_{0i}, or is used as an intermediate good by sector j, x_{ji}.

Given market clearing conditions along with firms' and households' first-order conditions, we can write CPI inflation as (see Appendix 7.2 for the full derivation):

$$\mathrm{d} \log \mathrm{CPI} = \boldsymbol{\Lambda}' \mathrm{d} \log \mathbf{W} - \boldsymbol{\lambda}' \mathrm{d} \log \mathbf{A}\,, \tag{4.3}$$

where $\mathrm{d} \log \mathbf{W}$ is a $N \times 1$ vector of wage changes and $\mathrm{d} \log \mathbf{A}$ is a $N \times 1$ vector of productivity changes, $\boldsymbol{\Lambda}$ represents the factor shares in the country GDP, and $\boldsymbol{\lambda}$ is a vector of Domar weights. Equation (4.3) shows that, up to a first-order approximation, inflation mimics the behavior of factor prices, weighted by their factor shares, and productivity changes, weighted by their Domar weights; i.e., the relative importance of each sector on aggregate value added.

In what follows, we assume there are no productivity shocks and therefore set $d \log \mathbf{A} = \mathbf{0}$ to focus on the role of changes in sectoral labor

[15] There are many potential factors that drove aggregate demand throughout the pandemic period, as this equation makes clear. As we describe in the quantitative section below, we ultimately back out the aggregate demand shock ζ to match observed CPI inflation. Therefore, we do not need to take a strong empirical stand on what is ultimately driving aggregate demand shifts.

and aggregate demand. Changes in CPI are then directly mapped to changes in factor prices:

$$\mathrm{d}\log \mathrm{CPI} = \mathbf{\Lambda}'\mathrm{d}\log \mathbf{W}\,. \tag{4.4}$$

This equation links aggregate inflation with wage inflation. As it is difficult to measure sectoral wages in the data, we follow Baqaee and Farhi (2022) and translate these wage changes to changes in factor usage in each sector and changes in aggregate demand as measured by nominal expenditure changes (nominal GDP).

To do so, note that share of factor f in value added is $\Lambda_f = \frac{w_f L_f}{GDP}$ and that $\sum_{f=1}^{N} \Lambda_f = 1$. In this definition, sector-specific labor L_f is an endogenous object that can change due to supply or demand forces and should not be confused with labor supply alone. Log-differentiating these expressions and replacing into Equation (4.5) we arrive at:

$$d\log \mathrm{CPI} = d\log \mathrm{GDP} - \mathbf{\Lambda}' d\log \mathbf{L}\,, \tag{4.5}$$

which is *Corollary 1* of Baqaee and Farhi (2022) under the assumption of no hand-to-mouth consumers and no productivity shocks. This equation tells us that inflation can be mapped to two key objects: (i) changes in nominal expenditures (= GDP in the closed economy), which capture changes in aggregate demand, and changes in equilibrium employment levels, which capture the supply side of the economy. As noted in Baqaee and Farhi (2022), it is irrelevant if these changes in equilibrium sectoral labor come from supply or demand forces: declines in these quantities are always inflationary.

In sections 4.2-4.3, we show how we map these objects to the data to compute inflation numbers for the Euro Area and the US. Before doing so, we provide the key intuition of the model using a two-sector example.

4.1.1 Two-sector stylized example

We provide a stylized example to highlight the main mechanisms in the model above. We use this example to incorporate the possibility of downward nominal wage rigidity and how it interacts with the production structure of the economy to create unemployment and, in doing so, how shocks can impact inflation.

Suppose we write Equation (4.1) for two goods (and therefore two factors) and assume there are no productivity shocks. This implies that the price equations for the two goods are

$$d\log p_1 = \tilde{\alpha}_{11}\mathrm{d}\log \mathrm{w}_1 + \tilde{\alpha}_{12} d\log \mathrm{w}_2, \tag{4.6}$$

$$d \log p_2 = \tilde{\alpha}_{21} d \log w_1 + \tilde{\alpha}_{22} d \log w_2 \,. \tag{4.7}$$

The tilde terms are network-adjusted exposures of each sector to changes in wages in the two sectors and satisfy $\tilde{\alpha}_{if} = \Psi_{ik}\alpha_{kf}$. Note that without multi-sectors and/or a production network, there would be no transmission of wages from one sector to the other. For example, if sector 1 does not use any input from sector 2 then $\tilde{\alpha}_{12} = 0$. The network exposures highlight the importance of the production network in transmitting wage changes from one sector to the other in a world where labor is sector specific. The production network thus acts as a mechanism that allows sectors to "demand" labor from every market, even though labor is sector-specific. Intuitively, by using intermediate goods from other sectors to produce, each sector is at the end demanding labor from other sectors. This does not mean that sector-specific labor supply in equilibrium is going to move due to the presence of intermediate inputs, this is fixed in our exercise. Rather, it implies that, although labor is sector-specific, some of this sector-specific labor will end up being used in other sectors i.e. being demanded by other sectors indirectly through intermediate input linkages.

Next, suppose that there is a relative shift in demand that increases the demand for sector 1 but decreases it for sector 2. To begin, we assume that wages are fully flexible, and thus focus on a scenario where labor supply is at its potential and all adjustment is through wages. Since sector 1 needs to increase production, it demands more of its factor, which puts upward pressure on wages in that sector. Sector 2 experiences a decrease in its demand and thus demands less of its sector-specific labor, putting downward pressure on its wage. Since both labor-specific supplies are fixed and production is constant returns to scale, we have off-setting impacts on wages in the two sectors: $\mathrm{d} \log w_1 = -\mathrm{d} \log w_2$.

The total effect of the wage changes on each sectoral prices depends on the sectors' relative network-adjusted exposure. To see this, use changes in wages to express everything in terms of the change in wage in sector 1 and equations (4.6) and (4.7):

$$d \log p_1 = (\tilde{\alpha}_{11} - \tilde{\alpha}_{12})\, d \log w_1,$$

$$d \log p_2 = (\tilde{\alpha}_{21} - \tilde{\alpha}_{22})\, d \log w_1.$$

Therefore, up to a first order, depending on whether each sector is relatively intensive in its use the labor of sector 1, prices can go up or down for a positive wage change in sector 1, i.e., we can have sectoral inflation or deflation. If each sector is intensive in sector 1's labor, prices in both sectors will increase. Otherwise, they will decrease. A similar logic applies to aggregate inflation since we can use Equation (4.4) to write

$$d \log CPI = (\Lambda_1 - \Lambda_2)\, d \log w_1.$$

Up to a first order, if the economy was relatively more intensive on labor of sector 1, a shift in demand towards sector 1 will translate into an aggregate price increase.

With *downward nominal wage rigidity* in both sectors, the story is different. The increase in demand for good 1 poses no problem: it raises wages in sector 1. However, since wages in sector 2 cannot go down, employment in sector 2 must fall, and thus at the current wage, demand does not equal supply in the labor market of sector 2. In the equation above, this means that we can write the change in the price of sector 1 and 2 as:

$$\mathrm{d} \log p_1 = \tilde{\alpha}_{11}\, \mathrm{d} \log w_1\,;\ \mathrm{d} \log p_1 = \tilde{\alpha}_{21}\, \mathrm{d} \log w_1.$$

Therefore, changes in sectoral prices arise from changes in wages in sector 1 because the increase in demand hit the factor supply constraint in that sector. In this scenario, we have inflation in both sectors regardless of whether the economy is relatively more intensive on the labor of sector 1 or not, as was the case in the fully flexible equilibrium. This economy then features inflation coupled with a decrease in labor quantity of sector 2. A single sector negative labor supply shock coupled with relative demand shifts can create country-wide inflation.

4.2 Data

We now discuss the data used to perform our quantitative exercise.

Nominal GDP. We use seasonally adjusted nominal GDP series from FRED for both the Euro Area (EUNNGDP) and the US (GDP). These are available at a quarterly frequency.

Total Hours Worked. We take the model literally and use changes in total hours worked as our sectoral supply shocks. We describe how we construct this data for the US and Euro Area, separately.

US data. We use Tables B1 and B2 provided by the Bureau of Labor Statistics. These tables contain information on employment and average weekly hours at a monthly frequency, respectively. Since hours in Table B2 are at a higher level of aggregation than those for employment in Table B1, we construct measures of L, by multiplying employment in a disaggregated sector by the hours of the aggregate sector. For example, the information sector contains six subsectors in Table B1 but it is only available as an aggregate information sector in Table B2. We thus multiply each subsector employment by the hours of the aggregate sector

in Table B2 to get a measure of total hours worked in each of the six subsectors separately.

Euro Area data. We collect data from EuroStat, which contains information on both hours and employment at the sectoral level for the entire Euro Area at a quarterly frequency. We follow the same procedure as in the US to construct changes in total hours worked in each sector.

Input-Output Matrices, Factor and Consumption Shares: $\mathbf{\Omega}$, $\mathbf{\Lambda}$, $\boldsymbol{\kappa}$

Since we assume two sector-specific factors in each sector in our quantitative exercise, labor and capital, we need to compute the respective share in nominal GDP of each of these objects. For our purposes, we only need to construct intermediate input expenditure, factor and consumption shares at some initial equilibrium.

US I-O matrix. We construct all objects for the US using the BEA Use-Before-Redefinitions producer prices tables for the year 2015. As it is typical with the input-output data from the BEA, we remove the following sectors to perform our analysis: government sectors (sectors 67 to 71 in the BEA IO Table), scrap, used and secondhand goods (sector 72) and noncomparable imports and rest-of-the-world adjustment (sector 73). This immediately provide us with enough information to compute the elements of $\mathbf{\Omega}$. We measure sectoral labor compensation as "compensation to employees" and sectoral capital compensation as "gross operating surplus." Our measure of nominal GDP for computing factor shares is simply the sum across sectors of these two items. This notion of nominal GDP coincides with a measure of gross value added at factor costs (Horvát and Webb, 2020) that we also use below when constructing the Euro Area numbers. For consumption shares, we use sectoral consumption of the 66 sectors that are also present in the BEA Input-Output table.

