The Global Component of Inflation Volatility*

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Abstract

Global developments play an important role for domestic inflation rates. Earlier literature has found that a substantial amount of the variation in a large set of national inflation rates can be explained by a single global factor. However, inflation volatility has been typically neglected, while it is clearly relevant both from a policy point of view and for structural analysis and forecasting. We study the evolution of inflation rates in several countries, using a novel model that allows for commonality in both levels and volatilities, in addition to country-specific components. We find that inflation stochastic volatility is indeed important, and a substantial fraction of it can be attributed to a global factor that is also driving inflation levels and their persistence. While various phenomena may contribute to global inflation dynamics, it turns out that since the early '90s the estimated global factor is correlated with the Chinese PPI and Oil inflation. The extent of commonality among core inflation rates and volatilities is substantially smaller than for overall inflation, which leaves scope for national monetary policies. Finally, we show that the point and density forecasting performance of the model is quite good, also relative to standard benchmarks.

Keywords: Inflation, Volatility, Global factors, Large Datasets, Multivariate Autoregressive Index models, Reduced Rank Regressions, Forecasting.

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1 Introduction

Global developments play an important role in the determination of inflation rates. Papers such as Borio and Filardo (2007) and Ciccarelli and Mojon (2010) find that a substantial amount of variation in a large set of national inflation rates can be explained by global factors. Quoting Borio and Filardo (2007): "*...proxies for global economic slack add considerable explanatory power to traditional benchmark inflation rate equations, even allowing for the influence of traditional indicators of external influences on domestic inflation, such as import and Oil prices. Moreover, the role of such global factors has been growing over time, especially since the 1990s. And in a number of cases, global factors appear to have supplanted the role of domestic measures of economic slack.*" This evidence has been recently challenged by Lodge and Mikolajun (2016), whose results suggest that the relevance of global factors for forecasting domestic inflation is related to their ability to capture slow-moving trends, like those emphasized by Stock and Watson (2007) in their decomposition of US inflation into trend and cyclical components. Other empirical contributions, as Bianchi and Civelli (2015) and Auer et al. (2017), show that financial openness and Global Value Chains are positively related to the effects of global slack on inflation. We do not take an a priori stance on this point, but we will use an econometric model where the relative contribution of global and country-specific factors as drivers of inflation developments is estimated and can vary over time and across countries.

Another point stressed by Stock and Watson (2007), which however dates back to at least Engle (1982), is the importance of allowing for conditional time-varying volatility when modelling inflation. While Engle introduced the ARCH specification as a model for inflation volatility, Stock and Watson (2007) used stochastic volatility, which is indeed more common in macroeconomics applications and more flexible since it permits to have different shocks as drivers of the level and volatility of an economic variable. Stock and Watson found that the introduction of SV improves the out of sample forecasting power of their model for US inflation, and it is preferable to both rolling estimation and regime switching to allow for heteroskedasticity. Besides forecasting, inflation volatility is also relevant for policy making as, for example, in periods of high volatility it is more difficult to understand whether inflation movements are temporary or persistent.

Volatility needs to be modeled properly in multi-country studies on inflation determinants. In particular, it seems interesting to understand whether and to what extent the cross-country commonality among inflation levels is also present among inflation volatilities.

Furthermore, recent macro-financial literature has considered stochastic volatility as a basis to construct measures of macro and financial uncertainty (see Jurado et al., 2015, and Carriero et al., 2017). From this perspective, it may be important for a policymaker to disentangle whether inflation uncertainty originates locally or globally.

Mumtaz and Surico (2008) investigate co-movements in an unbalanced panel of inflation rates from the 1970s to early 2000s for 11 countries, using a large dynamic factor model that incorporates time-varying coefficients and stochastic volatility in the unobservable factors' law of motions. Their decomposition does not show a large role for common components, since most of the time variation in levels and volatilities seems captured by the country-specific component and the residuals, which are left unexplained. They conclude that there has been a fall in level, persistence and volatility of inflation across countries, but with the drop in volatility not synchronized across nations.

Delle Monache et al. (2016) extend the model of Stock and Watson (2007) to a multivariate inflation setting for the euro area, where the permanent component is common among inflation rates of EMU members and the cyclical components are modeled as country-specific autoregressive processes with time-varying parameters. They document that the common permanent component has driven the general disinflation within the euro area, and the importance of common shocks to euro area inflation has increased relatively to idiosyncratic disturbances.

We have collected inflation rates for 20 OECD countries, over the period 1960Q1-2016Q4. Figure 1 reports the time series of CPI inflation rates for each country. From visual inspection, there emerges a non-trivial degree of commonality at low-medium frequencies, as pointed out by Lodge and Mikolajun (2016). A plot of the inflation rates together with their first principal component (PC), Figure 2, provides more evidence on their co-movement (the first PC explains about 70% of the variability of all inflation rates). However, the figure also highlights some country-specific movements in inflation rates, and changes in the volatility of inflation, which seems overall smaller in the later part of the sample. To provide descriptive evidence on commonality in inflation volatility, we have estimated AR-SV models for each inflation rate, and in Figure 3 we report the estimated volatilities together with their first principal component, which explains almost 60% of their time variation.

This evidence motivates the choice of decomposing inflation rates into a common component driven by a single global inflation factor, a country-specific component, and an error term featuring, in turn, common and idiosyncratic time-varying volatility.

Hence, we introduce a novel multivariate autoregressive index (MAI) model, with stochastic volatility (SV), and autoregressive (AR) terms. A MAI model is a VAR with a particular reduced rank structure imposed on the coefficient matrices, such that each variable is driven by the lags of a limited number of linear combinations of all variables (so called Indexes), which can be considered as observable common factors. The MAI model was introduced by Reinsel (1983) and further extended by Carriero et al. (2016b) to allow for a large number of variables. Stochastic volatility (SV) was introduced in the MAI model by Carriero et al. (2018), while Cubadda and Guardabascio (2017) allowed for the possibility of autoregressive (AR) terms to capture idiosyncratic components. We combine all these features into the MAI-AR-SV model, and develop a novel Bayesian MCMC estimation algorithm.

The proposed methodological framework is considerably different from Mumtaz and Surico (2008), who build upon the dynamic factor model of Stock and Watson (1989) and Forni et al. (2000), and estimate their model's stochastic volatilities using the univariate method of Jacquier et al. (2004). Our methodology is also substantially different from Delle Monache et al. (2016), who model multi-country inflation rates with a common permanent component and its own changing volatility, estimated in a non-Bayesian setting in which time variation is driven by likelihood scores.

We work with a single index model where the index (a linear combination of all the national inflation rates) represents the global factor that drives both levels and volatilities of all national inflation rates. Inflation levels and volatilities also have an idiosyncratic, country-specific, component, whose relative importance with respect to the global component is time-varying and empirically determined.

We find that the single common factor in the MAI-SV model explains on average about 70% of the variability of all inflation rates. Moreover, there is also substantial commonality in the inflation volatilities, increased in the last two decades. The average (across countries) share of stochastic volatility explained by the global component spans from 20% to 65% throughout the sample. While various sources can be behind the global inflation factor, it turns out that since the early '90s it is strongly correlated with Chinese PPI and Oil inflation.

We also find that the global inflation factor is highly persistent, and this persistence is transmitted to the global component of the national inflation rates, in line with Ciccarelli and Mojon (2010). Level components explained by the common factor show a larger degree of persistence than idiosyncratic components.

We then repeat the same analysis on a panel of non-Food and non-Energy inflation rates for the same set of OECD countries, using data available for the period 1979Q1-2016Q4, finding a smaller but non-negligible degree of commonality. The global core inflation factor explains roughly 25% of the variability of core CPI inflation levels and the average (across countries) share of stochastic volatility explained by the global component spans from 10% to 20% throughout the sample, without displaying sizable variation over time as in the case of headline inflation rates. The remaining substantial national component of core inflation level and volatility leaves scope for national monetary policies.

The evidence provided in this paper also contributes to the long standing debate on globalisation, inflation and monetary policy. Rogoff (2003) and Rogoff (2006) discuss how various structural elements accompanying the globalisation since the early 1990s may have lowered the global long term equilibrium of inflation rates, fostering the strong global co-movement of CPI and somehow diminishing the role of domestic slack and monetary policy in determining national inflation. However, as highlighted also in the recent speech by Carney (2017), core inflation seems to be less affected by global dynamics, already when looking at simple pairwise correlation. Our work and methodology allow to measure separately the degree of cross-country commonality in first and second moments of both headline and core inflation rates, providing precious information to monetary policy makers pursuing their inflation mandate in an increasingly global context.

Finally, point and density forecast evaluation shows that the MAI-AR-SV model has very good out of sample properties for inflation rates, when compared with a set of multivariate and univariate competitors, and the SV specification is particularly relevant for the proper calibration of density forecasts. These results hold for both all items inflation and core inflation rates, and provide further empirical support for our proposed model.

The paper is structured as follows. Section 2 introduces the econometric models and the volatility decomposition. Section 3 discusses the choice of prior distributions. Section 4 develops the MCMC estimation methodology, with additional details in the Appendix. Section 5 presents data and empirical results on the commonality in inflation rate levels and volatilities. Section 6 assesses the point and density forecasting performance of the MAI-AR-SV inflation model. Section 7 concludes.

2 The econometric model

2.1 The MAI-AR-SV model

We assume that the model for the n-dimensional zero mean process[1] y_t containing the inflation rates of interest is:

$$\underbrace{y_t}_{n\times 1} = \sum_{\ell=1}^{q} \underbrace{\Gamma_\ell}_{n\times n} \cdot y_{t-\ell} + \sum_{\ell=1}^{p} \underbrace{A_\ell}_{n\times r} \cdot \underbrace{B_0}_{r\times n} y_{t-\ell} + u_t, \tag{1}$$

where Γ_ℓ is a diagonal matrix:

$$\Gamma_\ell = \mathrm{Diag}(\gamma_\ell), \qquad \gamma_\ell = \begin{bmatrix} \gamma_{1,\ell} & \gamma_{2,\ell} & \dots & \gamma_{n,\ell} \end{bmatrix}'.$$

We can rewrite the model more compactly as

$$y_t = \sum_{\ell=1}^{s} (\Gamma_\ell + A_\ell \cdot B_0)\ y_{t-\ell} + u_t,$$

or

$$(I - C_1 L - ...C_s L^s) y_t = C(L) y_t = u_t, \tag{2}$$

where $s = \max(p, q)$ and $C_\ell = \Gamma_\ell + A_\ell \cdot B_0$, $\ell = 1, ..., s$.

Moreover, we assume that

$$u_t = G^{-1} \Sigma_t \varepsilon_t, \qquad \varepsilon_t \overset{iid}{\sim} \mathcal{MN}\left(\mathbf{0},\ I_n \right), \tag{3}$$

so that

$$u_t \overset{i}{\sim} \mathcal{MN}\left(\mathbf{0}, \underbrace{\Omega_t}_{n\times n} \right), \qquad \Omega_t = G^{-1} \Sigma_t \Sigma_t \left(G^{-1}\right)', \tag{4}$$

where G is a triangular matrix containing reduced form covariances[2], and $(\Sigma_t)_{t=1}^{T}$ is the

[1] A non-zero mean can be easily allowed by inserting an intercept in the model.

[2] The matrix G can be also made time-varying, but at the cost of a substantial increase in computational complexity when the number of variables is large.

history of diagonal matrices containing the stochastic volatilities:

$$\underbrace{G}_{n\times n} = \begin{bmatrix} 1 & 0 & \dots & \dots & 0 \\ g_1 & 1 & \ddots & \ddots & \vdots \\ g_2 & g_3 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ g_{m-n+2} & g_{m-n+3} & \dots & g_m & 1 \end{bmatrix}, \qquad \underbrace{g}_{m\times 1} \equiv \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_m \end{bmatrix}, \qquad m \equiv \frac{n\,(n-1)}{2},$$

$$\underbrace{\Sigma_t}_{n\times n} = \text{Diag}(\sigma_t), \qquad \underbrace{\sigma_t}_{n\times 1} \equiv \begin{bmatrix} \sigma_{1,t} \\ \sigma_{2,t} \\ \vdots \\ \sigma_{n,t} \end{bmatrix},$$

$$\log \sigma_t = \log \sigma_{t-1} + \nu_{\sigma,t}, \qquad \nu_{\sigma,t} \overset{iid}{\sim} \mathcal{MN}\left(\mathbf{0}, \underbrace{Q_\sigma}_{n\times n}\right).$$

The specification in (1)-(4) is a Multivariate Autoregressive Index (MAI) model with stochastic volatility (SV) and autoregressive (AR) terms, MAI-AR-SV. Each of the n variables in the MAI-AR-SV model is driven by its own lags, capturing in our case country-specific features of inflation, with associated coefficients Γ_ℓ, $\ell = 1, ..., q$; by the lags of r common observable factors ($B_0\ y_{t-\ell}$, the "indexes"), capturing in our case global features of inflation, with associated loading matrices A_ℓ, $\ell = 1, ..., p$; and by variable-specific errors, u_t, whose time-varying covariance matrix Ω_t is expressed as in Cogley and Sargent (2005).

With respect to an unrestricted VAR, the MAI-AR-SV specification leads to a substantial reduction in the number of parameters influencing the conditional means: we go from n^2p coefficients of the VAR to at most $n \cdot q + n \cdot r \cdot p + r \cdot n$ in the MAI-AR-SV case[3]. In our empirical application, we have $p = q = 4$, $r = 1$ and $n = 20$, so that there are 180 parameters in the MAI-AR-SV while there would be 1600 parameters in an unrestricted VAR.

[3]Assuming no restrictions in the matrix B_0.

2.2 An alternative representation of the MAI-AR-SV model

Let us define the obervable factors driving all variables as

$$F_t \equiv B_0 \cdot Y_t, \tag{5}$$

and note that the following decomposition holds:[4]

$$I_n = \Omega_t B_0' \Xi_t^{-1} B_0 + B_{0\perp}' \Xi_{\perp,t}^{-1} B_{0\perp} \Omega_t^{-1}, \tag{6}$$

where $B_{0\perp}$ is the $(n-r) \times n$ orthogonal matrix of B_0, such that the scalar product of any pair of rows of B_0 and $B_{0\perp}$ has zero value[5], $\Xi_t = B_0 \Omega_t B_0'$ and $\Xi_{\perp,t} = B_{0\perp} \Omega_t^{-1} B_{0\perp}'$. Let us also define

$$G_t = B_{0\perp} \Omega_t^{-1} y_t, \tag{7}$$

where G_t are $n-r$ variables that can be interpreted as idiosyncratic components, as there are many of them and, as we will see later on, they are driven by shocks uncorrelated with those driving the common factors F_t.

Using (5)-(7), we can now write the MAI-AR-SV model in (1)-(4) as

$$y_t = \sum_{\ell=1}^{q} \Gamma_\ell [\Omega_t B_0' \Xi_t^{-1} B_0 + B_{0\perp}' \Xi_{\perp,t}^{-1} B_{0\perp} \Omega_t^{-1}] \; y_{t-\ell} + \sum_{\ell=1}^{p} A_\ell \cdot B_0 \; y_{t-\ell} + u_t,$$

or

$$y_t = \sum_{\ell=1}^{q} \Gamma_\ell B_{0\perp}' \Xi_{\perp,t}^{-1} G_{t-\ell} + \sum_{\ell=1}^{\max(p,q)} (\Gamma_\ell \Omega_t B_0' \Xi_t^{-1} + A_\ell) F_{t-\ell} + u_t. \tag{8}$$

Next, we derive the model for the factors F_t implied by the MAI-AR-SV model. Starting

[4] See Carriero et al. (2016b) and the references therein for details.

[5] This is equivalent to state

$$B_0 B_{0\perp}' = \mathbf{0}_{r\times(n-r)}.$$

from (8) and multiplying both sides of it either by B_0 or by $B_{0\perp}\Omega_t^{-1}$, we obtain:

$$F_t = \sum_{\ell=1}^{q} B_0 \Gamma_\ell B_{0\perp}' \Xi_{\perp,t}^{-1} G_{t-\ell} + \sum_{\ell=1}^{\max(p,q)} B_0(\Gamma_\ell \Omega_t B_0' \Xi_t^{-1} + A_\ell) F_{t-\ell} + \omega_t, \tag{9}$$

$$G_t = \sum_{\ell=1}^{q} B_{0\perp}\Omega_t^{-1} \Gamma_\ell B_{0\perp}' \Xi_{\perp,t}^{-1} G_{t-\ell} + \sum_{\ell=1}^{\max(p,q)} B_{0\perp}\Omega_t^{-1}(\Gamma_\ell \Omega_t B_0' \Xi_t^{-1} + A_\ell) F_{t-\ell} + \psi_t,$$

where

$$\begin{bmatrix} \omega_t \\ \psi_t \end{bmatrix} = \begin{bmatrix} B_0 u_t \\ B_{0\perp}\Omega_t^{-1} u_t \end{bmatrix} \overset{i}{\sim} \mathcal{MN}\left(\mathbf{0}, \begin{bmatrix} \Xi_t & 0 \\ 0 & \Xi_{\perp,t} \end{bmatrix} \right), \tag{10}$$

since

$$E(\omega_t \psi_t') = E(B_0 u_t u_t' \, \Omega_t^{-1} B_{0\perp}') = B_0 \Omega_t \Omega_t^{-1} B_{0\perp}' = 0.$$

Hence, the r observable factors F_t and the $n - r$ variables G_t jointly evolve as a VAR, with block uncorrelated errors.

The model in (8)-(9) is similar to a factor augmented VAR (FAVAR) model, as for example in Bernanke et al. (2005), or Stock and Watson (2002a) who also allow for variable-specific AR terms. The model in (8)-(9) also features stochastic volatility both in the common (ω_t) and in the idiosyncratic (ψ_t) shocks, which is particularly relevant for modelling inflation, as we will see. Moreover, in the FAVAR model the factors are unobservable, while they are observable in the MAI case, which simplifies model estimation and interpretation of the results. Finally, in general unobserved factors should be modeled with a VARMA rather than a VAR model, as emphasized by Dufour and Stevanović (2013), while in our case we can analytically derive the VAR model followed by the observable factors F_t (jointly with the variables G_t).

2.3 Decomposing the volatilities

We decompose the stochastic volatility of the MAI-AR-SV errors u_t into two orthogonal components, one of them driven by the volatility of the common shocks ω_t, the other by that of the idiosyncratic shocks, ψ_t, orthogonal to ω_t.

Using again the decomposition in (6), we get:

$$u_t = \Omega_t B_0' \Xi_t^{-1} \omega_t + B_{0\perp}' \left(B_{0\perp} \Omega_t^{-1} B_{0\perp}' \right)^{-1} \psi_t,$$

with $\Xi_t = B_0 \Omega_t B_0^{'}$. Hence, due to the orthogonality of ω_t and ψ_t, we can then decompose the total error volatility into the volatility of the common component and that of the idiosyncratic component:

$$\Omega_t = \Omega_t^{com} + \Omega_t^{idio},$$

where

$$\begin{aligned} \Omega_t^{com} &= \Omega_t B_0' \Xi_t^{-1} B_0 \Omega_t, \\ \Omega_t^{idio} &= B_{0\perp}' \Xi_{\perp,t}^{-1} B_{0\perp}. \end{aligned}$$

2.4 Decomposing the levels and computing IRFs

It is interesting to decompose the inflation rates in y_t into their common and idiosyncratic components, where the common component is driven by the common shocks ω_t and the idiosyncratic component by the idiosyncratic shocks ψ_t. The decomposition can be also used to compute impulse response functions (IRFs) to common shocks.

We cannot directly use the model in (8), as both F_t and G_t are driven by both ω_t and ψ_t. If the restricted VAR in (2) is stationary, we can write the associated MA representation as

$$\begin{aligned} y_t &= C(L)^{-1} u_t = B(L) u_t \\ &= \underbrace{B(L) \Omega_t B_0' \Xi_t^{-1} \omega_t}_{\text{Common}} + \underbrace{B(L) B_{0\perp}' \left(B_{0\perp} \Omega_t^{-1} B_{0\perp}' \right)^{-1} \psi_t}_{\text{Idiosyncratic}}. \end{aligned}$$

As an alternative, we can use the projection approach, proposed for example by Jordà (2005) for IRF computation, to obtain a similar decomposition:

$$y_t = B_1(L) \omega_t + B_2(L) \psi_t.$$

Note that, in both cases, the common and idiosyncratic components are orthogonal at all leads and lags, due to temporal independence and orthogonality of ω_t and ψ_t. Therefore, empirically, we can obtain the common component as the fitted value in a regression of y_t on contemporaneous and lagged values of the (estimated) common shocks ω_t (and the idiosyncratic component as y_t minus the estimated common component), while the IRFs to common shocks are computed from the elements of $B_1(L)$.

In our empirical application on inflation, we have a single factor ($r = 1$), so that ω_t is a scalar, which further simplifies the computation of the common component of inflation rates, and their impulse response functions to global shocks.

The MAI-AR-SV model is estimated by means of Bayesian techniques. The next section describes the specification of prior distributions for model parameters, while section 4 presents the MCMC estimation algorithm, with additional details in the Appendix. Readers not interested in technical details can go directly to the empirical results in Section 5.

3 Estimation of the MAI-AR-SV model

3.1 Specification of the prior distributions

The prior is constructed in various steps, which generally require the use of a training sample $\{-T^*, \ldots, -1, 0\}$.

3.1.1 Prior on B_0 for the Metropolis step

Prior knowledge for the unrestricted elements of B_0 is elicited with a Normal distribution. To define these prior distributions, let us decompose the n variables in r blocks, so to have as many blocks as factors (r).

$$\underbrace{y_t}_{n\times 1} = \left[\underbrace{y_t^{1\prime}}_{1\times n_1} \quad \underbrace{y_t^{2\prime}}_{1\times n_2} \quad \vdots \quad \underbrace{y_t^{r\prime}}_{1\times n_r} \right]', \qquad n = \sum_{j=1}^{r} n_j.$$

For each $j \in \{1, \ldots, r\}$, we compute the largest eigenvalue score from the Principal components analysis, so to obtain a final set of r score series $\left(S_t^j\right)_{t\in\{1,\ldots,T\}}^{j\in\{1,\ldots,r\}}$. Once obtained the scores, we consider the following $n-r$ univariate regression models:

$$\forall j \in \{1,\ldots,r\}\,,\ \forall k \in \{2,\ldots,n_j\}\,,\quad S_t^j = B_{0,j,k}\cdot y_{t,k}^j + u_{j,k,t},\quad u_{j,k,t} \overset{iid}{\sim} \mathcal{N}\left(0,\sigma_{j,k}^2\right)$$

To normalize the first element of each $B_{0,j}$, $B_{0,j,1}$ is set at 1. Defining :

$$\forall j \in \{1,\ldots,r\}\,,\qquad \underbrace{\widetilde{B}_{0,j}}_{1\times(n_j-1)} \equiv \begin{bmatrix} B_{0,j,2} & \ldots & B_{0,j,n_j}\end{bmatrix},$$

for each $\widetilde{B}_{0,j}$, we compute the OLS estimate and its variance.

The prior distribution for $\underbrace{B_0}_{r\times n}$ can be then centered at

$$\underbrace{B_0}_{r\times n} = \begin{bmatrix} 1 & \widetilde{B}_{0,1} & 0 & \mathbf{0}_{1\times(n_2-1)} & \ldots & 0 & \mathbf{0}_{1\times(n_r-1)} \\ 0 & \mathbf{0}_{1\times(n_1-1)} & 1 & \widetilde{B}_{0,2} & \ldots & 0 & \mathbf{0}_{1\times(n_r-1)} \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \mathbf{0}_{1\times(n_1-1)} & 0 & \mathbf{0}_{1\times(n_2-1)} & \ldots & 1 & \widetilde{B}_{0,r}, \end{bmatrix}$$

and the respective variances are coming from each separate regression. Prior covariances among elements are set to zero.

3.1.2 Prior on the loadings A

Defining $A \equiv \begin{bmatrix} A_1 & \ldots & A_p \end{bmatrix}$, the prior on $a = vec\,(A')$ is multivariate Normal, centered on $\mathbf{0}$, and with diagonal variance V_a resembling a Minnesota prior.

$$V_a = \begin{bmatrix} \widehat{\sigma}_{y,1}^2 & 0 & \ldots & 0 \\ 0 & \widehat{\sigma}_{y,2}^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \ldots & 0 & \widehat{\sigma}_{y,n}^2 \end{bmatrix} \otimes \begin{bmatrix} \Upsilon_1 & 0 & \ldots & 0 \\ 0 & \Upsilon_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \ldots & 0 & \Upsilon_p \end{bmatrix},$$

$$\forall \ell \in \{1,\ldots,p\}\,,\qquad \Upsilon_\ell = \frac{\lambda_a}{\ell^d}\cdot \begin{bmatrix} \frac{1}{\widehat{\sigma}_{F,1}^2} & 0 & \ldots & 0 \\ 0 & \frac{1}{\widehat{\sigma}_{F,2}^2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \ldots & 0 & \frac{1}{\widehat{\sigma}_{F,r}^2} \end{bmatrix},$$

where $\widehat{\sigma}^2_{y,j}$ and $\widehat{\sigma}^2_{F,s}$ are the residual variances of a univariate AR(1) for, respectively, each variable j and each factor s (computed using the prior mean of B_0). λ_a is a tightness parameter.

3.1.3 Prior for the elements of the residual variance

The prior for the elements of G is a multivariate Normal distribution centered at zero, with large diagonal covariance matrix. The prior for σ_0 is a multivariate Normal, centered at $\begin{bmatrix} \widehat{\sigma}^2_{y,1} & \widehat{\sigma}^2_{y,2} & \dots & \widehat{\sigma}^2_{y,n} \end{bmatrix}'$, with identity covariance matrix, as in Primiceri (2005). Prior distributions for the innovation covariance matrix Q_σ is calibrated as in Primiceri (2005).

3.1.4 Prior for the AR coefficients γ

The prior distribution of the AR coefficients in γ is a multivariate Normal distribution. In the spirit of a Minnesota Prior, we choose an a priori unitary mean for the first lag of each variable whose dynamics resemble a random walk, and a zero mean for the higher lags. Regarding the a priori covariance matrix, we assume no correlation across coefficients of different lags and variables, and we set a prior structure for the variances which resembles the Minnesota prior, using the tightness and decay parameters.

$$\bar{\gamma} = \begin{bmatrix} \bar{\gamma}_1 \\ \bar{\gamma}_2 \\ \vdots \\ \bar{\gamma}_q \end{bmatrix} = \begin{bmatrix} \mathbf{1}_{n\times 1} \\ \mathbf{0}_{n\times 1} \\ \vdots \\ \mathbf{0}_{n\times 1} \end{bmatrix}, \qquad V_\gamma = \lambda_\gamma \cdot \begin{bmatrix} 1^{-d} & 0 & \dots & 0 \\ 0 & 2^{-d} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & q^{-d} \end{bmatrix} \otimes I_n.$$

3.2 Gibbs Sampler

This subsection describes each step of the Gibbs Sampler (GS) used to simulate from the joint posterior distribution of both parameters $\{\gamma, A, B_0, G, Q_\sigma\}$ and unobservable states $(\sigma_t)_{t=1}^T$ of the MAI-AR-SV model. Moreover, the Omori et al. (2007) procedure requires drawing the indexes of Normal components of the mixture approximating the $\log \chi^2_1$, contained in the matrix S. This approach is needed as the joint posterior distribution cannot be analytically determined. The steps are the following:

1. Draw the AR coefficients $\gamma \left| A, B_0, G, Q_\sigma \, (\sigma_t)_{t=1}^T \right.$,

2. Draw the loadings $A \left| \gamma, B_0, G, Q_\sigma, (\sigma_t)_{t=1}^T \right.$,

3. Draw the factor weights $B_0 \left| \gamma, A, G, Q_\sigma, (\sigma_t)_{t=1}^T \right.$,

4. Draw the off-diagonal elements in $G \left| \gamma, A, B_0, Q_\sigma, (\sigma_t)_{t=1}^T \right.$,

5. Draw the indexes of the mixture in $S \left| \gamma, A, B_0, G, Q_\sigma, (\sigma_t)_{t=1}^T \right.$,

6. Draw a history of volatilities $(\sigma_t)_{t=1}^T \,| S, \gamma, A, B_0, G, Q_\sigma$,

7. Draw the covariance of volatilities' innovations $Q_\sigma \left| \gamma, A, B_0, G, (\sigma_t)_{t=1}^T \right.$.

It is important to note[6] that steps 2 and 3 have $y_t - \mathcal{X}_t \cdot \gamma$ as dependent variable in order to draw A and B_0, while in step 1 we use $y_t - A \cdot Z_t$ to draw the AR coefficients[7] [8].