Euro Area I-O matrix. We compute the input-output matrix using the Inter-Country Input-Output (*ICIO)* database from the OECD that we already used in section 3 for the baseline year 2018. Also, we use this same dataset to construct consumption shares. We collapsed all Euro Area countries into one single entity to perform our analysis.

To construct the vector of factor shares, we use the 2018 OECD Structural Analysis (STAN) Database. It contains information on labor compensation and gross operating surplus at the sectoral level matching the ICIO sectoral classification for 17 out of the 19 countries (except for Cyprus and Malta) that composed the Euro Area. We add up these items within sectors across Euro Area countries to construct sectoral measures for the Euro Area. We then proceed as in the US and take nominal GDP to be the sum of these two items across sectors. We divide each item by this nominal GDP measure to get our factor shares at the sectoral level.

We also aggregate the sectoral input-output data in the US and the Euro Area into three sectors: durables, non-durables, and services, as in Section 3 to assess whether these different levels of aggregation matter for our findings.

Price indices and Nominal Wages. We collect three type of price indices: headline consumer price index, core (headline minus food and energy) consumer price index and the GDP deflator. For both the US and Euro Area data are sourced from FRED (https://fred.stlouisfed.org/). We also collect nominal wage indices for the US and Euro Area. For the US, we use the Employment Cost Index: Wages and Salaries, Private Industry Workers, also available from FRED (code ECIWAG). For the Euro Area, we use the wage part from the labour cost index available from EuroStat (code D11).

Calibrated shocks. The quantitative exercise requires three sets of shocks: (i) an aggregate demand shock (which maps to $d \log \zeta$), (ii) relative demand shocks (which maps to $d \log \kappa_i$'s), and (iii) sectoral supply shocks (which maps to $d \log L_i$).

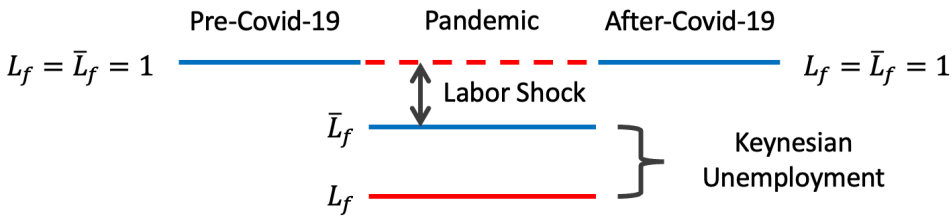
We back out the aggregate demand shock out from observed changes inflation and total hours worked using Corollary 1 of Baqaee and Farhi (2022) and (4.5) above. Rather than using changes in nominal GDP as a measure of the aggregate demand shock, we measure the aggregate demand shift as

$$d \log \zeta = d \log \mathrm{CPI} + \mathbf{\Lambda}' d \log \mathbf{L}. \tag{4.8}$$

In doing so, our aggregate demand shock is the part of inflation that is not explained by observed employment changes in the data. We adopted this backed-out strategy for two reasons. First, while our stylized model provides a one-to-one mapping between nominal GDP changes and $d \log \zeta$, it does so under several assumptions, and importantly the assumption that there are no hand-to-mouth consumers, which Guerrieri, Lorenzoni, Straub, and Werning (2022) and Baqaee and Farhi (2022) show generate a negative pressure on aggregate demand following drops in sectoral employment. Our backed out aggregate demand shock thus incorporates all other forms of aggregate demand shifts that are not necessarily accounted for by changes in nominal GDP alone in a parsimonious way. Second, this approach allows us to get more sensible numbers to match inflation observed in the data as nominal GDP changes are extremely large in the data, which are in part due to base effects and will imply unplausible large inflation numbers. Operationally, to feed this aggregate demand shock into the model, we assume that the discount factor β change is consistent with the observed changes in ζ.

We measure changes in relative demand as changes in sectoral consumption. In the US, we use information from the BEA, which contains information for the 66 sectors. For the Euro Area, unfortunately, the level of sectoral disaggregation is quite poor for the time frequency we require. For that reason, we use information on nominal consumption expenditure for three sectors: durables, non-durables and services as in Section 3, which are available from the OECD Quarterly National Accounts. This information is available for 10 out of the 19 countries that composed the Euro Area: Austria, Estonia, Finland, France, Germany, Ireland, Italy, Latvia, Luxembourg and the Netherlands.

Figure 11. Schematic of the labor shock

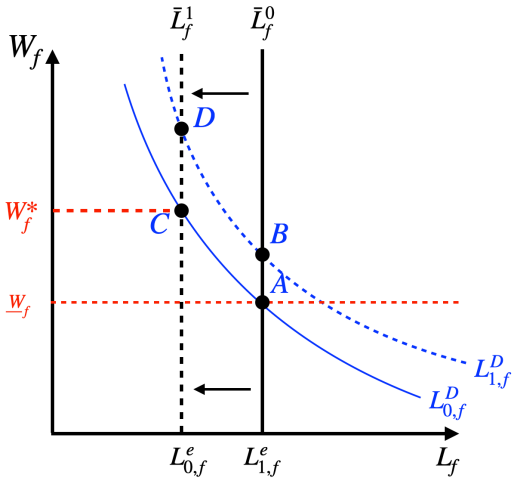


Notes: Before and after the shock, we assume there is full employment. During the pandemic, there is a decline in the potential labor, $\bar{L}_f$, and employment, L_f. For capital, we assume that the levels do not change throughout the pandemic.

Identifying supply shocks. For sectoral supply shocks, we take the Baqaee and Farhi (2022) model literally. Figure 11 shows the schematic of the labor shocks that we model. Before and after the pandemic, we assume that all factors are fully employed with employment normalized at 1. During the early phase of the pandemic, potential labor goes down to $\bar{L}_f$. But due to wage rigidities and demand changes, there is also Keynesian unemployment, potentially bringing down employment to L_f, below $\bar{L}_f$. This is the situation analyzed in Baqaee and Farhi (2022).

In contrast, the period we focus on (2019Q4-2021Q4) exhibits decreases in $\bar{L}_f$ with increases in demand for most sectors. We plot this situation in Figure 12 below. We assume that these changes occur starting from the initial pre-Covid-19 equilibrium where $L_f = \bar{L}_f = 1$. Since wages are only rigid downward, changes in equilibrium employment in those sectors where employment decreases exactly match changes in $\bar{L}_f$, movements from point A to point C in Figure 12. If this change in potential factor supply is also accompanied by an increase in labor demand, wages will move from point C to point D in Figure 12, ultimately increasing wages more.

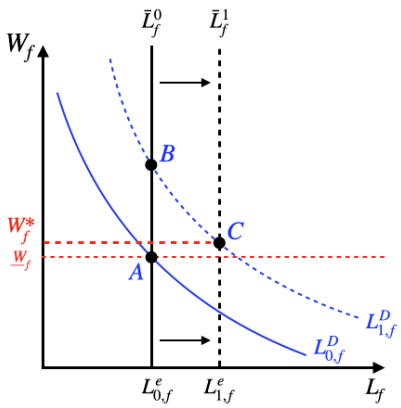
Figure 12. Decreases in potential labor supply and increase in labor demand



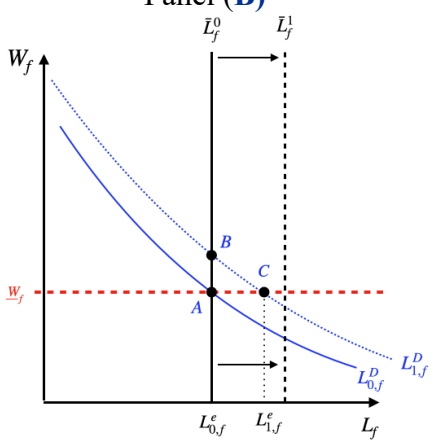
Things are more complicated in those sectors where equilibrium employment increases. The two possible outcomes can be seen in Figure 13. Panel (A) shows a situation where at the given lower bound on the wage, demand is larger than potential supply, i.e., there is excess demand at this wage. This puts upward pressure on the wage such as to eliminate this excess demand. Since the wage is not bounded above, it increases restoring the equilibrium in the market. Hence, we observe an increase in employment in the data bounded by the shift in potential labor supply.

Figure 13. Increases in potential labor supply and labor demand

Panel (A)



Panel (**B**)



Panel (B) plots a situation where at the lower bound of the wage, potential supply is larger than labor demand, i.e., there is excess supply. Contrary to the earlier case, the wage mechanism cannot clear the market: the excess supply requires a decrease in the wage to equate supply and demand. As this is not possible due to the downward nominal wage rigidity, this market features Keynesian unemployment due to insufficient demand. The new "equilibrium" is thus at point C.

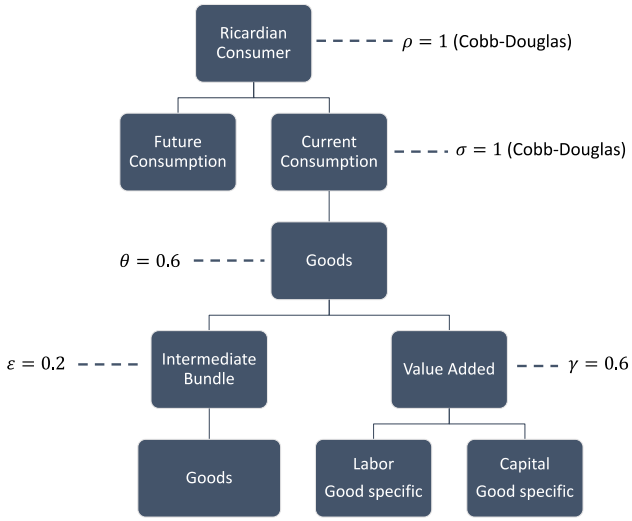
Note, however, that for employment to increase, the market needs a shift on potential labor supply as large as the increase in demand at the given lower bound of the nominal wage. Without the potential labor supply shift, the economy would move from point A to point B featuring the same employment level but higher wages. That is, we should observe no changes in equilibrium employment. Therefore, we assume that observed changes in employment in a situation like panel B comes from a shift in both supply and demand in the same amount and thus moving from point A to C features no Keynesian unemployment. Hence, we can also say that changes in potential labor supply and observed changes in employment exactly match in our analyzed period.