Each step of the GS for the MAI-AR-SV is described in detail in section A of the Appendix.

4 The global component of inflation volatility

4.1 Data

Following the literature on global inflation (e.g. Ciccarelli and Mojon, 2010 and Borio and Filardo, 2007) we gathered a panel of Consumer Price Indices for a set of 20 OECD countries[9], downloaded from the *OECD main economic indicators* database. The dataset includes 228 observations at quarterly frequency, covering the period from 1960-Q1 to 2016-Q4. We then constructed inflation rates as year on year changes of the indexes[10].

[6] See the Appendix for further details.

[7] $Z_t \equiv (I_p \otimes B_0) \cdot vec\left(\begin{bmatrix} y_{t-1} & \dots & y_{t-p} \end{bmatrix}\right)$.

[8] $\mathcal{X}_t = \begin{bmatrix} Diag(y_{t-1}) & Diag(y_{t-2}) & \dots & Diag(y_{t-q}) \end{bmatrix}$.

[9] USA, Australia, Austria, Belgium, Canada, Finland, France, Germany, Greece, Italy, Japan, Luxembourg, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, UK

[10] Ciccarelli and Mojon (2010) use Year on Year changes of CPI inflation rates for the bulk of their analysis. O'Reilly and Whelan (2005) adopt the same transformation stressing that is cited in the ECB's official inflation mandate. Lodge and Mikolajun (2016) point out that using YoY changes in CPI is preferable since this transformation produces no seasonal pattern by construction.

4.2 MAI-SV and MAI-AR-SV

We start with a MAI-SV specification (that is with $\Gamma_\ell = \mathbf{0}, \ \forall \ell$), with $p = 4$ lags and with a single global factor ($r = 1$), similar to the preferred specification of Ciccarelli and Mojon (2010). The resulting model is estimated by a simplified version of the MCMC algorithm presented in Section 4, see Carriero et al. (2018) for details.

Figure 4 reports the inflation rates for each country along with the posterior bands and median of the estimated common global inflation factor. The model is clearly able to capture the substantial co-movement of national inflation rates.

Figure 5 reports the data compared with the in-sample fit of the MAI-SV model for each country, as well as the percentage share of variance explained. On average (across countries) the estimated common component explains roughly 73% of the variance, which is in line with the Principal Component Analysis.

Next, as the residuals of the MAI-SV model are clearly serially correlated at least over parts of the sample, we estimate a MAI-AR-SV model with $p = 4$ lags for the common part, as for the MAI-SV, and $q = 4$ lags for the country-specific AR components. The in-sample fit for the various countries is presented in Figure 6. The fit of the MAI-AR-SV is systematically higher than that of the MAI-SV specification, reaching an average explained variance of about 94%. In particular, the MAI-AR-SV specification is able to capture both the low and the high frequency variation of each inflation series, due to the presence of both common and country-specific autoregressive components.

Notwithstanding the differences mentioned above, the estimated global factor from the MAI-SV and MAI-AR-SV models are very similar, see Figure 7. They are also very similar to the first PC of the inflation rates. The latter is used to form the prior on the B_0 coefficients in the MAI models, but the prior variance is large enough so that results are data driven rather than dictated by the prior. All such measures of common components are also comparable, though with some differences, to an OECD measure of global inflation, also reported in Figure 7. These results are in line with the findings of Ciccarelli and Mojon (2010), even though their sample stops in 2008. As reported also by Ferroni and Mojon (2016), our analysis suggests strong commonality in inflation developments across OECD continues also in the more recent period, and, actually, it has been particularly high during the last financial crisis[11].

[11] Using a more recent sample of inflation rates (1993-2014), Ferroni and Mojon (2016) find that the

Finally, while there can be many drivers of the global inflation factor, Figure 8 shows that after the 90's it is correlated with the Chinese PPI inflation rate and the Oil inflation rate.

4.3 Levels decomposition and persistence

Using the level decomposition discussed in section 2, we are able to disentangle the observed inflation series of each country into orthogonal components driven, respectively, by common and idiosyncratic shocks. Moreover, for each country we measure how much variation is explained by each component.

Figures 9 and 10 report, respectively, the common and idiosyncratic components, compared with the actual series. The common components tend to explain more than 50% of almost all countries' inflation rates, and are particularly important in large economies like the US, UK, Germany and Japan.

Stock and Watson (2007) discuss the persistence of US inflation, using as a measure of persistence the largest autoregressive root of the levels' process. Inference about this measure of persistence is made possible by the Stock (1991) method, which is appropriate when dealing with series displaying high levels of persistence. Stock and Watson (2007) do not find strong evidence of persistence changes in US inflation from the 1970s onwards, reporting the largest AR root of US CPI inflation comprised between 0.85 and 1.05 (as 90% confidence interval). O'Reilly and Whelan (2005) report little evidence of instability for inflation persistence in the Euro Area since the 1970s; they report rolling confidence intervals for the largest AR root of Euro Area CPI inflation that are centered around 0.9 across almost the entire sample.

In light of this literature, using the entire sample, we computed the 90% confidence intervals (CI) for the largest AR roots of all national CPI inflation series, of their common and idiosyncratic components, and of the global factor. Figure 11 compares the CI for the largest AR root of the observed series, their components and the global factor, separately for each country. The picture clearly shows how the common global components tend to preserve the high persistence of the observed series, while the idiosyncratic country-specific components display wider confidence intervals centered on slightly smaller values. The global factor shows a very narrow CI centered on 0.99.

fraction of national inflation rates' variance that is explained by Global Inflation remains dominant.

These results are in line with what reported by Ciccarelli and Mojon (2010), who argue that "*the global component captures the most persistent and possibly nonstationary part of inflation*". Indeed, using a different methodology, they report smaller persistence for the so called "national" components; interpreting such results, they consider the global factor as an attractor and the main driver of persistence coming from the observed data. However, for this specific exercise they use annualized quarter on quarter inflation rates, which is a transformation that tends to display a smaller degree of persistence than the year on year transformation. Performing our analysis using QoQ CPI changes, we measure a degree of persistence in line with Ciccarelli and Mojon (2010) for both global and national inflation components.

4.4 Time-varying residual volatility decomposition

Figure 12 reports the posterior bands of the estimated reduced form conditional inflation volatilities of all countries for the MAI-AR-SV. The estimated volatilities display a relevant degree of commonality. Indeed, the first principal component of the volatilities explains on average about 50% of their variation.

Principal component analysis is however not so suited in this context, due to the time-varying covariance matrix of the errors. Hence, to better understand what is driving the volatilities, we can apply the decomposition discussed in Section 2. Figures 13 and 14 present the decomposition of the estimated volatilities in their common and idiosyncratic components. More specifically, Figure 13 presents results in absolute terms and Figure 14 in relative terms. It turns out that the contribution of the common component is non trivial, reaching values above 50% for some countries and time periods, especially during the last decades, in particular during the Great Recession.

In this multi-country context, it is complex to understand the drivers of the common inflation volatility component. However, for a single country this can be done. Carriero et al. (2018), focusing on the US, find that supply shocks are particularly important, with demand shocks ranked second and monetary/financial shocks third.

Figure 15 shows the posterior bands of the global factor volatility, that is $(\Xi_t)_{t=1}^{T}$. Global inflation volatility was moderate during the 1960s, increased dramatically during the 1970s before the sharp reduction starting in the 1980s associated with the change in monetary policy to fight inflation occurred in several countries. These results are in line with the

US inflation volatility estimated by Stock and Watson (2007). Global inflation volatility has remained very low until mid 2000s, reaching a new spike during the Great Recession, before turning back to the historically low values of the last 3/4 years. Time variation is significant and relatively large throughout the entire estimation sample.

In order to understand which global forces may correlate with global CPI inflation volatility, we estimated two measures of stochastic volatilities from separate univariate AR-SV models for Oil inflation, measured by the WTI price ($/barrel), and for Chinese PPI inflation, available only from the early 1990s. A comparison of median volatilities is reported in Figure 16. From visual inspection, a clear co-movement between Oil and global CPI inflation volatility stands out, showing a correlation around 0.5 from the early 1970s and almost 0.8 from the early 1990s. Also the Chinese PPI inflation volatility displays a positive correlation with global CPI uncertainty: the correlation is around 0.7 from the early 1990s.

4.5 Commonality in core inflations

In light of the correlation (in both levels and volatilities) between the global component of headline CPI inflation and Oil, it is important to detect how much core components of the CPIs remain correlated. To this end, the same exercises of this section have been performed using the non-Food and non-Energy Consumer Prices Indices for the same set of countries, downloaded from the *OECD main economic indicators* database. These data are available only from the late '70s onwards.

Non-Food and non-Energy inflation tends to display a lower degree of commonality, already from a quick graphical inspection. Performing our decompositions, results collected in Appendix C indicate a smaller importance of the common component both in volatilities and in levels: the global core inflation factor explains roughly 25% of the variability of core CPI inflation levels, while the average (across countries) share of stochastic volatility explained by the global component spans from 10% to 20% throughout the sample. The fact that core inflation remains mostly a national phenomenon leaves ample scope for national monetary policies.

5 Forecasting inflation with the MAI-AR-SV model

To provide further evidence on the usefulness of the MAI-AR-SV as a model for multi-country inflation, we now evaluate its out of sample properties, also in comparison with a set of standard competitors.

Using the same inflation series employed in the structural analysis, several models are recursively estimated on a forecasting window of 101 quarterly vintages (forecasting window starts from 1990Q1). The associated out of sample forecasts are produced for six different models and 8 horizons, from 1 to 8 quarters ahead.

The models under evaluation are the following:

- the Multivariate Autoregressive Index model with AR components and Stochastic Volatility (MAI-AR-SV)
- the Multivariate Autoregressive Index model with AR components (MAI-AR)
- the univariate Autoregressive model (AR)
- the univariate Autoregressive model with Stochastic Volatility (AR-SV)
- the Vector Autoregressive model (VAR)
- the Vector Autoregressive model with Stochastic Volatility (VAR-SV)

All models are estimated using Bayesian techniques. AR and VAR priors are constructed using the standard Litterman (1986) a priori assumption of univariate random walk processes. The SV prior in all models is calibrated as in Primiceri (2005). The MAI prior is specified as shown in section 3.

Diagnostics are then computed both in terms of point forecasting and density forecasting, following the evaluation framework of Clark and Ravazzolo (2015).

Specifically, to evaluate the accuracy in terms of point forecasting, we compute the forecasts posterior medians for all vintages, models, variables and horizons. Then, we compute the Root Mean Squared Forecast Error (RMSFE) for each model, variable and horizon, using the variation across vintages. Hence, for each variable $j \in \{1, \dots, n\}$, each horizon $h \in \{1, \dots, H\}$ and each model $m \in \{1, \dots, M\}$ we compute:

$$RMSE^m_{j,h} = \sqrt{\frac{1}{T^*} \sum_{t=T+1}^{T+T^*} \left(y_{j,t+h} - \widehat{y}^m_{j,t+h}\right)^2},$$

where $\widehat{y}^{m}_{j,t+h}$ is the median of the posterior distribution $\left(\widehat{y}^{m,i}_{j,t+h}\right)_{i=1}^{L_c}$ (L_c is the length of the discretized posterior distribution). To test for significance of the squared forecast errors differences across models, we compute the Diebold and Mariano (1995) t-tests for equality of the average loss.

To evaluate models in terms of density forecasting, we use two measures of accuracy: the average log-predictive score and the average Continuous Ranked Probability Score (CRPS). Even in this case, to test for significantly different performances we employ the Diebold and Mariano test, following Clark and Ravazzolo (2015).

Log Predictive Scores are obtained via non-parametric kernel smoothing density estimators. Adopting a normal kernel $\mathcal{K}_{\mathcal{N}}(\cdot)$ and following an optimal selection strategy of the bandwith parameter $\widehat{\mathcal{H}}$, we can compute for each variable, model, horizon and vintage the empirical density evaluted at the actual observation $y_{j,t+h}$, that is:

$$\widehat{f}_m\left(y_{j,t+h},\widehat{\mathcal{H}}\right)=\frac{1}{\widehat{\mathcal{H}}\cdot L_c}\sum_{i=1}^{L_c}\mathcal{K}_{\mathcal{N}}\left(\frac{y_{j,t+h}-\widehat{y}^{m,i}_{j,t+h}}{\widehat{\mathcal{H}}}\right).$$

Then, applying logarithms and computing the average across forecasting vintages yields the average log score for each variable, model and horizon:

$$\overline{logScore}^{m}_{j,h}=\frac{1}{T^*}\sum_{t=T+1}^{T+T^*}\log\widehat{f}_m\left(y_{j,t+h},\widehat{\mathcal{H}}\right).$$

To compute the average CRPS, following Clark and Ravazzolo (2015), we first compute the CRPS per each variable, model, horizon and vintage, making use of the actual observations, the posterior distribution $\left(\widehat{y}^{m,i}_{j,t+h}\right)_{i=1}^{L_c}$ and a random permutation of the latter $\left(\widehat{y}^{m,i'(i)}_{j,t+h}\right)_{i=1}^{L_c}$ where $i':\{1,\ldots,L_c\}\to\{1,\ldots,L_c\}$ is randomly drawn without replacement. Lastly, we simply compute the average across time vintages:

$$CRPS^{m}_{j,t+h}=\frac{1}{L_c}\sum_{i=1}^{L_c}\left|\widehat{y}^{m,i}_{j,t+h}-y_{j,t+h}\right|-\frac{1}{2\cdot L_c}\sum_{i=1}^{L_c}\left|\widehat{y}^{m,i}_{j,t+h}-\widehat{y}^{m,i'(i)}_{j,t+h}\right|,$$

$$\overline{CRPS}^{m}_{j,h}=\frac{1}{T^*}\sum_{t=T+1}^{T+T^*}CRPS^{m}_{j,t+h}.$$

Figure 17 portrays the relative performance of the competing set of models against the benchmark model MAI-AR-SV, for each country and four selected horizons. Models' point forecasting performance is reported as ratio between their own Root Mean Squared Errors and the benchmark's, so that values larger than one imply that the MAI-AR-SV produces more accurate point forecasts. The MAI-AR-SV model improves significantly upon its counterparts on most variables, especially at short horizons, even though in a smaller number of cases this is reversed. The AR-SV shows competitive point forecasting performance, especially at longer horizons. The highly parametrized VARs generally achieve a lower degree of point forecasting accuracy than the benchmark. Tables 1a and 1b in the appendix report detailed results.