4.3 Quantitative Exercise

In this section, we perform our quantitative exercise. Recall that our model, based on Baqaee and Farhi (2022), is parsimonious and stylized though it will help us to separate the demand and supply shocks underneath the observed inflation. We calibrate the model to the 2019Q4-2021Q4 period. The key result coming out of the calibration exercise is that the sectoral heterogeneous nature of the Covid-19, coupled with immobile labor and complementarities, played a key role in driving aggregate inflation. Next, we describe how we calibrate the parameters and the shocks.

4.3.1 Calibrating Parameters

Figure 14. Schematic of the model



Notes: We assume that each node is an aggregation of the nodes below with the constant elasticity of substitution function with the corresponding elasticities.

As shown in Figure 14, each sector's production function is a nested CES aggregators of labor, capital, and intermediates good. As both labor and capital are sector-specific, there are three different layers of substitution in the model. The first one is between labor and capital. We set the elasticity of substitution between these factors to $\gamma = 0.6$. The second substitution is that between value added, the one produced using labor and capital, and intermediates. We set this elasticity to $\theta = 0.6$. Finally, there is substitution across intermediate goods. We set this elasticity to $\varepsilon = 0.2$. All these values are in line with estimates in the literature (see Atalay, 2017; Boehm, Flaaen, and Pandalai-Nayar, 2019; Oberfield and Raval, 2021; Oberfield, 2013), and are the same as the ones used in Baqaee and Farhi (2022), which points to complementarity in production across different inputs. For consumption, as we already highlighted in the previous sections, we assume a Cobb-Douglas intratemporal utility function. Thus, current consumption is an aggregate of goods with elasticity of substitution $\sigma = 1$. Finally, we set the intertemporal elasticity of substitution $\rho = 1$.

4.3.2 Calibrating Sectoral Demand and Supply Shocks

As the intertemporal elasticity of substitution equals one, households' expenditure shares do not depend on relative prices and hence we feed the sectoral consumption expenditures in the data to the model directly for sectoral demand shocks.

The model envisions the labor supply shocks affecting the quantity of potential labor used by each industry. As there is no labor mobility across sectors, changes in labor hours will match potential unemployment if the model is true, especially in the recovery phase, our focus. During the lockdown phase if there are sectors that are demand constrained and slack (Keynesian Unemployment) then reduction in potential labor hours will have no effect on observed outcomes.[16]

For sectoral variables such as labor hours worked and consumption, we calculate monthly/quarterly changes at time t relative to a baseline period t_0. For a given variable, X_t, we have

$$\Delta_{t-t_0} = \frac{1}{t-t_0} \sum_{\tau=t_0+1}^{t} \frac{X_\tau}{X_{t_0}} - 1,$$

where Δ_{t-t_0} is the observed average change in variable X between periods t and t_0. For total hours worked, we aggregate sectoral changes weighting by each sector labor compensation over total value added. For consumption, we aggregate using each sectoral consumption share over total consumption.

4.3.3 Calibrating Aggregate Demand Shocks

The model-specific concept for the aggregate demand shock is not changes in nominal GDP but rather a discount shock, we back this aggregate demand shock using equation (4.12) above. To calculate aggregate demand shock, we use observed inflation and labor hour changes in the specified period. We feed all these numbers into the model.

For nominal GDP, inflation, and nominal wages, we compute the change between the baseline and the period-end value as a growth rate. That is, we construct

$$g_{(t-t_0)} = \frac{X_t}{X_{t_0}} - 1,$$

where X can be nominal GDP or a CPI index, and $g_{(t-t_0)}$ is the growth rate between period t and t_0. We use two different price indices to compute inflation: headline CPI or core CPI (headline CPI minus food and energy).

[16] As Baqaee and Farhi (2022) focus on the lockdown phase, they set the labor supply shocks to zero in demand-constrained sectors.

4.3.4 Data

We show realized data in Table 4 below, measurements done as explained above, where we use all the sectors in Panel A and we use 3-sector aggregation as described in the data section in Panel B.

Table 4. Observed data, 2019Q4-2021Q4 (in % changes)

Panel A. All sectors						
	Nominal GDP	Cons.	Hours	Headline CPI	Core CPI	Nominal Wages
United States	10.64	-0.72	-2.14	8.47	7.16	7.85
Euro Area	4.42	-7.54	-1.48	4.69	2.86	5.01

Panel B. Three sectors						
	Nominal GDP	Cons.	Hours	Headline CPI	Core CPI	Nominal Wages
United States	10.64	-1.23	-2.58	8.47	7.16	7.85
Euro Area	4.42	-7.54	-1.93	4.69	2.86	5.01

Notes: We compute nominal GDP, inflation measures and nominal wages using period-end month and quarters. Nominal GDP measures for the US and Euro Area come from the Federal Reserve Economic Data (FRED) of the Federal Reserve bank of St. Louis. US inflation measures come from FRED, while Euro Area inflation measures come from EuroStat. Nominal wages for the Euro Area come from EuroStat, while nominal wages in the US come from. For consumption and total hours worked, we compute them as cumulative changes between the baseline and end period. In Panel A, for the US, we use information on all 66 sectors for both consumption and total hours worked. For the Euro Area, we use information on 45 sectors for total hours worked and for three sectors only for consumption. We aggregate sectoral consumption and total hours worked using consumption shares and labor shares, respectively. In Panel B, we aggregate to the durable, non-durable, and service sectors. For consumption and total hours worked, we compute them as cumulative changes between the baseline and end period.

When we focus on the entire period from 2019Q4 to 2021Q4, both in the US and Euro Area, nominal GDP, wages, and inflation go up, whereas consumption and labor hours go down.

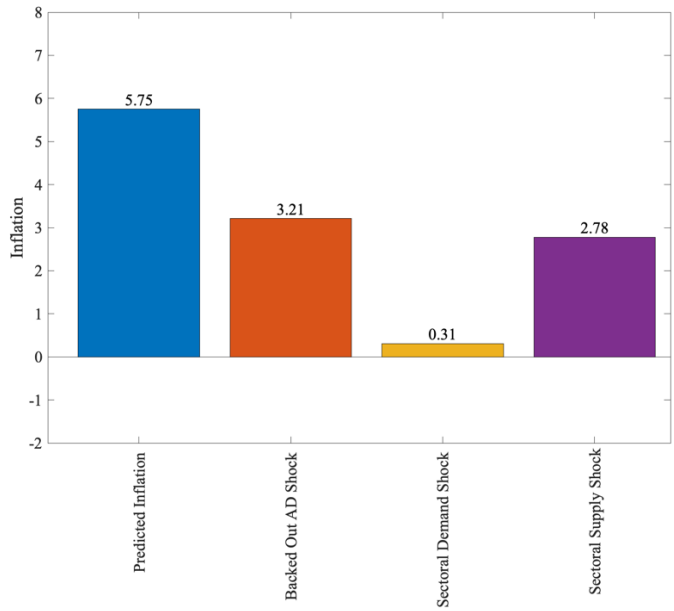
4.3.5 Predicted Inflation

Euro Area Calibration. Figure 15 below shows the baseline calibration for predicted inflation for the Euro Area based on 45 sectors of data for labor hours, whereas consumption data is only for 3 sectors. Predicted inflation of 5.75 percent includes all the shocks (sectoral demand, supply, aggregate demand) and is largely due to the aggregate demand shock but the sectoral labor supply constraint plays still plays a large role. Put it differently, without the negative sectoral labor supply shock, inflation would have been only 3.21 percent. This is also reflected in the fact that out of the 45 sectors in our model that consider all shocks, 34 of them are supply constrained.

Figure 16 examines the robustness of aggregating all labor hours to 3 sectors from 45 and shows the same result as the 45-sector calibration: inflation would have been 2.76 instead of 4.81 if we only considered aggregate demand shocks. Therefore, regardless of the level of

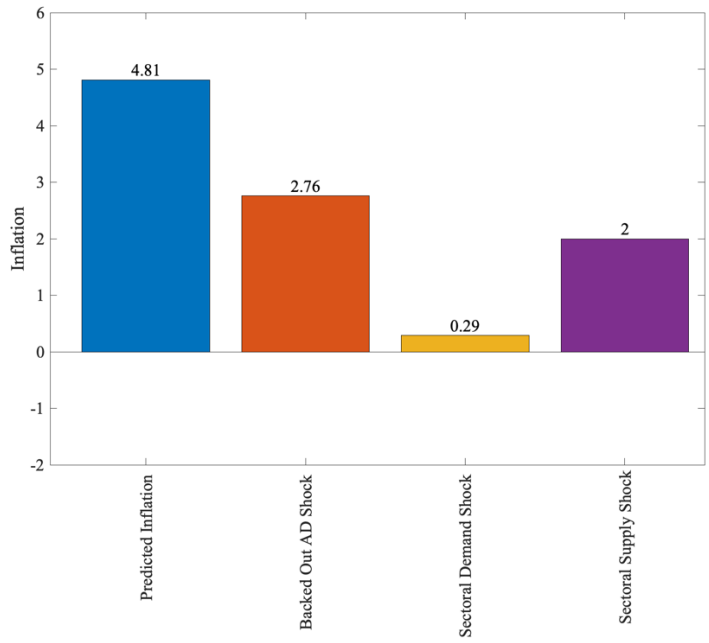
aggregation, sectoral supply bottlenecks played a key role in explaining Euro Area inflation over the 2019Q4-2021Q4 period. Further, notice that the fit of the model is also very good during this period as the observed inflation in the Euro Area is 4.69.

Figure 15. Euro Area 45 Sectors: 2019Q4-2021Q4



Notes: The first (blue) bar shows predicted inflation considering all shocks (demand and supply). The second (orange) bar considers the aggregate demand shift only. The third (yellow) bar uses only sectoral demand shocks. Finally, the fourth (purple) bar uses sectoral supply shocks.