Moving to density forecast evaluation, Figure 18 reports the relative average log predictive scores for the chosen set of models and horizons. Alternative models' performance is reported in terms of log-scores differences with the benchmark MAI-AR-SV, so that negative values favor the MAI-AR-SV. The benchmark model clearly improves upon its competitors: the difference is negative and significant in most cases. Eventually, Figure 19 reports the CRPS reported comparatively as a ratio, where values greater than one indicates a worse density forecasting performance with respect to the MAI-AR-SV. Results are in line with the log-scores, with the benchmark model improving significantly upon its competitors[12].

To conclude, MAI-AR-SV is also a good forecasting model for inflation rates. The introduction of SV is particularly relevant to improve density forecasts. This evidence is in line with findings reported by Clark and Ravazzolo (2015) and D'Agostino et al. (2013). On the other hand, even though the AR-SV shows already good point and density forecasting power for inflation rates, the introduction of a MAI component proves to be quite beneficial. Furthermore, notwithstanding the smaller number of coefficients due to the reduced rank restriction imposed by the MAI structure, the benchmark model attains a higher degree of forecasting accuracy with respect to the standard unrestricted VAR estimated using a Minnesota Prior as shrinkage device.

6 Conclusions

Global developments play an important role in the determination of inflation rates, and indeed earlier literature has found that a substantial amount of the variation in a large

[12]More detailed forecasting results are reported in Appendix B.

set of national inflation rates can be explained by a single global factor. This literature has typically neglected inflation (conditional) volatility, while volatility is clearly relevant both from a policy point of view and for structural analysis and forecasting.

In this paper we study the evolution of inflation rates in many countries, using a novel model that allows for commonality in both levels and volatilities, in addition to country-specific components. We find that allowing for inflation volatility is indeed important, and a large fraction of it can be attributed to a global factor that is also driving the inflation levels.

While other sources can be behind this global factor, it turns out that since the early '90s it is strongly correlated with the Chinese PPI and Oil prices. Moreover, also the global factor stochastic volatility is highly correlated with that of Chinese PPI and Oil prices.

Repeating the same analysis on core inflation rates for the same set of OECD countries, the model finds a smaller but non-negligible degree of commonality. The substantial national component of core inflation level and volatility leaves ample scope for local monetary policies.

The MAI-AR-SV shows also very good out of sample properties, achieving comparatively better forecasting performances when compared with a set of prominent alternative models, especially in terms of density forecasting.

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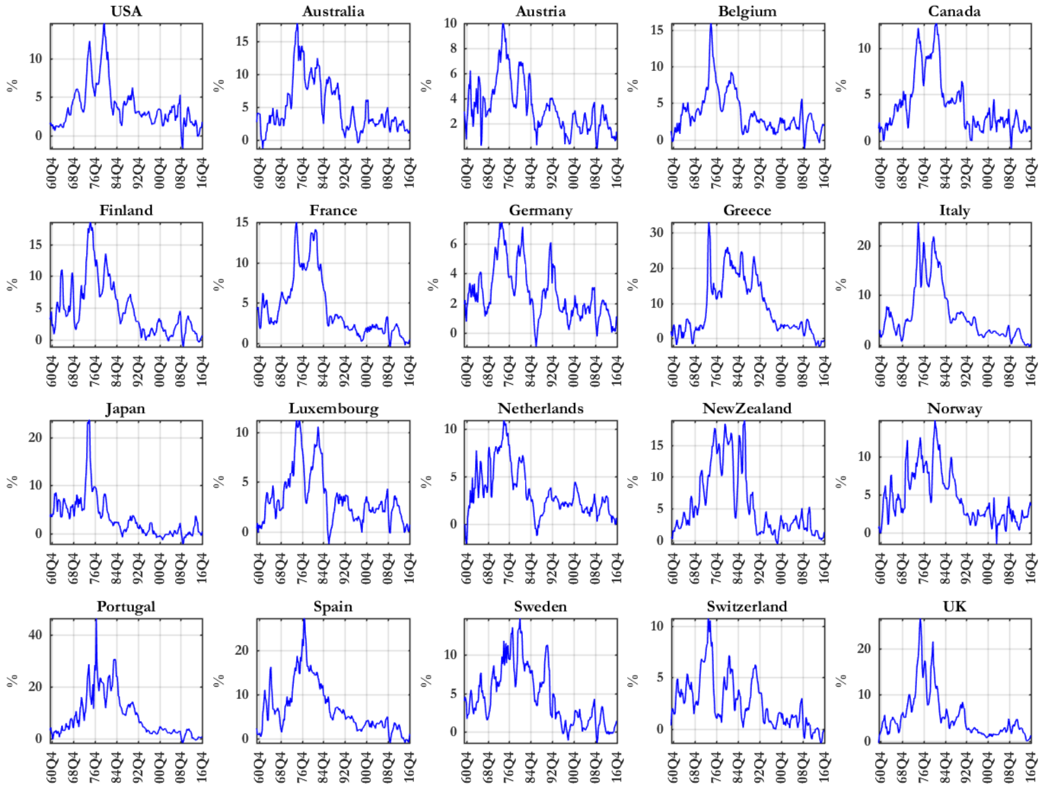


Figure 1: Inflation rates (year on year growth rates in quarterly CPIs)

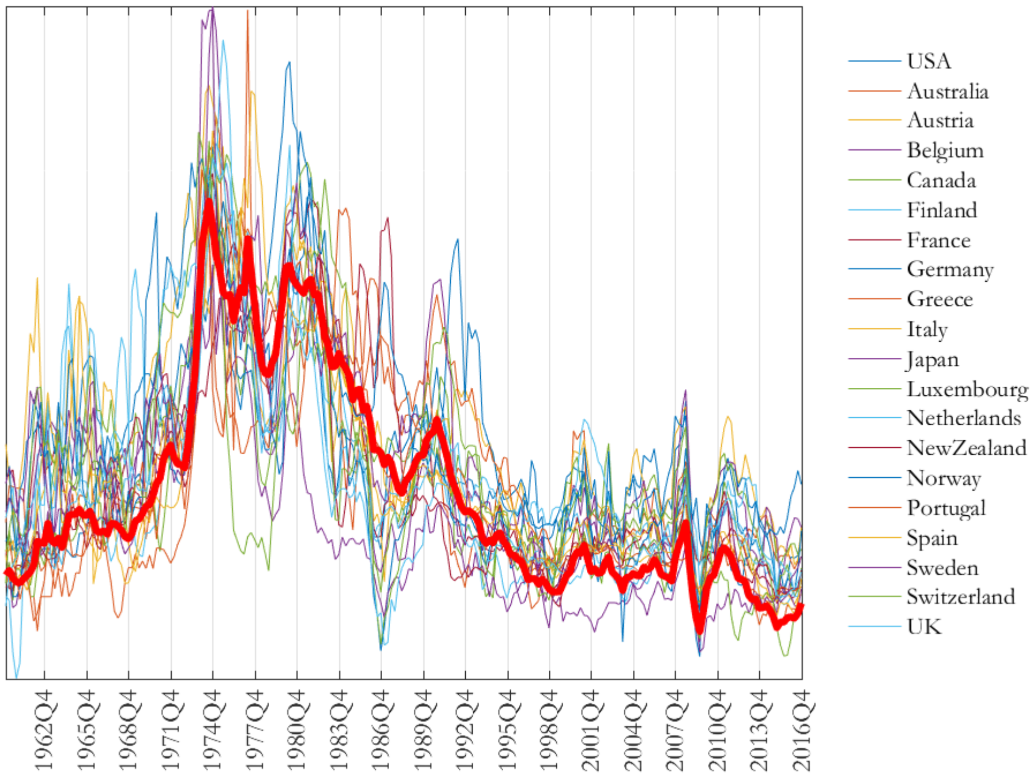


Figure 2: Inflation rates and their first principal component (thick red line)

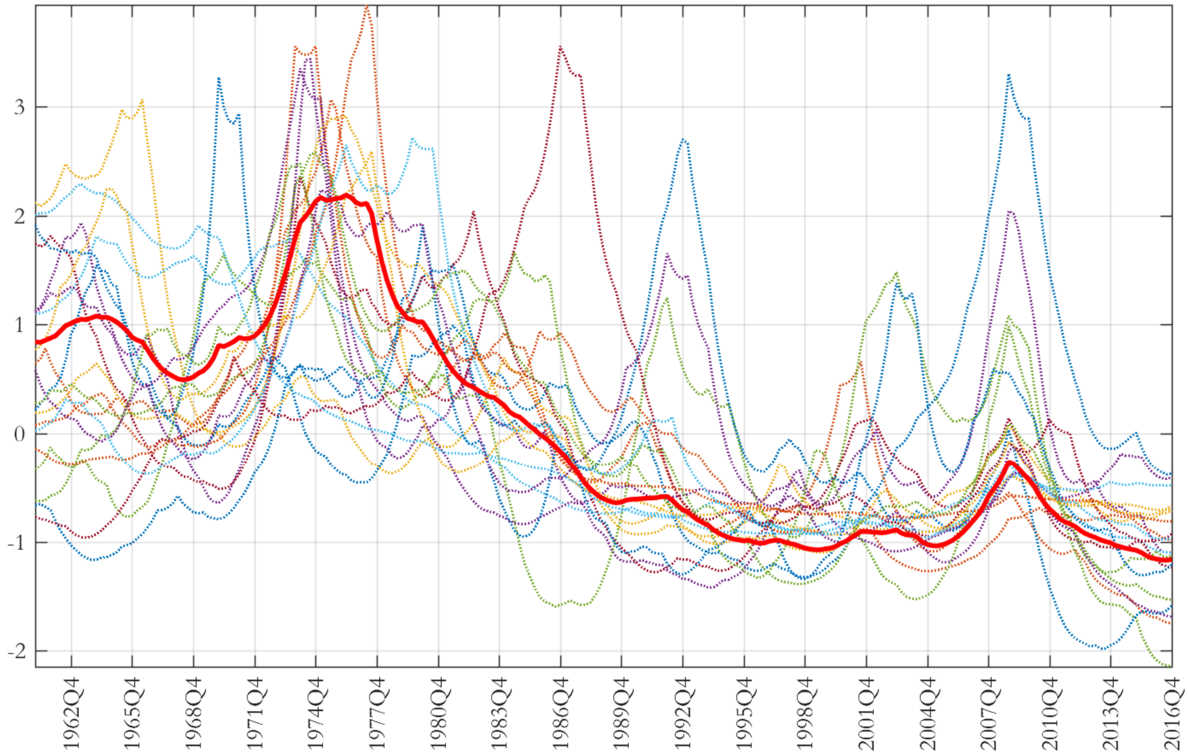


Figure 3: CPI inflation rates Stochastic Volatilities estimated from univariate AR-SV, and their first Principal Component (thick red line)

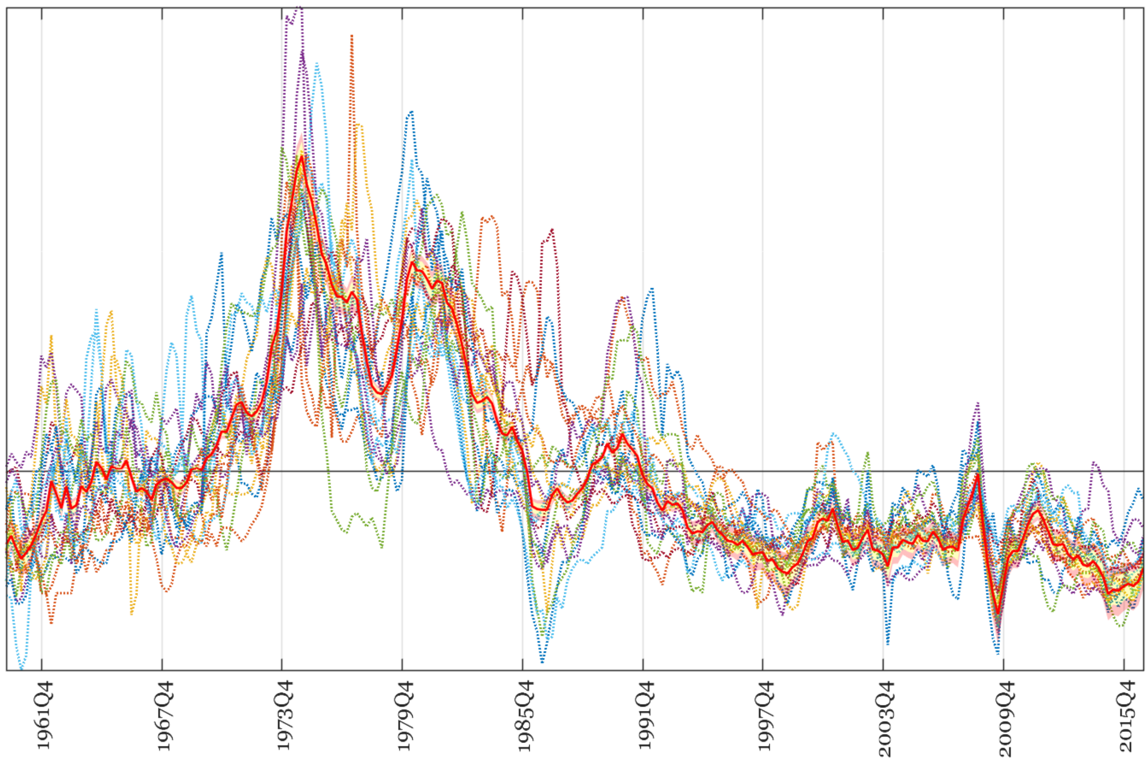


Figure 4: MAI-SV estimated common factor (with posterior bands) Vs Data

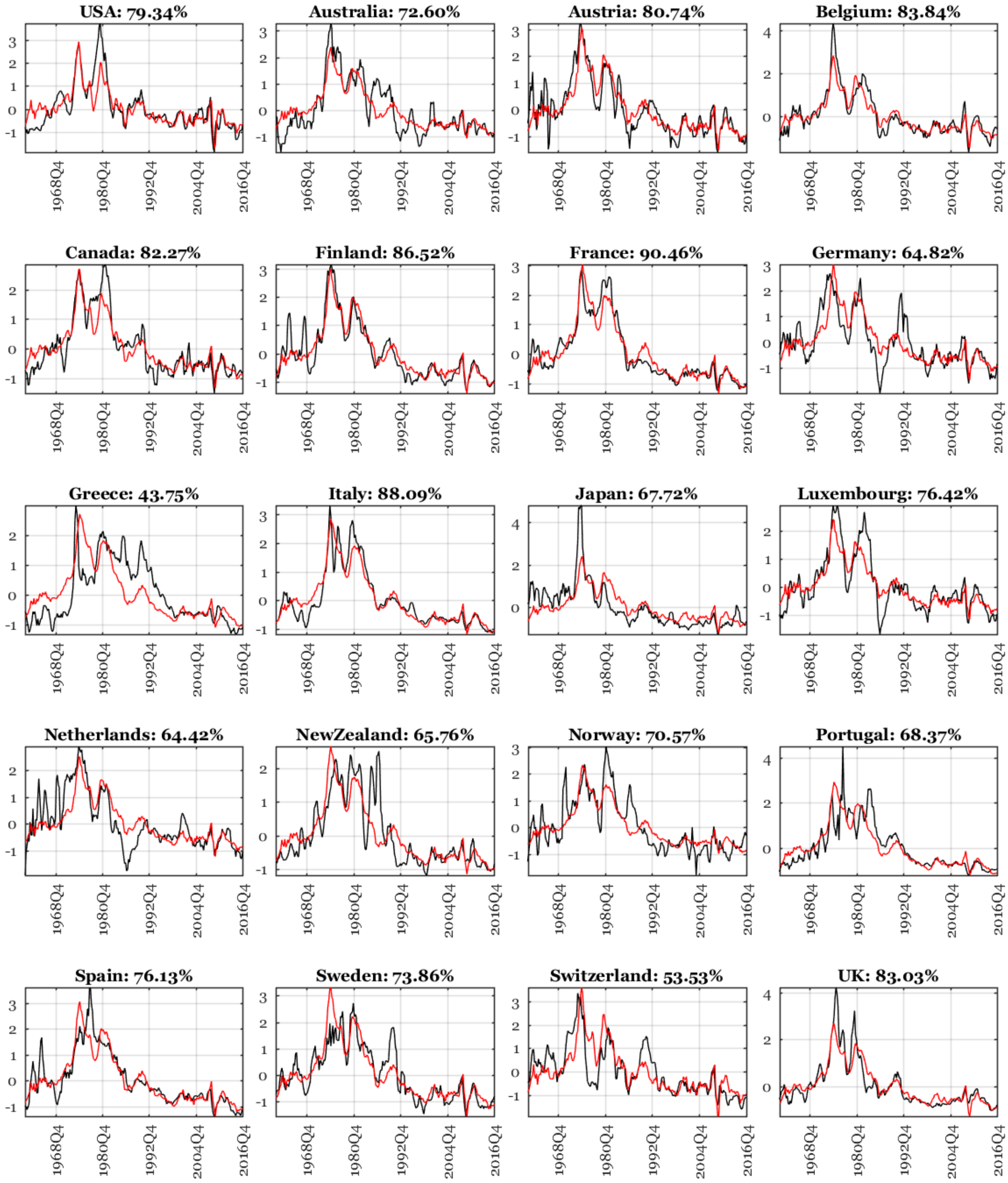


Figure 5: MAI-SV in-sample fit

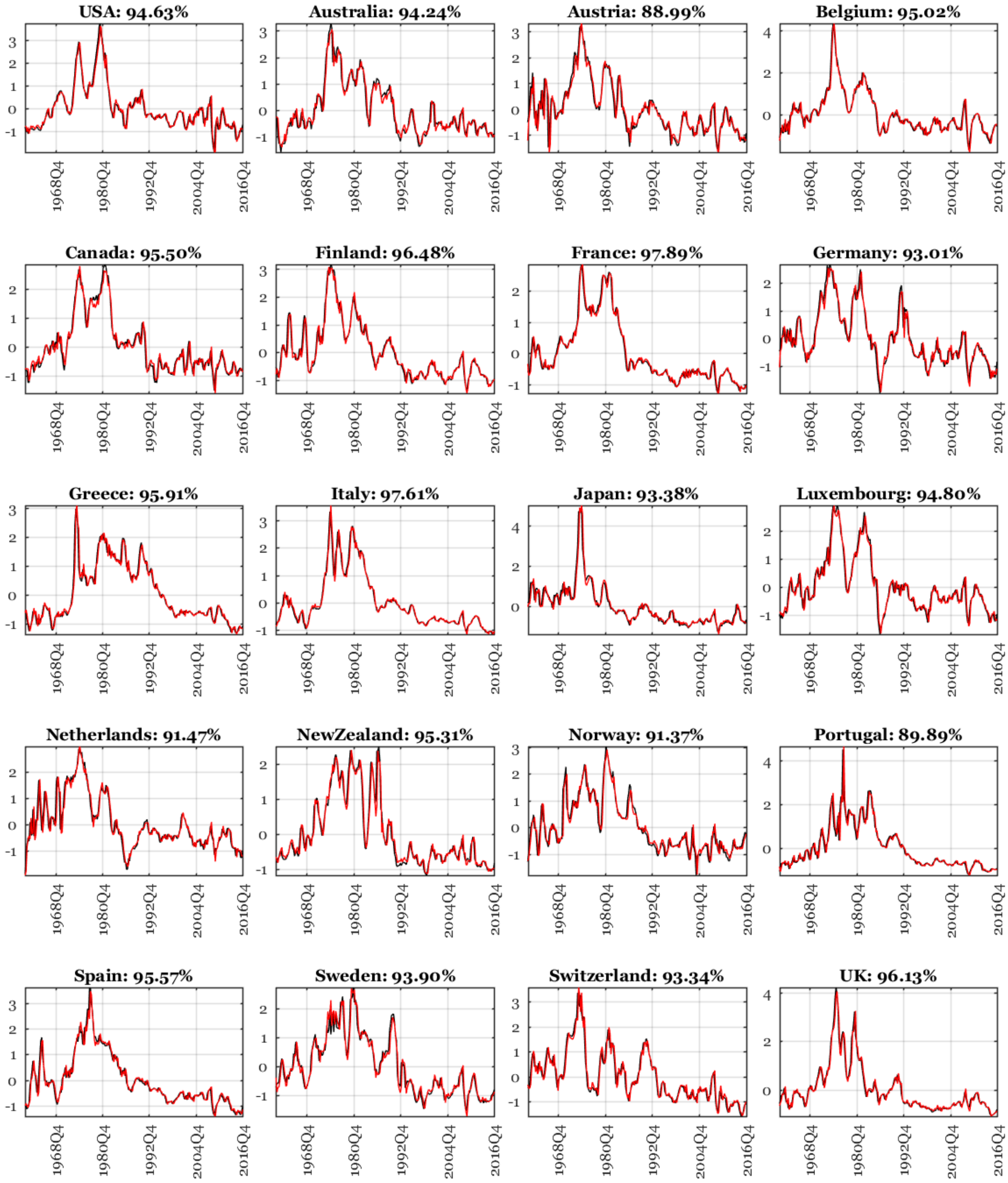


Figure 6: MAI-AR-SV in-sample fit

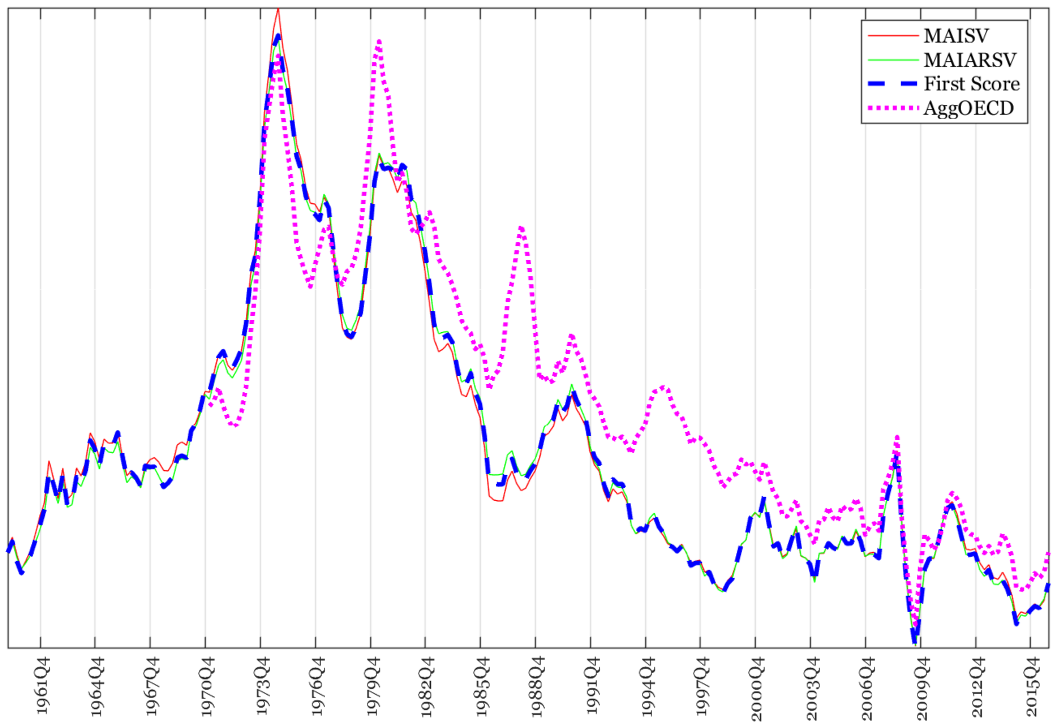


Figure 7: Comparing common factor

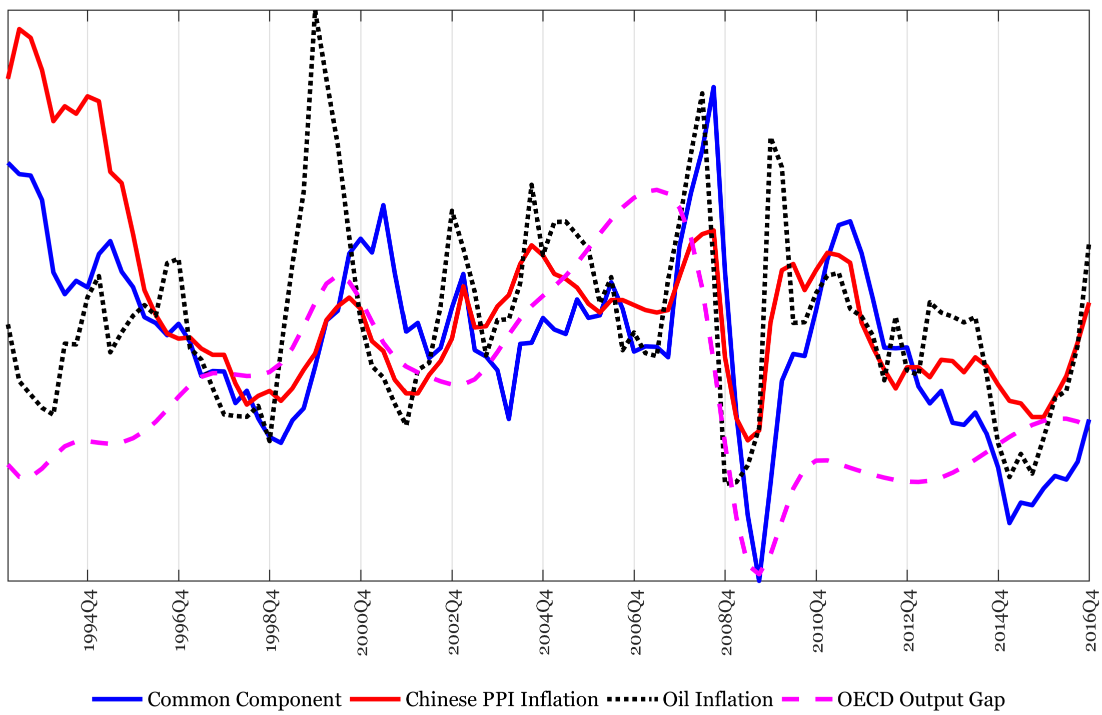


Figure 8: Comparing MAI component with Chinese PPI, Oil Inflation and Global Output Gap

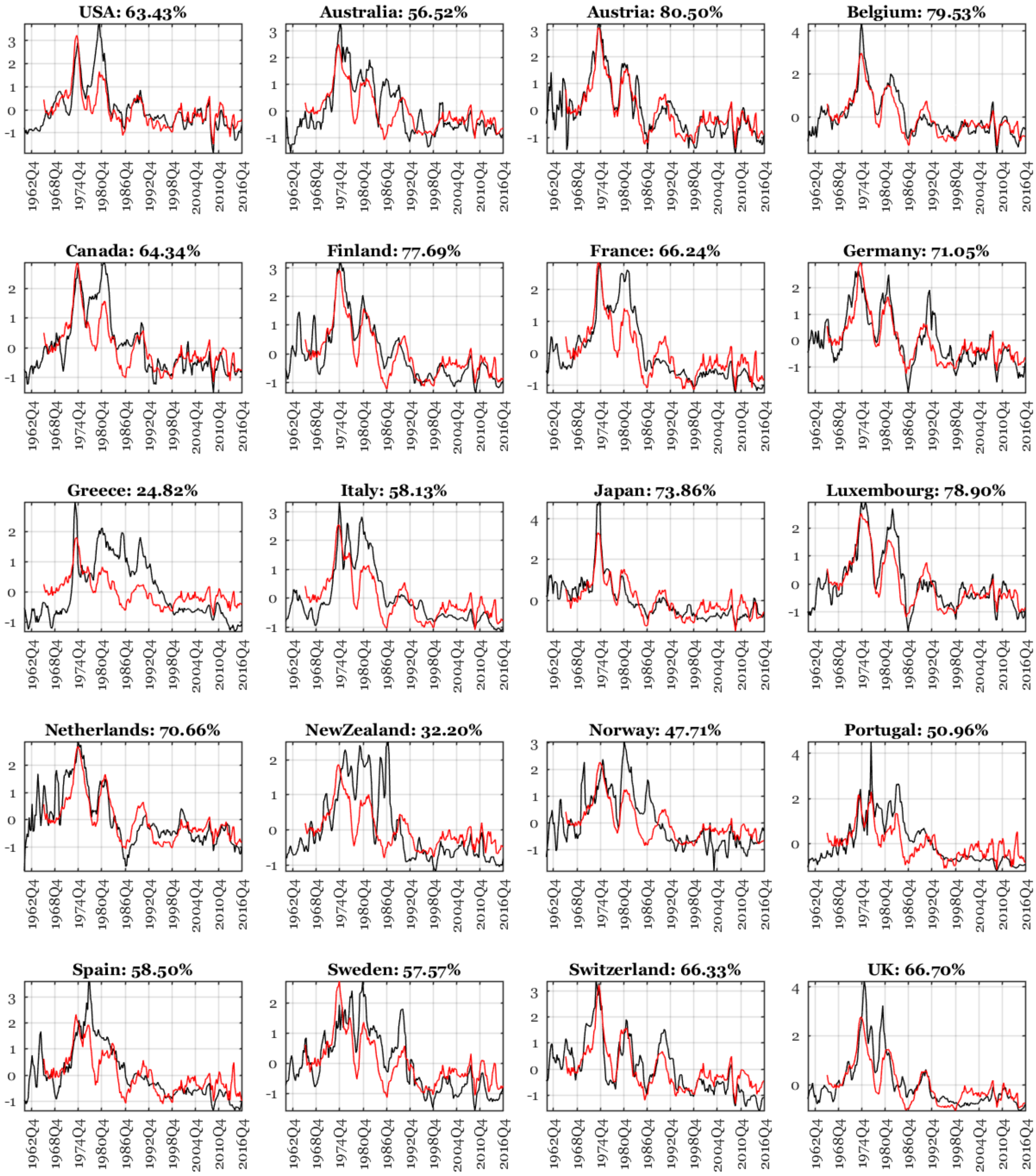


Figure 9: MAI-AR-SV, Actual series and Common component (red)