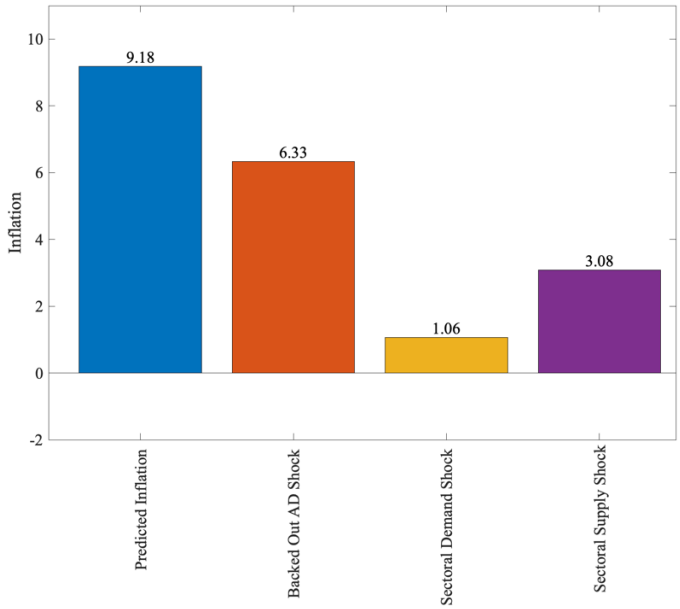
Figure 16. Euro Area 3 Sectors: 2019Q4-2021Q4



Notes: The first (blue) bar shows predicted inflation considering all shocks (demand and supply). The second (orange) bar considers the aggregate demand shift only. The third (yellow) bar uses only sectoral demand shocks. Finally, the fourth (purple) bar uses sectoral supply shocks.

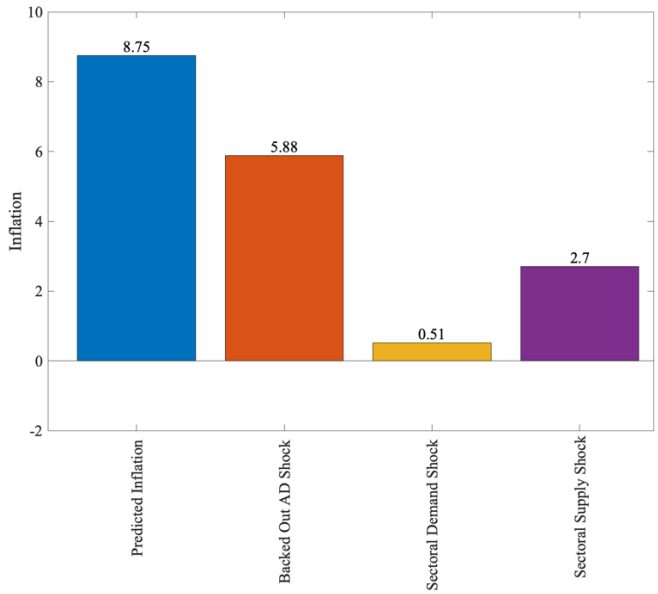
US Calibration. Figure 17 replicates the same exercise for the US, using the same set of 66 sectors as in Baqaee and Farhi (2022). Supply constraints also play a role here, with a predicted inflation of 9.18 instead of 6.33 due to sectoral labor supply constraints. As shown in Figure 18, using only 3 sectors does not change the results. The observed inflation in the US during this period is 8.47. However, in contrast to the Euro Area results, aggregate demand shocks play a greater role in explaining US inflation. Consequently, most sectors in the US are supply-constrained in our main specification (blue bar): 58 out of 66 sectors are supply-constrained. This result is consistent with Gourinchas, Kalemli-Özcan, Penciakova, and Sander (2021), where expansionary fiscal policy (an aggregate demand shock) increases the share of sectors classified as supply constrained. Further, while the relative contribution in the change of sectoral demand shifts (the yellow bars) is small both for the Euro Area and US calibrations, these shifts play a relatively larger role in explaining observed US inflation (roughly double the importance for the US).

Figure 17. US 66 Sectors: 2019Q4-2021Q4



Notes: The first (blue) bar shows predicted inflation considering all shocks (demand and supply). The second (orange) bar considers the aggregate demand shift only. The third (yellow) bar uses only sectoral demand shocks. Finally, the fourth (purple) bar uses sectoral supply shocks.

Figure 18. US 3 Sectors: 2019Q4-2021Q4



Notes: The first (blue) bar shows predicted inflation considering all shocks (demand and supply). The second (orange) bar considers the aggregate demand shift only. The third (yellow) bar uses only sectoral demand shocks. Finally, the fourth (purple) bar uses sectoral supply shocks.

Sensitivity Analysis. Thus far, our results suggest that the model, although close to observed data, overpredicts inflation. We now conduct sensitivity analysis on these results using the more disaggregated data for the US and the Euro Area, i.e., 66 and 45 sectors, respectively.

We consider three scenarios where we vary the degree of substitutability across factors (labor and capital, γ), across intermediate goods (ϵ), and between factors and intermediates goods (θ). This way, we can study in a transparent manner how complementarities can affect our earlier results. In all our exercises, we maintain the consumer's side untouched, meaning the consumer still has Cobb-Douglas preferences both within and across periods.

Table 5 shows inflation numbers under the three different scenarios and a different set of shocks, as we did in the earlier section. The Baseline column reproduces our earlier inflation numbers. Recall that that scenario uses an elasticity of substitution of 0.6 between factors of production, 0.6 between value-added and intermediate goods, and 0.2 across intermediate goods. In the Cobb-Douglas column, we set these three different elasticities to be equal to 1. Finally, the Leontief model set these three elasticities to equal 0.2.

Quantitatively, aggregate demand shocks are more important in the US than in the Euro Area in all three cases. Indeed, they account for around two-thirds of inflation in the US, while only half of inflation in the Euro Area. Instead, sectoral supply shocks in the US account for around one-third of inflation and the other half of inflation in the Euro Area. Sectoral

demand shocks alone play a minor role in driving inflation, which is more limited in the Euro Area than in the US.

Moving from a model with high complementarity (Leontief column) to a model with no complementarity (Cobb-Douglas column), reduces overall inflation in both the US and the Euro Area. The behavior of sectoral supply shocks mainly drives this result. The intuition for this result is as follows. When a negative sectoral supply shock occurs, and labor and capital are complements in production, the rise in wages that follows the sectoral supply shock reduces the own sector's capital demand. Since sector-specific capital quantity is fixed and its price is fully flexible, this reduces its price, with this effect being stronger when elasticities of substitution are lower. When coupled with the respective shares of each factor on value-added, this increase in wages and decrease in capital prices implies a higher level of inflation under complementarities. The decline in capital demand is absent when goods are neither gross complements nor gross substitutes, as in the Cobb-Douglas case, which explains why inflation is lower under this scenario.

The earlier discussion suggests that inflation under our baseline scenario, which features complementarities, should mainly come from sectoral wage inflation and less from capital price inflation. Indeed, a simple decomposition of the model when all shocks are fed in shows that wage inflation is 9.14 percent and capital price inflation is 1.53 percent in the Euro Area. In comparison, it is 11.75 and 5.32 percent in the US, respectively. When compared to the actual data on nominal wage growth during this period (last column in Table 4), we can see that we are over predicting them, as they were 7.85 percent in the US and 5.01 percent in the Euro Area. Yet, the model's notion of "factors" is broader than we have in the data. As a result, factor prices in the model may capture other issues that we do not explicitly model. These include things such as markups, other sector-specific factors (e.g., human capital), or open economy aspects that we do not formally include in these exercises but briefly explore in our open economy exercise below.

On the other hand, sectoral demand shocks do not have a clear monotonic relationship with the elasticities of substitution. This result is expected, however, as by construction sectoral demand shocks shift demand from some sectors to other sectors while keeping aggregate expenditure fixed. In doing so, it pressures prices downwards in some sectors and upward in others. Ultimately, it is a quantitative question of how much inflation they generate at the aggregate level. In our case, not so much.

Finally, aggregate demand shocks generically raise inflation in the same proportion in all cases as prices of goods and factors are fully allowed to adjust upwards. Therefore, elasticities of substitution have no role on the pass-through from aggregate demand shocks to inflation.

Table 5. Inflation under Different Substitution Patterns

Panel A. United States			
	Calibration Model		
Shocks	Cobb-Douglas (1)	Baseline (2)	Leontief (3)
All	8.93	9.18	9.68
Aggregate Demand	6.33	6.33	6.33
Sectoral Demand	1.01	1.06	0.77
Sectoral Supply	2.70	3.08	3.56

Panel B. Euro Area			
	Calibration Model		
Shocks	Cobb-Douglas (1)	Baseline (2)	Leontief (3)
All	5.40	5.75	6.16
Aggregate Demand	3.21	3.21	3.21
Sectoral Demand	0.28	0.31	0.22
Sectoral Supply	2.56	2.78	3.04

Notes: This table shows overall inflation under three different specifications of the production function. The baseline model, column (1), corresponds to the same model used in the previous section, which uses a elasticity of substitution of 0.6 between factors of production, 0.6 between value added and intermediate goods, and 0.2 across intermediate goods. The Cobb-Douglas model of column (2) sets all these elasticities of substitution to equal 1. The Leontief model, column (3), sets all these elasticities to equal 0.2. We use 66 sectors for the US and 45 sectors for the EA and the period 2019Q4-2021Q4 to compute these numbers in the calibrations.