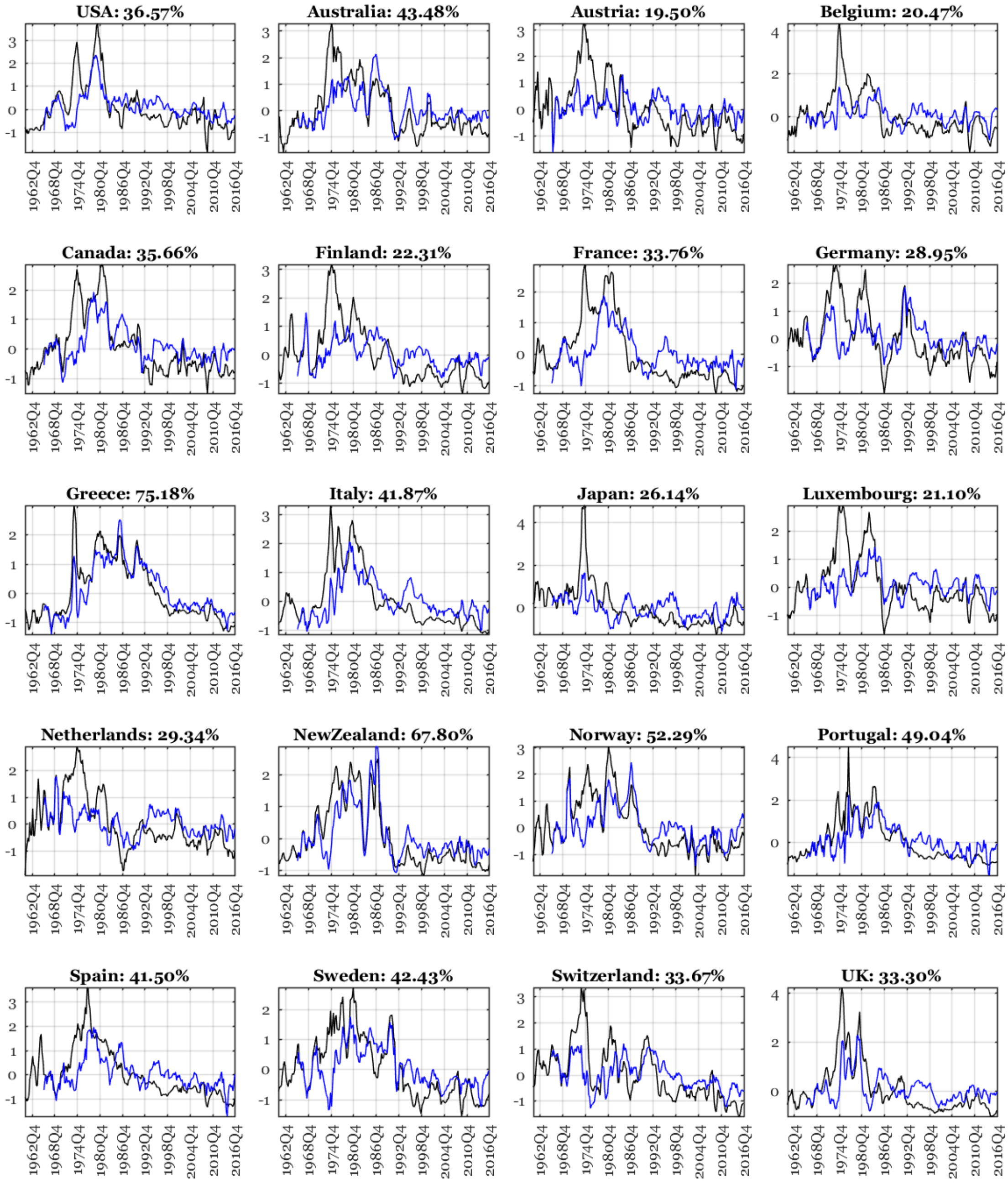


Figure 10: MAI-AR-SV, Actual series and Idiosyncratic component (blue)

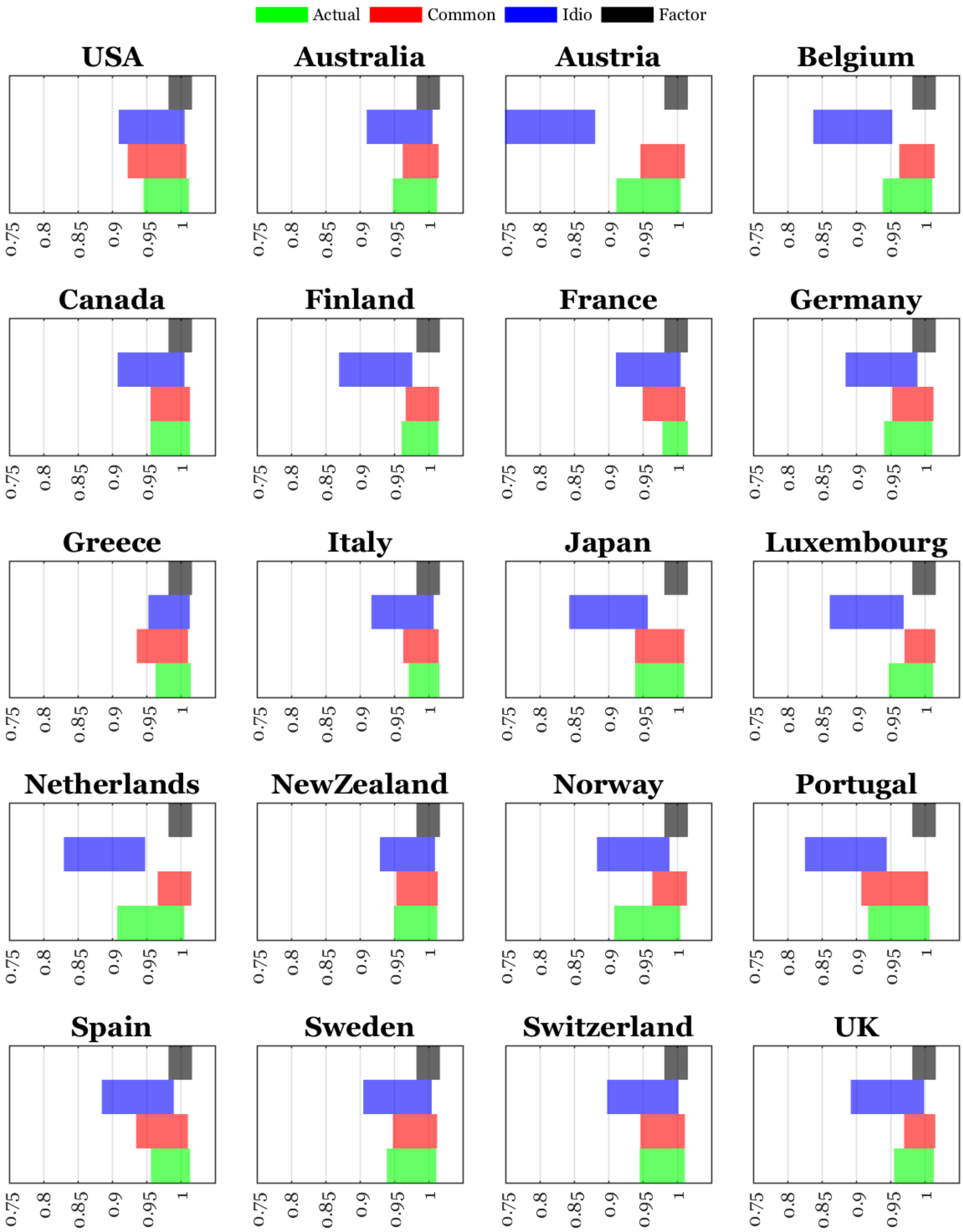


Figure 11: Largest Autoregressive Root (90% confidence intervals) CPI inflation levels, components, and global factor.