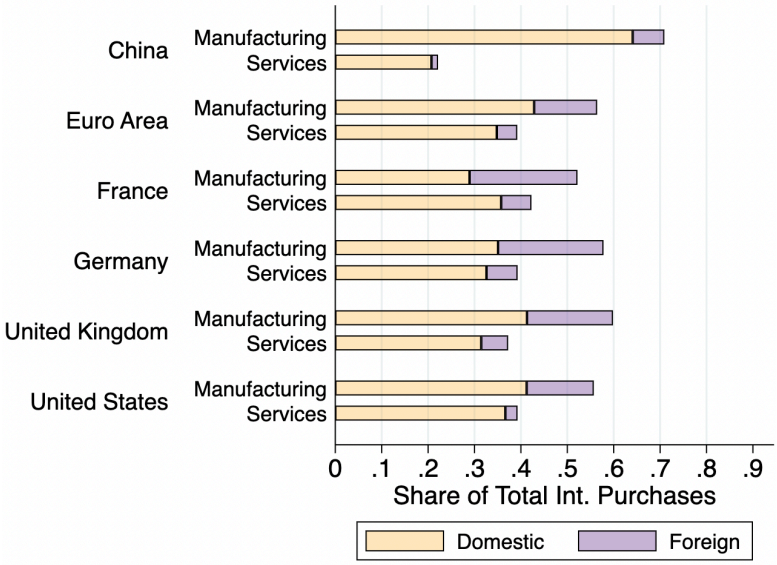
To summarize, this section shows the important role that different shocks played in driving inflation throughout the Covid-19 period to the end of 2021. Crucially, both demand and supply shocks played quantitatively important roles in driving inflation in both the Euro Area and the US. However, the relative importance of aggregate demand and sectoral supply (supply chain bottleneck) shocks differed across the two regions. Whereas aggregate demand shocks appear to have played a larger role in explaining US inflation, supply chain bottlenecks play a larger role in explaining European inflation. This finding is perhaps not surprising given the differing nature of government support during the crisis across the two regions as well as the consumption behavior of consumers in the two regions. Further, the impact of shocks and bottlenecks spilled over across countries, which may have also played an important role in explaining observed inflation. We tackle this issue in the following section.

5 Inflation under Supply Chain Bottlenecks: International Linkages Model

Following Çakmaklı, Demiralp, Kalemli-Ozcan, Yesiltas, and Yildirim (2022) we extend the closed-economy model presented in Baqaee and Farhi (2022) to analyze the effects of the global inter-industry linkages on inflation observed in Euro Area. We use three sectors (durables, non-durables and services) and three regions (Euro Area, US and Rest of the World).

We first illustrate how different the shares of sourced inputs domestically and from abroad can be. Figure 19 shows input sourcing for the manufacturing sector in different countries. The height of each bar shows how much each sector below the bars account of total intermediate purchases by the manufacturing sector. We then split these shares into what comes from domestic and foreign sources. For example, take China: 70 percent of manufacturing intermediate purchases comes from the manufacturing sector (height of the first bar, the sum of the purple and yellow bars). Of this 70 percent, near 65 percentage points (yellow bar) come from domestic manufacturing

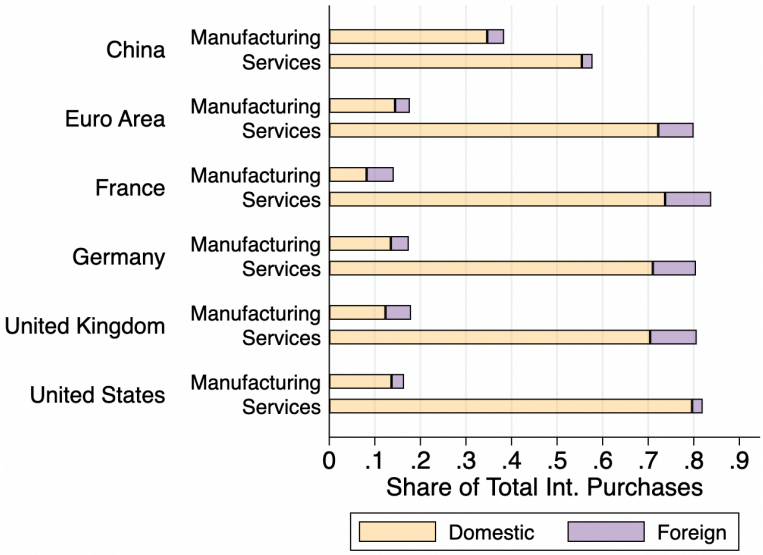
Figure 19. Domestic and Foreign Content of Intersectoral Trade (%, Total Intermediate Purchases): Manufacturing



Notes: Based on ICIO Tables Year 2018. Sectors are classified as: Agriculture, Manufacturing, Services and Others.

Figure 20 shows the same figure for services. For example, take China: 40 percent of services intermediate purchases comes from the manufacturing sector (height of the first bar, the sum of the purple and yellow bars). Of this 40 percent, near 37 percentage points (yellow bar) come from domestic manufacturing.

Figure 20. Domestic and Foreign Content of Intersectoral Trade (%, Total Intermediate Purchases): Services



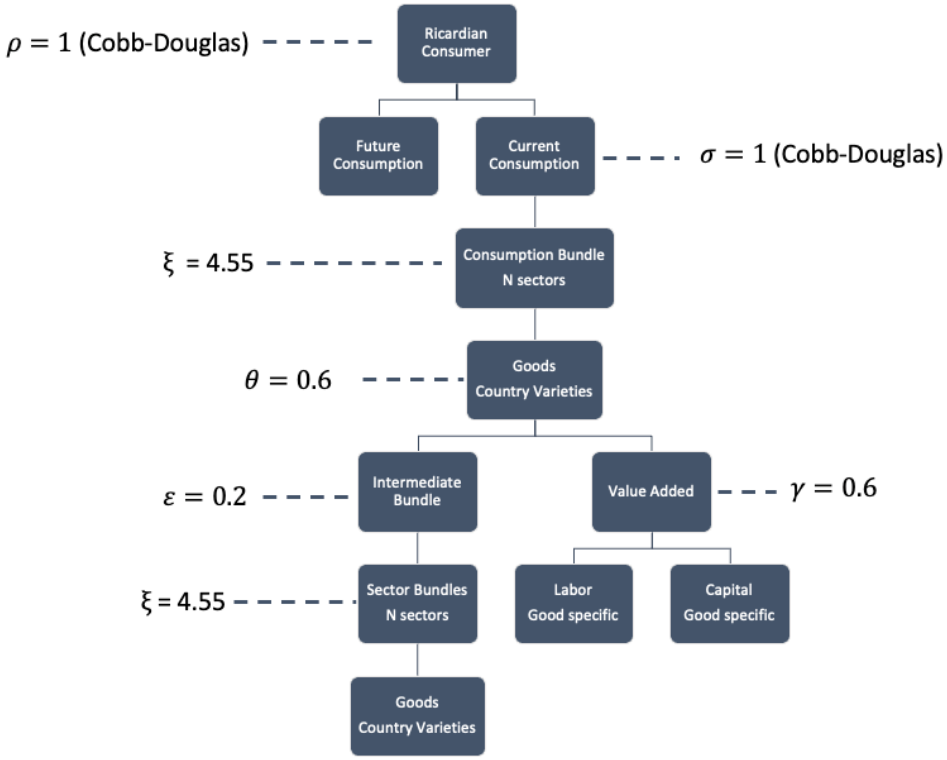
Notes: Based on ICIO Tables Year 2018. Sectors are classified as: Agriculture, Manufacturing, Services and Others.

The derivation of the full model is in Appendix Section 7.2, which we refer the interested reader to. For the sake of saving space, we note that the model structure is like the closed-economy model of Section 4, though now with households and firms being able to source goods from abroad.

Figure 21 shows the structure of our model with international linkages. In contrast to the closed economy-model outline in Figure 14, we have additional layers of sector bundles in both the consumption and production sides. Specifically, each sectoral bundle is made of goods from sourced from different countries, and the sectoral bundles are then aggregated to a final consumption or production goods. These elasticities for this sectoral aggregation are set to the aggregate elasticity of 4.55 reported by Caliendo and Parro (2015). The rest of the parameters are the same as the closed-economy case.

Our multi-country framework assumes that countries all have balanced trade. In Section 7.3 of the Appendix, we show how we operationalize this assumption to match the observed input output linkages by adjusting observed value-added levels (similar to inventory wedge approach in trade literature). Finally, our model incorporates exchange rate dynamics by assuming that central banks are inflation targeters. With this assumption, we are able to model a country-specific downward nominal wage rigidity.

Figure 21. Schematic of the model.



Notes: We assume that each node is an aggregation of the nodes below with the constant elasticity of substitution function with the corresponding elasticities.

In our multi-country framework, we use the same labor shocks, demand shocks and aggregate demand shocks for the Euro Area and the US as in Section 4. We aggregate all the data for "Rest of the World" into a fictive country that we denote by RoW. We create the sectoral demand shock for RoW by aggregating the demand changes that we used in Section 3 for countries outside the Euro Area and US. For the aggregate demand shock for the Euro Area and US, we use the same backed out values reported in Figures 16 and 18. For the RoW, we calibrate its GDP change over the period to 2 percent so that the predicted inflation in the Euro Area falls between the values reported in Figures 15 and 17. For the labor shock, we do not have data on the sectoral labor hour changes for RoW. Hence, we use the population weighted Oxford stringency index (Hale et. al, 2021) and compare it to the labor declines obtained for the US and Euro Area. Given this strategy, we calculate a 2.5 percent fall in labor for RoW over 2019Q4-2021Q4. Finally, we build the labor shares of value-added for each sector from Baqaee and Farhi (2021), who obtain these values from the World Input Output Database.

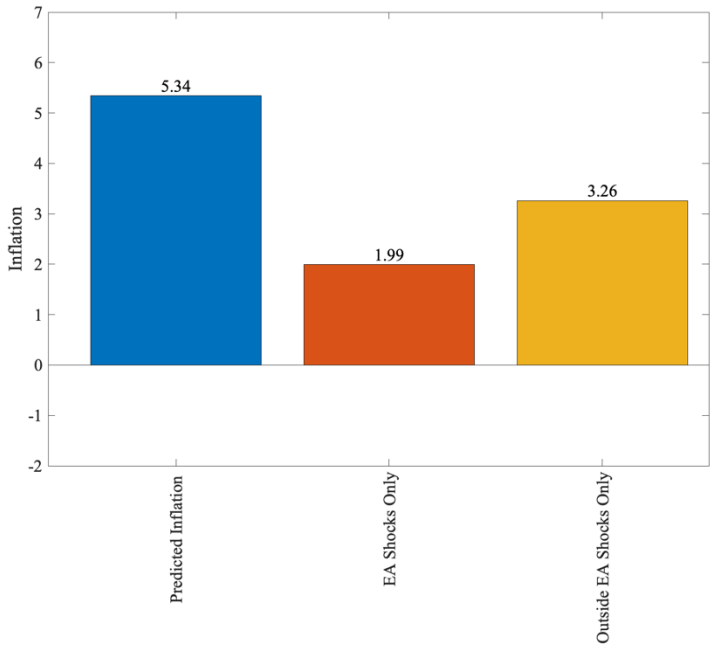
Using these values, we run three alternative scenarios. In **scenario 1**, we run the model with the full shocks present in Euro Area, the US and the RoW. In **scenario 2**, we only apply shocks to the Euro Area, and in

scenario 3 we do the opposite and only apply shocks to outside of the Euro Area. For all scenarios, we compare the inflation implications.

Figure 22 shows the results for Euro Area inflation for the calibration of the three scenarios. Scenario 1 (blue bar) gives a similar inflation level to the closed-economy counterparts presented in Section 4. In scenario 2 (red bar), where we assume shocks are present only in the Euro Area, the predicted inflation goes down by more than 3.3 percent. This fall in predicted inflation arises because domestic goods demanded by Euro Area households can be substituted with the goods produced abroad, and these regions (the US and RoW) have not been hit by expansionary demand shocks or contractionary labor supply shocks, thus keeping prices of their goods (which are reflected in Euro Area import prices) lower than domestic prices in the Euro Area. In scenario 3, even though there are no shocks present in the Euro Area, shocks abroad increase prices faced by Euro Area households significantly, resulting an inflation level of 3.26 percent in the Euro Area.

Therefore, given that the Euro Area is quite open to trade with the rest of the world and was also subject to a relatively smaller domestic aggregate demand shock, the impact of foreign demand and supply shocks played a larger role relative to domestic shocks in explaining observed inflation over the 2019Q4-2021Q4 period.

Figure 22. Euro Area Inflation over 2019Q4-2021Q4 in a 3 Sectors-3 Countries Model with I-O Linkages: Scenario Analysis



Notes: The first (blue) bar shows predicted inflation considering all shocks in the US, Euro Area and Rest of the World (demand and supply). The second (orange) bar considers the case when all shocks occur only in the Euro Area shutting down shock in the US and Rest of the World. The third (yellow) bar uses feed in shocks for the Rest of World and the US shutting down any shocks occurring starting in the Euro Area.

6 Conclusion

Our results point to several factors underlying the persistent inflation that the Covid-19 pandemic has generated. Interestingly, the relative importance of these factors varies across countries, with marked differences in the Euro Area and United States. While global supply bottlenecks have played a key role in generating inflation across all countries, our analysis shows that the relative importance of these negative supply shocks (domestic and foreign) is larger for the Euro Area than the US, where aggregate demand shocks played a comparatively greater role in explaining the observed inflation between 2019Q4-2021Q4. These findings present a mixed view on the potential potency of monetary policy in taming current inflation. While our model-based calibrations imply that a contraction in aggregate demand will help dampen inflation, there will remain upward pressure on price growth as long as global supply bottlenecks persist.

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Appendix

6.2 Effects of sectoral shocks on trade elasticities

To better understand the role of the sectoral composition of demand, Table A1 utilizes the model framework to perform several "counterfactual" exercises for the GFC and Covid-19 periods. We feed in observed consumption changes for all countries but only for a subset of sectors. Column (1) labeled 'Dur.' feeds in observed changes only for consumption growth in the Durables sector while setting other sectors consumption growths to be zero, column (2) labeled 'Dur. + NDur.' feeds in observed changes for consumption growth in the Durables and Non-Durables sectors while setting the Services sector consumption growth to be zero, while column (3) labeled 'Serv.' feeds in observed changes only for consumption growth in the Services sector while setting other sectors consumption growths to be zero.

Table A1. Composition Effect: Great Financial Crisis and Covid-19 Pandemic

	Panel A. Great Financial Crisis Collapse						**Panel B.** Covid-19 Pandemic Collapse					
	Imports			Exports			Imports			Exports		
	Dur. (1)	Dur.+NDur. (2)	Serv. (3)	Dur. (4)	Dur.+NDur. (5)	Serv. (6)	Dur. (1)	Dur.+NDur. (2)	Serv. (3)	Dur. (4)	Dur.+NDur. (5)	Serv. (6)
United States	4.36	3.99	0.55	2.63	2.31	0.85	1.45	1.61	0.55	10.23	8.90	0.73
Euro Area	3.17	2.69	0.59	7.10	4.47	0.71	2.49	2.40	0.65	2.24	2.14	0.55
United Kingdom	4.82	4.17	0.69	2.46	1.84	0.09	4.57	4.12	0.71	2.10	2.49	0.42
World	3.01	2.90	0.61	3.01	2.90	0.61	2.90	2.37	0.64	2.90	2.37	0.64
	Panel C. Great Financial Crisis Recovery						**Panel D.** Covid-19 Pandemic Recovery					
	Imports			Exports			Imports			Exports		
	Dur. (1)	Dur.+NDur. (2)	Serv. (3)	Dur. (4)	Dur.+NDur. (5)	Serv. (6)	Dur. (1)	Dur.+NDur. (2)	Serv. (3)	Dur. (4)	Dur.+NDur. (5)	Serv. (6)
United States	3.71	3.44	0.55	4.46	4.18	0.77	4.73	4.08	0.55	3.32	2.95	0.75
Euro Area	0.89	1.31	0.59	6.51	4.72	1.02	2.17	2.13	0.67	2.52	2.45	0.73
United Kingdom	2.85	2.60	0.69	4.06	3.69	0.16	3.95	3.57	0.71	2.59	2.68	0.42
World	2.29	2.01	0.60	2.29	2.01	0.60	2.93	2.51	0.59	2.93	2.51	0.59

Notes: This table presents model-based elasticities for real imports and real exports viz. real GDP, where we plug in observed consumption growth rate for all countries across a subset of sectors during the Great Financial Crisis and the Covid-19 Pandemic: (i) 'Dur.': the Durables sector only; (ii) 'Dur.+NDur.': the Durables and Non-Durables sectors only; 'Serv.': the Services sector only. All other sectors assume zero consumption growth. Panels A presents results for the GFC Collapse period (2008Q2-2009Q2), Panel B presents results for the Covid-19 Collapse period (2019Q2-2020Q2), Panels C presents results for the GFC Recovery period (2009Q2-2010Q2), and Panels D presents results for the Covid-19 Recovery period (2020Q2-2021Q2). Columns (1)-(3) present results for import elasticities and column (4)-(6) for export elasticities.

Panels A and B of Table A1 compare the trade collapse of the two episodes, while Panels C and D compare the recovery year. First, looking at within a Panel, whether it be for a fall or rebound in trade, it is evident that trade elasticities are always larger when applying the growth rates only to the goods' sectors, and particular to Durables goods. This is true both for the GFC and the Covid-19 health shock and reflects several key points highlighted by Bems, Johnson, and Yi (2010). First, goods are more tradable and thus changes in final demand in these sectors will have a larger direct impact on trade than a change in final demand for services.

Second, given global production linkages, this change in goods demand will amplify across borders given intermediates trade thus increasing trade further as the volume of trade is measured in gross output terms rather than value added. This in turn will lead to a larger elasticity viz. GDP, which is measured in value added. Finally, given that Services contribution to GDP (country's value added in Table 1) is much larger than its contribution to exports or imports, an equally sized change in consumption of services will have a larger impact on GDP than trade, while the opposite holds true for the goods' sectors.

Except for decompositions where we only allow consumption to change in the Services sector (columns (3) and (6)), results vary across countries when turning to comparing the decompositions across the two crises. Looking at the import elasticities during the Collapse periods in Panels A and B, we see these elasticities tend to be smaller in columns (1) and (2) during the Covid-19 crisis than the GFC trade collapse. This difference is particularly noticeable for the US relative to Euro Area or other countries and reflects the fact that household consumption of goods (durables goods in particular) did not fall as much during Covid, which in turn implied a smaller transmission across countries being picked up in international trade via production linkages. A similar story holds when looking at exports in columns (4) and (5). One exception is that the implied export elasticity for the US was larger during Covid, which reflects a fall in the demand for US goods by the Euro Area – this result follows from shocking only Euro Area consumption of goods in an unreported exercise.

The results for the Recovery periods in Panels C and D present a different picture than the Collapse decompositions. Looking at import elasticities across countries, we see that they are larger during the Covid-19 recovery than the GFC. This fact reflects the extremely fast recovery of goods' consumption starting in mid-2020. This final demand change in goods is amplified to total imports via imports given domestic and foreign production linkages, while its impact on GDP is relatively muted. Note further that the elasticity is more than twice as large for the US than for the Euro Area, as US consumption of goods surged. Meanwhile, export elasticities are in fact larger during the GFC than the Covid-19 period for the US and Euro Area, which reflects the depressed demand for these countries' growth from the rest of the world in the Recovery period as other countries' demand lagged due to delayed vaccination and less of boom for consumption goods. For example, European demand for US consumption goods remained muted, while demand for Euro Area countries' durables (e.g., Germany) also remained muted. Meanwhile, Americans' demand for goods from around the world help prop up exports for the world.

6.3 Closed-economy model details

Firm's cost minimization problem. Under perfect competition, firms take good and factor prices as given and solve the following cost minimization problem:

$$\min_{L_i, \mathbf{X_i}} \sum_{j=1}^{N} p_j\, x_{ij} + w_i L_i$$
$$s.t.$$
$$A_i \mathrm{G}(L_i, \mathbf{X}_i) \geq \bar{y}_i,$$

where $\bar{y}_i$ is a given level of output produced in sector i. In equilibrium, sectoral good prices equal sectoral marginal costs:

$$p_i = MC_i(A_i, w_i, \mathbf{p}). \tag{A1}$$

Given constant returns to scale production, a firm's marginal costs are a function of productivity in that sector, A_i, the wage of the factor it employs, w_i, and a vector of intermediate good prices, denoted by $\mathbf{p}$.