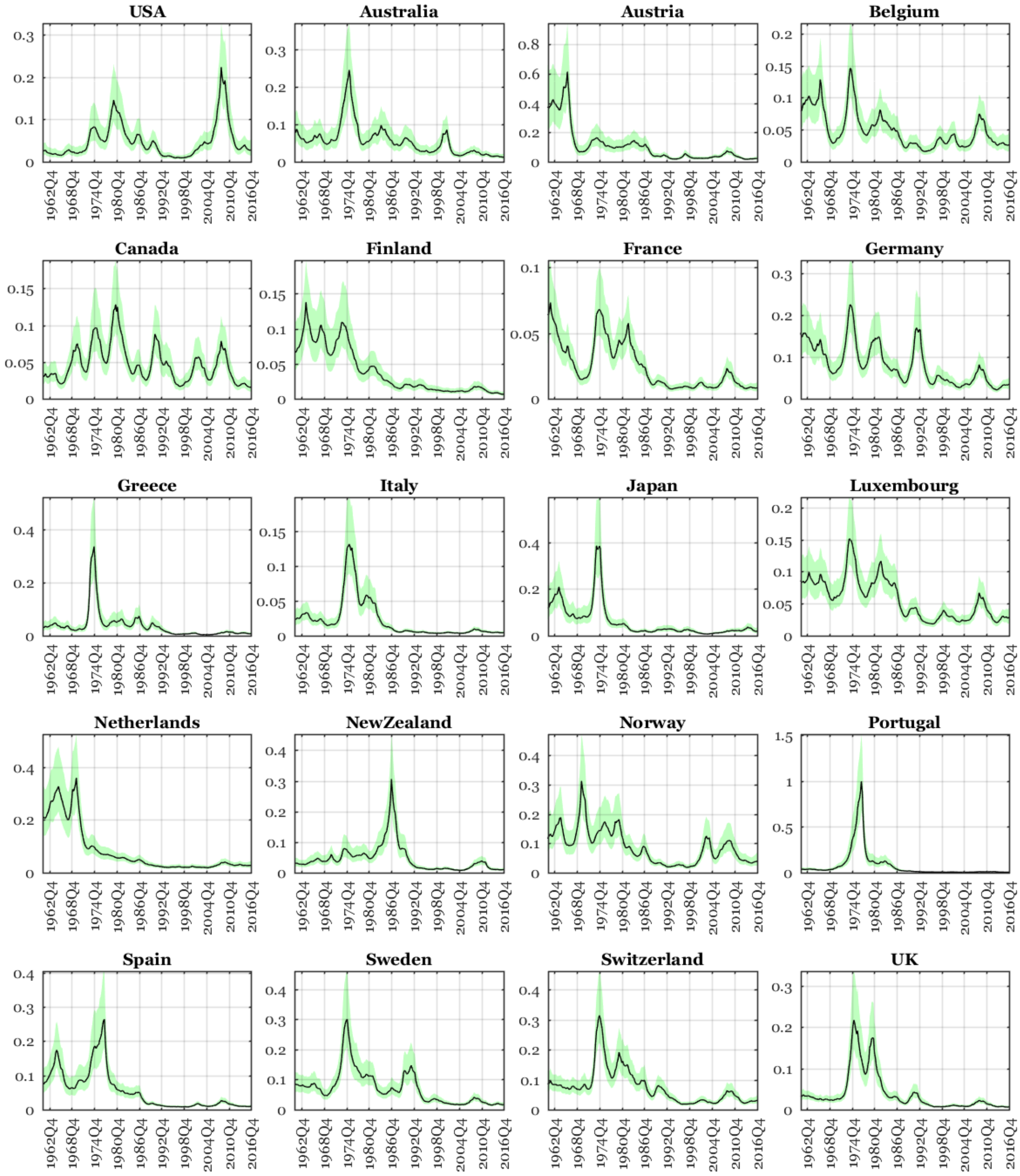


Figure 12: MAI-AR-SV, Residuals' Volatility, posterior bands

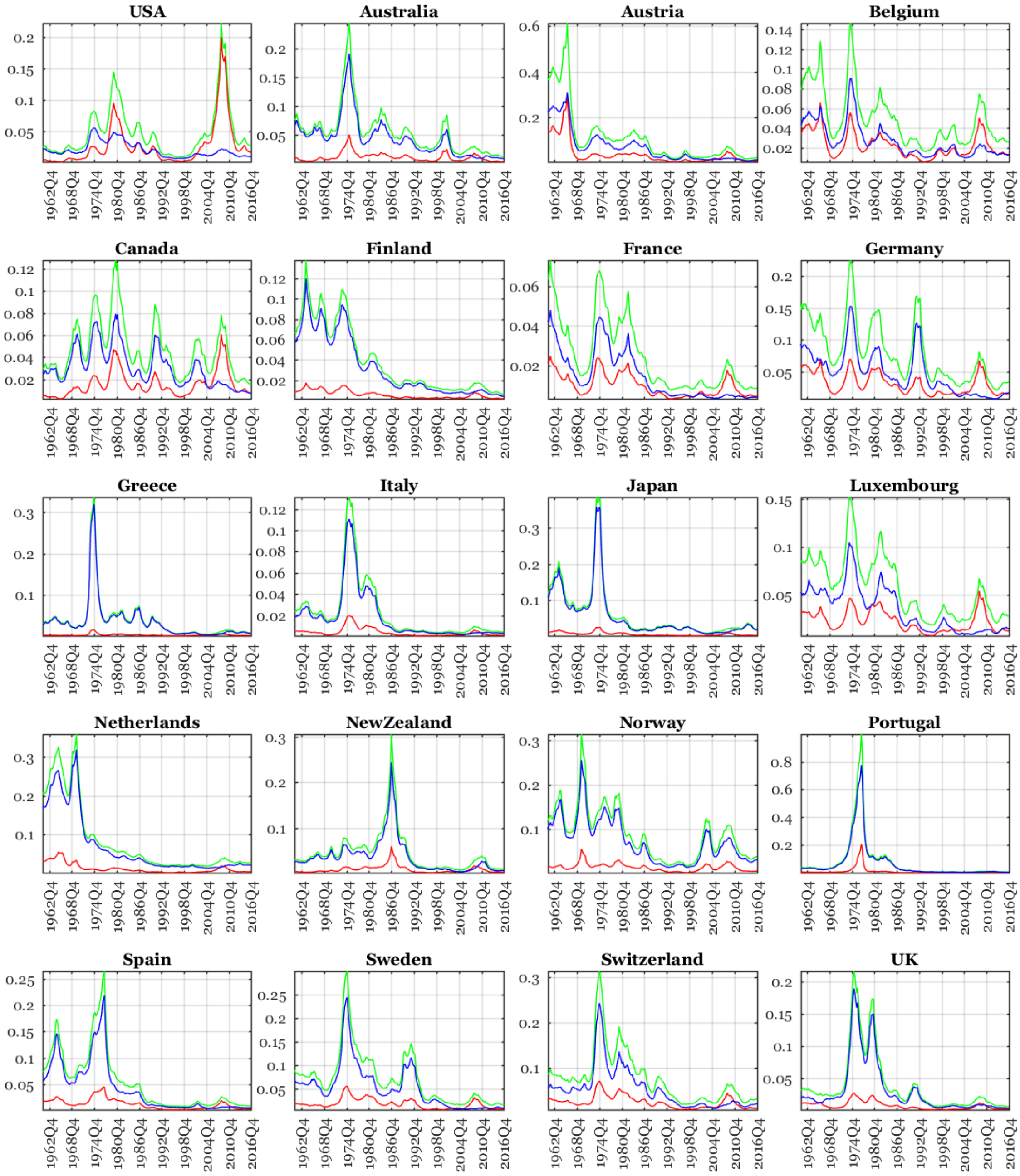


Figure 13: MAI-AR-SV, Residuals' Volatility, TV decomposition, Common (red), Idio (blue), total (green)

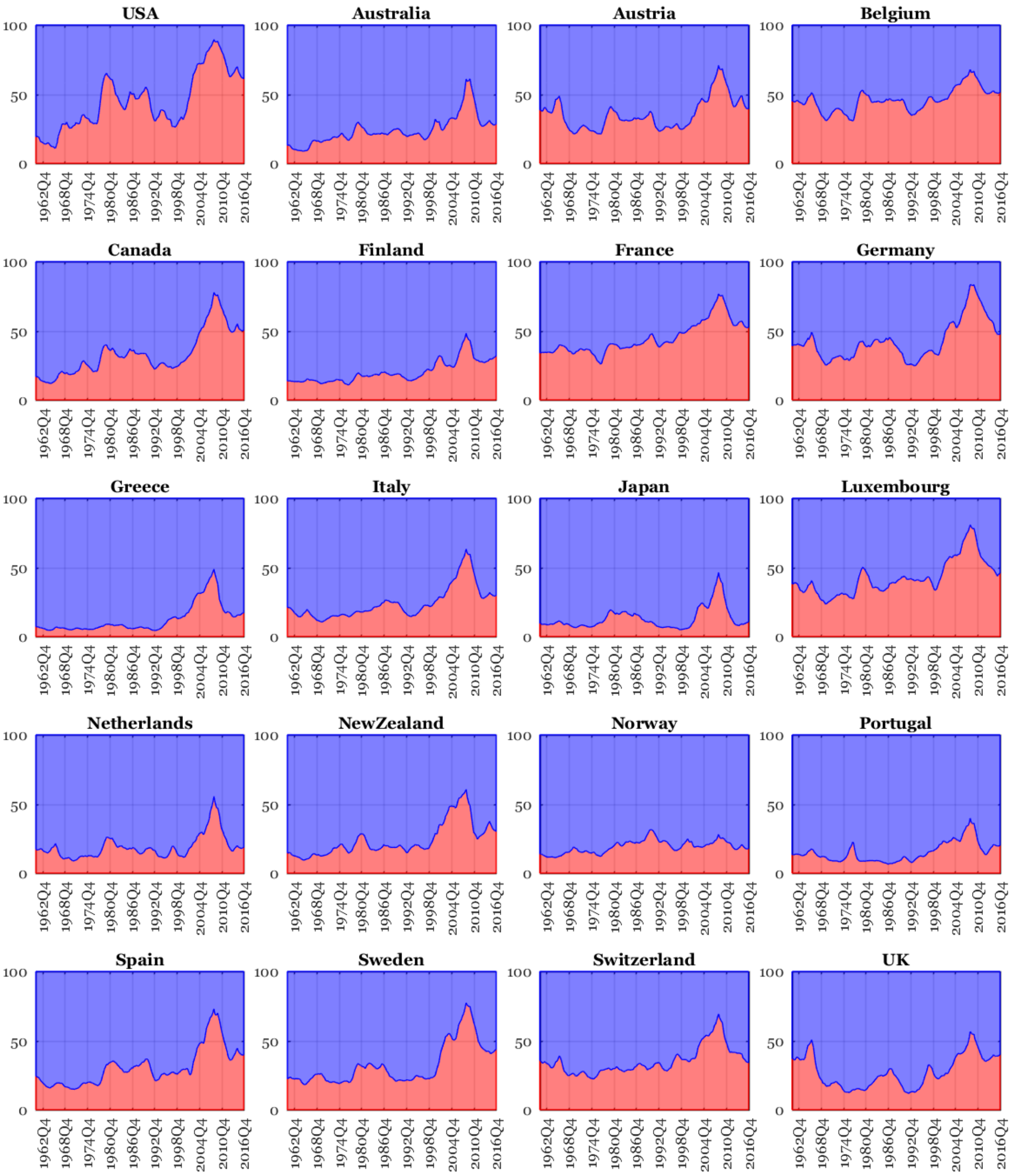


Figure 14: MAI-AR-SV, Residuals' Volatility, TV decomposition shares (%), Common (red), Idio (blue)

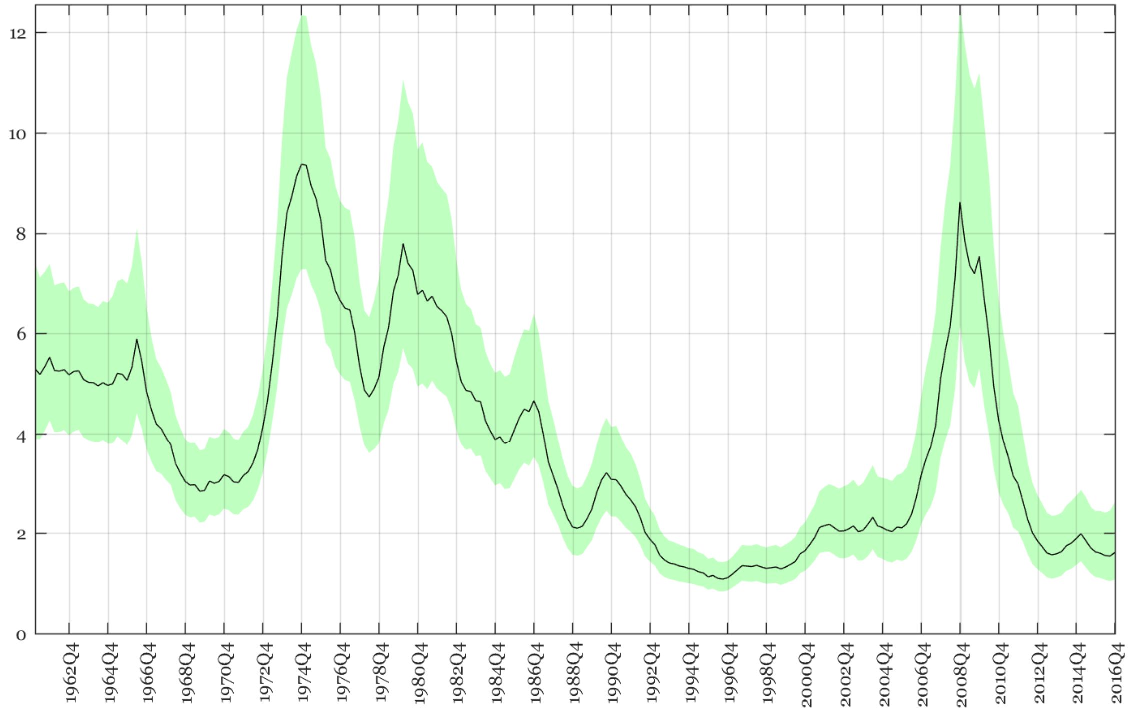


Figure 15: MAI-AR-SV, Global Factor Volatility, Posterior bands.

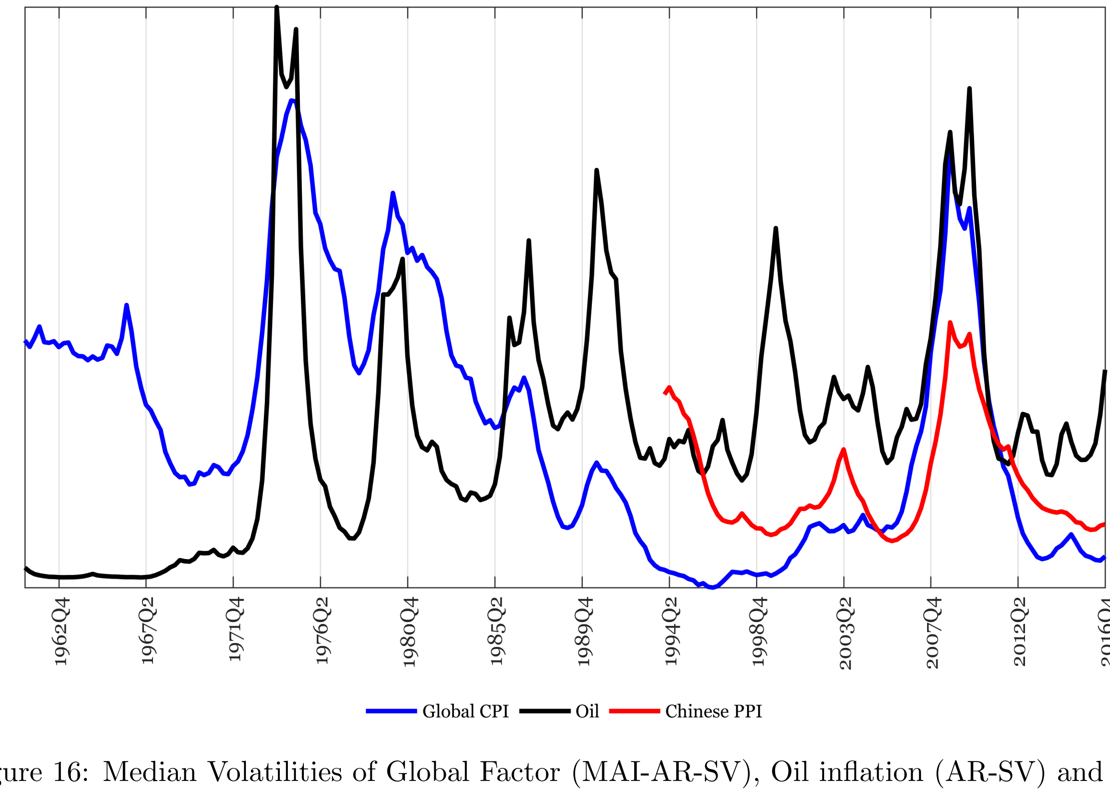


Figure 16: Median Volatilities of Global Factor (MAI-AR-SV), Oil inflation (AR-SV) and Chinese PPI (AR-SV)