Log differentiating Equation (A1) implies that the log change in sector i's price is related to technology and factor and intermediate goods price changes by

$$d \log p_i = \alpha_i\, d \log w_i + \sum_{j=1}^{N} \Omega_{ij}\, d \log P_j - d \log Z_i, \tag{A2}$$

where

$$\Omega_{ij} = \frac{p_j x_{ij}}{p_i y_i}, \text{and } \alpha_i = \frac{w_i L_i}{p_i y_i}.$$

Ω_{ij} represents sector i's expenditures on goods from sector j as a share of sector i output, which is referred to as an input-output coefficient. A_i is the expenditure on the specific factor by sector i again as a fraction of its total output and captures the value-added share.

Using equation (A2) to solve for prices as a function of factor prices and productivity changes yields

$$\mathrm{d} \log p_i = \sum_{k=1}^{N} \Psi_{ik}\, \alpha_k\, \mathrm{d} \log w_k - \sum_{k=1}^{N} \Psi_{ik}\, \mathrm{d} \log A_k\,. \tag{A3}$$

Household's problem. The household maximizes utility both intertermporally and intratemporally as follows.

Intertemporal Problem. Utility maximization implies the following Euler equation

$$C = \frac{(1-\beta)}{\beta}\frac{P^*C^*}{P}\frac{1}{1+i}$$

Taking logs and rearranging the terms, we can express consumption as a function of an aggregate demand shifter and the price index:

$$\log C = \log\zeta - \log P \implies \log P + \log C = \log\zeta$$

$$\log\zeta = \log\big((1-\beta)/\beta\big) + \log P^*C^* - \log(1+i)$$

where $log\,\zeta = \log P + \log C$ is an aggregate demand shifter and coincides with nominal expenditure in the present period. A decrease in the discount factor (β), the interest rate (i) or an increase in nominal expenditure in the future (P^*C^*), generate an aggregate demand shifts in the current period such that, given the price index, consumption today goes up.

Intratemporal Problem. Taking goods prices p_i, and total expenditure in the present PC as given, the consumer maximizes the intratemporal Cobb-Douglas utility function choosing consumption quantities C_i subject to the intratemporal budget constraint

$$\sum_{i=1}^{N} p_i x_{0i} = PC$$

Solving the maximization problem simply gives

And thus, the representative consumer spends a fraction κ_i of its total expenditure on good i.

$$p_i x_{0i} = \kappa_i\, PC \tag{A4}$$

Equilibrium conditions and CPI Inflation. The full derivation of CPI inflation follows from combining first-order conditions and market clearing conditions. To begin, it is convenient to rewrite the market clearing condition (4.2) in terms of observables. Multiplying Equation (4.2) by the price of good i and dividing by nominal gross domestic product, GDP, we arrive at

$$\frac{p_i y_i}{GDP} = \frac{p_i x_{0i}}{GDP} + \sum_{j=1}^{N} \frac{p_i x_{ji}}{p_j y_j} \frac{p_j y_j}{GDP}. \tag{A5}$$

Next, we define a sector's Domar weight:

$$\lambda_i = \frac{p_i y_i}{GDP},$$

which measures the importance (size) of each sector for total value added of the economy. Further, note that we have written the summation in Equation (4.3) as a function of a sectors' input-output coefficients, $\Omega_{ji} = \frac{p_i x_{ji}}{p_j y_j}$. We use these expressions, along with Equation (4.2) to re-write Equation (A5) as:

$$\lambda_i = \kappa_i + \sum_{j=1}^{N} \Omega_{ji} \lambda_j. \tag{A6}$$

We next stack the N market clearing conditions into a vector-form and invert the system to arrive at

$$\boldsymbol{\lambda}' = \boldsymbol{\kappa}' \boldsymbol{\Psi} \tag{A7}$$

where

$$\boldsymbol{\Psi} = (\mathbf{I} - \boldsymbol{\Omega})^{-1},$$
$$\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \dots, \lambda_N)',$$
$$\boldsymbol{\kappa} = (\kappa_1, \kappa_2, \dots, \kappa_N)'$$

$\boldsymbol{\Psi}$ is the Leontieff inverse matrix, which is a $N \times N$ matrix that records the direct and indirect exposure of each sector to other sectors in the economy via intermediate input usage.

Factor shares in this framework can be written as

$$\Lambda_f = \frac{w_f L_f}{GDP} = \alpha_{i=f} \lambda_{i=f},$$

which we can stack into the $N \times 1$ vector

$$\boldsymbol{\Lambda} = diag(\boldsymbol{\alpha})\boldsymbol{\lambda},$$

and the diagonal matrix

$$diag(\boldsymbol{\alpha}) = \begin{pmatrix} \alpha_1 & 0 & 0 & \cdots & 0 \\ 0 & \alpha_2 & \cdots & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & \ddots & \alpha_N \end{pmatrix}.$$

records the factor usage of each sector.

Weighting price changes with $\boldsymbol{\kappa}'$, the sectoral consumption shares, and using Equations (4.1) and (A7), we can write CPI inflation as

$$\mathrm{d}\log \mathrm{CPI} = \boldsymbol{\Lambda}'\mathrm{d}\log \mathbf{W} - \boldsymbol{\lambda}'\mathrm{d}\log \mathbf{A}\,, \tag{A8}$$

where $\mathrm{d}\log \mathbf{W}$ is a $N \times 1$ vector of wage changes and $\mathrm{d}\log \mathbf{A}$ is a $N \times 1$ vector of productivity changes.

6.4 Model with International Linkages

Following Çakmaklı, Demiralp, Kalemli-Ozcan, Yesiltas, and Yildirim (2022), we extend Baqaee and Farhi (2022)'s framework to a multi-country multi-industry setting. In essence, instead of assuming a single country with a closed economy, we consider the world as a closed economy. One major difference is that now we need to also aggregate the varieties coming from different countries in both production and consumption. Let's take German auto industry as an example. It might use steel imports from Turkey, China, Russia, etc. It also uses plastics from different countries. Steel from different countries are most likely to be substitutes for each other but steel and plastic are complements. We incorporate these differences in our model using sector (industry) bundles that are aggregates of country varieties.

Analytic framework is similar to the closed economy case with the caveat that we need to keep track of between country heterogeneity. Suppose there are C countries and N industries. Next, we will define the changes in the consumption and production.

Consumption: Consumption is a Cobb-Douglas aggregator over industry consumption bundles present in the country. Industry consumption bundles are aggregates of country-industry varieties with a CES aggregator with elasticity of substitution of $\xi = 4.55$ (Caliendo and Parro, 2015).

Production: Each country produces a variety in industry j using the intermediate bundle and value-added with a constant elasticity of substitution of $\varepsilon = 0.6$. Value-added is a bundle of country-industry specific labor and capital with a constant elasticity of substitution of $\eta = 0.6$. Intermediate bundle is composed of industry specific input sector bundles a constant elasticity of substitution of $\theta = 0.2$. Industry specific sector bundles are bundles of goods coming from all over the world with elasticity of substitution of $\xi = 4.55$.

Input-Output Matrix: Here, we will create $\widetilde{\Omega}$ matrix that incorporates the rich structure explained above. Denote the observed Ω with:

$$\Omega_{kc,jm} = \frac{p_{jm}\mathrm{x}_{kc,jm}}{p_{kc}\mathrm{y}_{kc}}$$

Where p_{kc} is the price of good k produced by country c. Note that k could also be the consumption good. For simplifying the notation, we will index all country-industry or country-factor pairs with a single index whenever we can. Let kI denote the intermediate bundle and kVA denote the value-added for industry k. We define:

$$\widetilde{\Omega}_{kI} = \sum_{jm} \Omega_{k,jm} = 1 - \alpha_{k,VA} \quad \text{and} \quad \widetilde{\Omega}_{kVA} = \alpha_{k,VA}$$

We will index the sector bundle for industry j that enters to the production of industry k with kj. Hence:

$$\widetilde{\Omega}_{kIj} = \frac{\sum_j \Omega_{k,jm}}{1 - \alpha_{k,VA}}$$

Each industry bundle is formed by different varieties from countries with:

$$\widetilde{\Omega}_{kjm} = \frac{\Omega_{k,jm}}{\sum_j \Omega_{k,jm}}.$$

In total, there are C consumption aggregates, $C \times N$ consumption bundles, $C \times N$ goods, $C \times N$ value-added bundles, $C \times N$ intermediate bundles and $C \times N^2$ sector bundles.

Value-added is composed of capital and labor. Each share will be denoted by:

$$\widetilde{\Omega}_{kL} = \alpha_{kL} \quad \text{and} \quad \widetilde{\Omega}_{kK} = \alpha_{kK} \quad \text{with} \quad \alpha_{kL} + \alpha_{kK} = 1.$$

Note that capital and labor are industry specific. Hence, there are $2C \times N$ factors.

Intertemporal Choice: To model the temporal choice, we will have a country specific Ricardian consumer who bridges the current consumption and future consumption decision. Country specific future consumption is denoted by an aggregate factor $* c$. Therefore, there are C Ricardian consumers and C future consumption aggregates. $\widetilde{\Omega}_{Rc,0c}$ and $\widetilde{\Omega}_{Rc,*c}$ denotes the Cobb-Douglas weights that this Ricardian consumer gives to the current and future consumption, respectively.

Input-Output Matrix Structure: With these additions of consumption, sector bundles, goods, intermediates, value-added and factor, the total size of the $\widetilde{\Omega}$ matrix becomes $(3C + 6CN + CN^2) \times (3C + 6CN + CN^2)$. The rows and the columns of this matrix are:

Description	Row / Column Indices
Consumption	$1 \rightarrow C$
Consumption Bundles	$C+1 \rightarrow C+CN$
Goods	$C+CN+1 \rightarrow C+2CN$
Intermediates	$C+2CN+1 \rightarrow C+3CN$
Sector bundles	$C+3CN+1 \rightarrow C+3CN+CN^2$
Value-Added	$C+3CN+CN^2+1 \rightarrow C+4CN+CN^2$
Labor	$C+4CN+CN^2+1 \rightarrow C+5CN+CN^2$
Capital	$C+5CN+CN^2+1 \rightarrow C+6CN+CN^2$
Ricardian consumer	$C+6CN+CN^2+1 \rightarrow 2C+6CN+CN^2$
Future consumption	$2C+3CN+CN^2+1 \rightarrow 3C+4CN+CN^2$

Solving the model: With these definitions, we can solve prices and Domar weights with the following equations and constraints implemented in AMPL/Knitro. Below, with an abuse of notation, instead of showing country-industry varieties separately, we will use a single index to address to address rows or columns of $\widetilde{\Omega}$ matrix.

Prices for goods (including sector bundles and value-added bundles):

$$p_k = \left[\sum_{j \notin F} \Omega_{kj} p_j^{1-\theta_k}\right]^{\frac{1}{1-\theta_k}}$$

Price indices for consumption goods:

$$\log p_{0c} = \sum_{j \in N} B_{Oc,j} \Omega_{Oc,j} \log p_j^{0c}$$

Where p_{0c} is the consumption price index and p_j^{0c} is the price of consumption bundle of industry j in country c.

Price Index for Ricardian consumers:

$$\log p_{Rc} = B_{Rc,0c} \Omega_{Rc,0c} \log p_{0c} + B_{Rc,*c} \Omega_{Rc,*c} \log p_{*c},$$

where p_{*c} is the price of future consumption in country c.

Domar weights for all goods, factors and future consumption:

$$\lambda_k = \sum_{j \notin F} \lambda_j \Omega_{jk} B_{jk}^{\theta_j} p_k^{1-\theta_j} p_j^{\theta_j - 1}$$

In particular:

$$\lambda_{0c} = \lambda_{Rc}\Omega_{Rc,0c}B_{Rc,0c}$$

And

$$\lambda_{*c} = \lambda_{Rc}\Omega_{Rc,*c}B_{Rc,*c}$$

Hence:

$$\lambda_{0c} = \frac{\Omega_{Rc,0c}B_{Rc,0c}}{\Omega_{Rc,*c}B_{Rc,*c}}\lambda_{*c} = \frac{B_{Rc,0c}}{B_{Rc,*c}}\lambda_{*c} = \frac{\beta}{1-\beta}\lambda_{*c}$$

This pins down the aggregate shock in terms of the expenditure share in the normal times. We assume that the future consumption levels are the same as pre-shock levels and the prices are also normalized. Hence:

$$\lambda_{*c} = \sum_{j \in F_c} \lambda_j^i \text{ and } p_{*c} = 1$$

Domar weights for Ricardian consumers:

$$\lambda_{Rc} = \sum_{j \in F_c \cup \{*c\}} \lambda_j.$$

Factor clearing conditions for capital: Since the factor levels do not change, we have the following identity:

$$\lambda_{fc} = p_{fc}\lambda_{fc}^i.$$

Factor clearing conditions for Labor:

$$[p_{fc} - 1]\left[\frac{\lambda_{fc}}{p_{fc}} - A_{fc}\lambda_{fc}^i\right] = 0,$$

Maximum labor could be $\bar{L}_{fc}$:

$$\frac{\lambda_{fc}}{p_{fc}} \leq A_{fc}\lambda_{fc}^i = \bar{L}_{fc}.$$

All factor prices are downward rigid:

$$p_{fc} \geq 1.$$

Here, this downward rigidity is assumed to be imposed at the US dollar level. Following Baqaee and Farhi (2021) this price rigidity should be implemented using the exchange rates. Let $\hat{p}_{fc}$ denote the wage paid to factor fc in local currency. We can impose the downward wage rigidity with:

$$d\log\hat{p}_{fc} = d\log\lambda_{fc} - d\log L_f + d\log e_c + d\log \mathrm{GDP} \geq 0. \qquad \text{(A9)}$$

0

where e_c is the exchange rate of the country c and GDP is the nominal gross domestic product in base country's units. The exchange rate is pinned down depending on a country's monetary policy.

We assume that the countries target inflation. Therefore:

$$d\log(\mathrm{p}_{0c} e_c \mathrm{GDP}) = 0,$$

where p_{0c} is the price of consumption good in country c. Plugging back this to the Equation A.9 for downward wage rigidity implies:

$$d\log p_{fc} = d\log\lambda_{fc} - d\log L_f \geq d\log \mathrm{p}_{0c}$$

Model Output: After solving for prices and Domar weights, we can calculate the CPI and GDP growth for each country.

Generalizing Corollary 1. Change in real GDP of country c can be written as:

$$d\log \mathrm{Y_c} = d\log \mathrm{E_c} - \sum_j \Omega_{0c,j}\, d\log p_j \qquad \text{(A.10)}$$

where $\mathrm{E_c}$ is the expenditure of country c and $\Omega_{0c,j}$ is the share of country-industry variety j in country c's final expenditures. We know that:

$$\mathrm{p_i y_i} = \sum_{\mathrm{c}} \mathrm{p_i x_{0c,i}} + \sum_{\mathrm{j}} p_i x_{ji}$$

Dividing both sides with E and rearranging the terms, we arrive at:

$$\lambda_i = \sum_c \frac{E_c}{E}\Omega_{0c,j} + \sum_j \lambda_j \Omega_{ji}$$

Using the matrix notation, we arrive at:

$$\lambda' = \sum_c \frac{E_c}{E}\Omega_{0c}\,\Psi$$

Combining with equation A.3, we find the factor shares to be:

$$\Lambda' = \sum_c \frac{E_c}{E} \Omega_{0c}\, \Psi \text{ diag}(\alpha).$$

Using this relationship, we can write:

$$\lambda_f = \sum_c \frac{E_c}{E} \Omega_{0c,j} \Psi_{jk}\, \alpha_f = \sum_c \frac{E_c}{E} \Psi_{0c,f}.$$

Let's define the country specific Domar weight as:

$$\lambda_f^c \equiv \Psi_{0c,f} = \Omega_{0c,j} \Psi_{jk}\, \alpha_f, \qquad \text{(A. 11)}$$

Essentially, λ_f^c captures the portion of factor L_f's output that ends up directly or indirectly with the consumers in country c. We can represent this share as:

$$\lambda_f^c = \frac{w_f L_f^c}{E_c}$$

Plugging A.3 and Equation A.11 into Equation A.10, we arrive at:

$$\begin{aligned} d \log \mathrm{Y_c} &= d \log \mathrm{E_c} - \sum_f \Psi_{0c,f}\, d \log w_f \\ &= d \log \mathrm{E_c} - \sum_f \Psi_{0c,f} \left(d \log \lambda_f^c + d \log \mathrm{E}_c - d \log L_f^c \right) = \\ &= -\sum_f \lambda_f^c \left(d \log \lambda_f^c - d \log L_f \right) = \sum_f \lambda_f^c\, d \log L_f \quad \text{(A. 12)} \end{aligned}$$

Here, we used the fact that: $\sum_f \Psi_{0c,f} = 1$, $\sum_f \lambda_f^c\, d \log \lambda_f^c = \sum_f \lambda_f^c\, d\, \lambda_f^c = 0$ and $d \log L_f^c = d \log L_f$.

Replacing Equation A.12 into the Proposition 1 would give us the multi-country version of Corollary 1.

Creating a balanced input-output network. International version of our model requires that the expenditures and income of a country to be equal to each other. However, in the current ICIO matrix, the sum of the final consumption of countries do not necessarily add up to the total value-added of the country once we include the heterogeneity in sectoral spending. To circumvent this issue, OECD uses taxes to make the expenditure and the production sides equal to each other at the sectoral level. Nevertheless, incorporating taxes into our model would result in intractability. Therefore, we will use the following equations to recover the self-consistent input-output tables.

Let's assume that we know $\mathrm{x}_{kc,jm}$ and $\Omega_{0c,jm}$. We would like to find value-added levels va_{kc} such that the final expenditures and value-added levels of a country match. The expenditure of each country is equal to its total value-added:

$$E_c = \sum_k \mathrm{va}_{kc}.$$

Total output of each industry should equal to each other both from the consumption and production side:

$$\mathrm{va}_{kc} + \sum_{jm} \mathrm{x}_{kc,jm} = \sum_m E_m \Omega_{0m,kc} + \sum_{jm} \mathrm{x}_{jm,kc}$$

Hence:

$$\mathrm{va}_{kc} = \sum_m \sum_j \mathrm{va}_{jm} \Omega_{0m,kc} + \sum_{jm} \mathrm{x}_{jm,kc} - \sum_{jm} \mathrm{x}_{kc,jm}\,. \qquad (\mathrm{A.}\,12)$$

These equations give us $\mathrm{C} \times \mathrm{N}$ equations and $\mathrm{C} \times \mathrm{N}$ unknowns. Note that, if we sum up both sides of this equation with respect to kc, we arrive at:

$$\sum_{kc} \mathrm{va}_{kc} = \sum_m \sum_j \mathrm{va}_{jm} \sum_{kc} \Omega_{0m,kc} + \sum_{kc} \sum_{jm} \mathrm{x}_{jm,kc} - \sum_{kc} \sum_{jm} \mathrm{x}_{kc,jm},$$

which is a tautology which would make the system un-invertible. We replace one of the equations with matching the world GDP:

$$\sum_{kc} \mathrm{va}_{kc} = \mathrm{GDP}, \qquad (\mathrm{A.}\,13)$$

which we assume to be given. Combining aquations A.12 and A.13, and using a matrix notation with matrices A and B given by replacing the relevant terms of these equations, we can write:

$$\mathrm{va}' = \mathrm{va}'\mathrm{A} + \mathrm{B}.$$

Solving this equation gives us va values which are balanced under given expenditure patterns and input-output linkages..