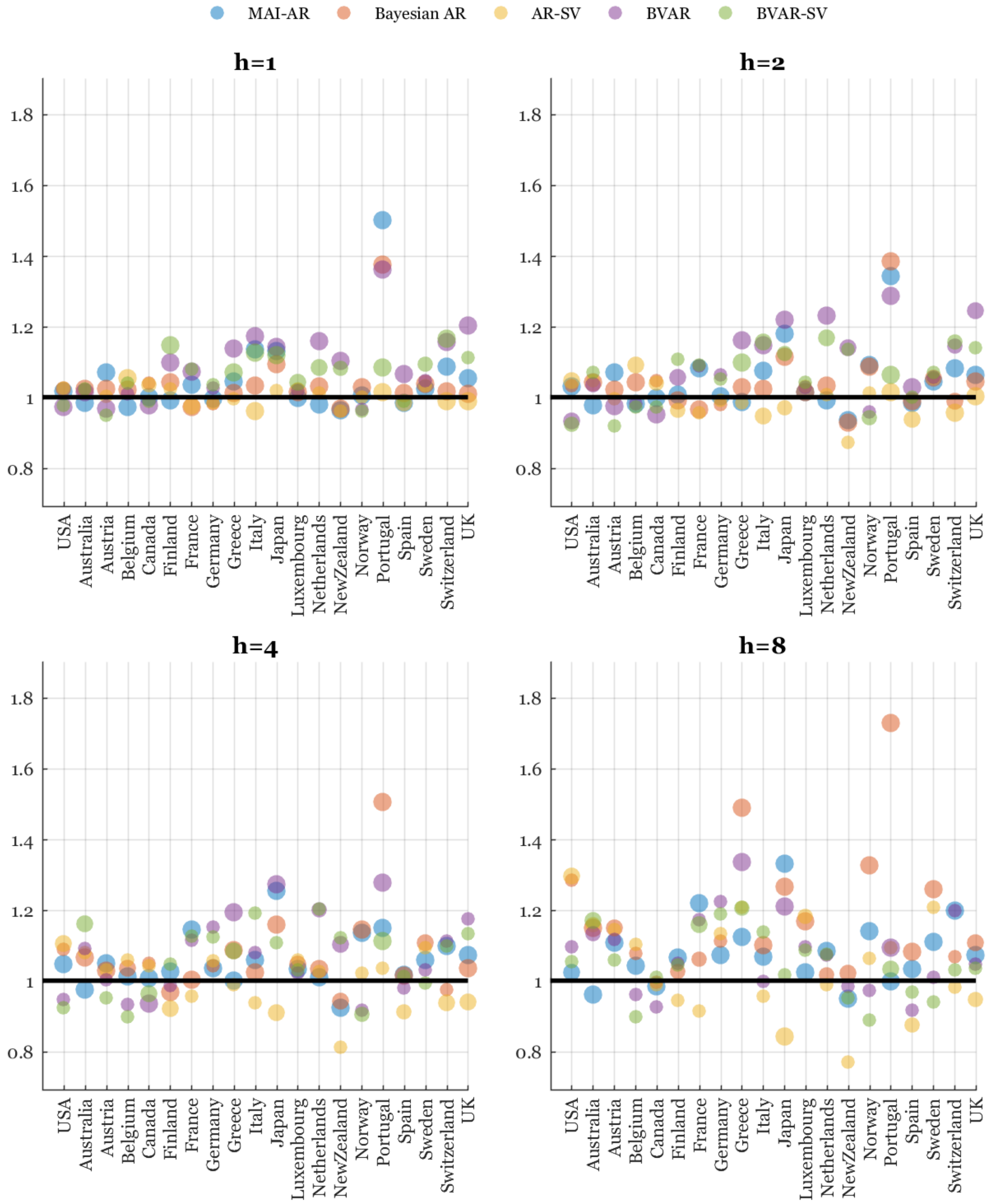
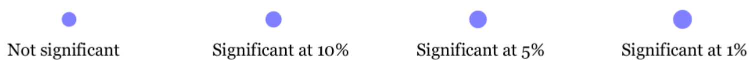


Figure 17: Relative Root Mean Squared Forecast Errors (ratios with MAI-AR-SV)

The round filled marker is larger when the difference is significant according to the Diebold Mariano t-statistic, see legend below.



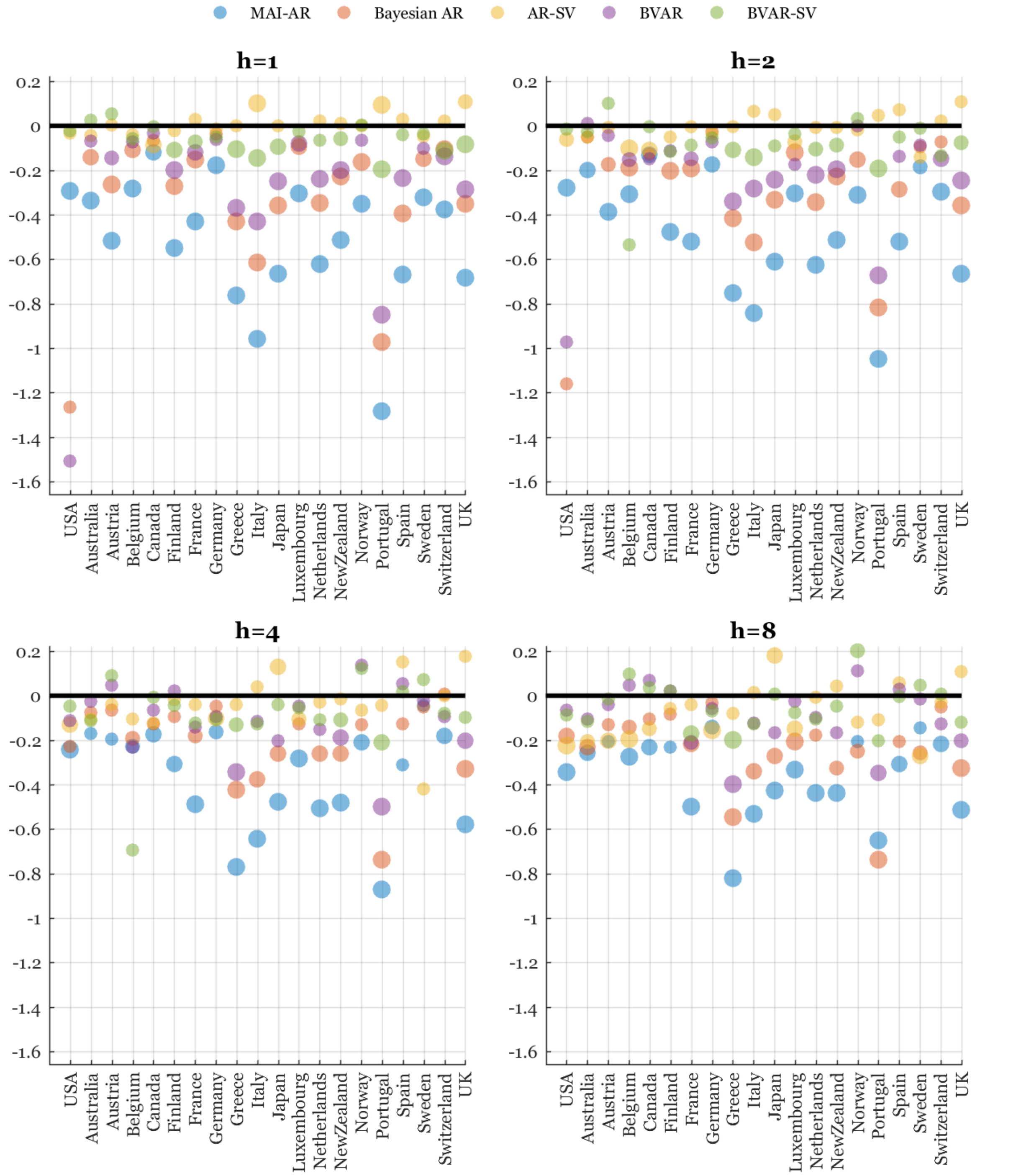
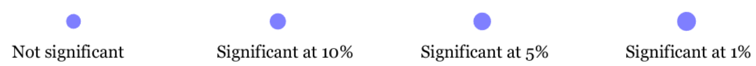


Figure 18: Relative Log Predictive Scores (differences with MAI-AR-SV)

The round filled marker is larger when the difference is significant according to the Diebold Mariano t-statistic, see legend below.



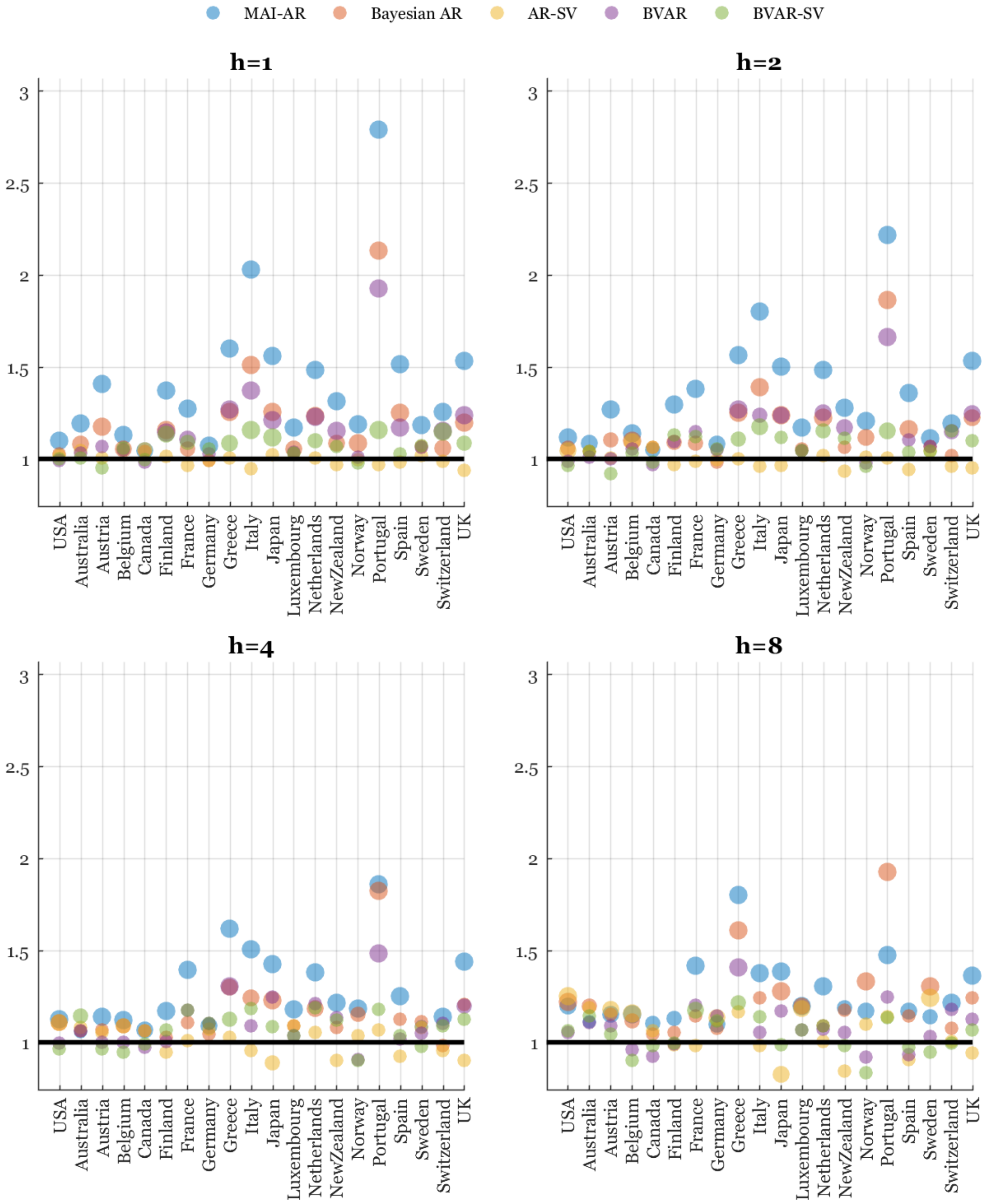
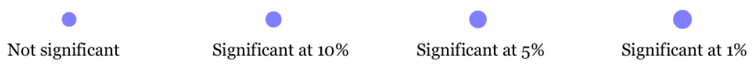


Figure 19: Relative Continuous Rank Probability Scores (ratios with MAI-AR-SV)

The round filled marker is larger when the difference is significant according to the Diebold Mariano t-statistic, see legend below.



A Gibbs Sampler for estimation of the MAI-AR-SV model

A.1 Step 1: draw AR-coefficients γ

The AR components included in the MAI-AR-SV are stacked in the following way:

$$
\begin{aligned}
y_t &= \sum_{\ell=1}^{q} \Gamma_\ell \cdot y_{t-\ell} + A \cdot Z_t + u_t, \\
y_t &= \sum_{\ell=1}^{q} \begin{bmatrix} \gamma_{1,\ell} & 0 & \dots & 0 \\ 0 & \gamma_{2,\ell} & \ddots & 0 \\ 0 & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \gamma_{n,\ell} \end{bmatrix} \cdot y_{t-\ell} + A \cdot Z_t + u_t, \\
y_t &= \sum_{\ell=1}^{q} \begin{bmatrix} y_{1,t-\ell} & 0 & \dots & 0 \\ 0 & y_{2,t-\ell} & \ddots & 0 \\ 0 & \ddots & \ddots & 0 \\ 0 & \dots & 0 & y_{n,t-\ell} \end{bmatrix} \cdot \begin{bmatrix} \gamma_{1,\ell} \\ \gamma_{2,\ell} \\ \vdots \\ \gamma_{n,\ell} \end{bmatrix} + A \cdot Z_t + u_t, \\
y_t &= \sum_{\ell=1}^{q} \mathcal{Y}_{t-\ell} \cdot \gamma_\ell + A \cdot Z_t + u_t, \\
y_t &= \begin{bmatrix} \mathcal{Y}_{t-1} & \mathcal{Y}_{t-2} & \dots & \mathcal{Y}_{t-q} \end{bmatrix} \cdot \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_q \end{bmatrix} + A \cdot Z_t + u_t,
\end{aligned}
$$

so that we can eventually write a fully stacked form:

$$
y_t = \underbrace{\mathcal{X}_t}_{n \times nq} \cdot \underbrace{\gamma}_{nq \times 1} + \underbrace{A}_{n \times rp} \cdot \underbrace{Z_t}_{rp \times 1} + u_t.
$$

Given B_0 and A, we can transform the matrix form to have the following linear regression model with common coefficients and variable specific regressors:

$$\begin{aligned} y_t &= \mathcal{X}_t \cdot \gamma + \ A \cdot \ Z_t + u_t, \\ y_t - A \cdot Z_t &= \mathcal{X}_t \cdot \gamma + u_t, \\ y_t^\circ &= \mathcal{X}_t \cdot \gamma + u_t, \end{aligned}$$

with

$$u_t \overset{i}{\sim} \mathcal{MN}\left(\mathbf{0},\ \underbrace{\Omega_t}_{n\times n} \right), \qquad u_t = G^{-1}\Sigma_t \varepsilon_t, \qquad \varepsilon_t \overset{iid}{\sim} \mathcal{MN}\left(\ \mathbf{0},\ I_n \ \right).$$

Considering separate equations to estimate the AR coefficients contained in γ would ignore the cross-correlations of the innovations in u_t. Considering that within the GS we draw directly the elements g in the matrix G and the stochastic volatilities σ_t in Σ_t, for efficiency purposes we can compute the following transformation of the equation:

$$\begin{aligned} y_t^\circ &= \mathcal{X}_t \cdot \gamma + u_t, \\ y_t^\circ &= \mathcal{X}_t \cdot \gamma + G^{-1}\Sigma_t \varepsilon_t, \\ \Sigma_t^{-1} G \cdot y_t^\circ &= \Sigma_t^{-1} G \cdot \mathcal{X}_t \cdot \gamma + \underbrace{\Sigma_t^{-1} G \cdot G^{-1} \Sigma_t}_{I_n} \cdot \varepsilon_t, \\ \Sigma_t^{-1} G \cdot y_t^\circ &= \Sigma_t^{-1} G \cdot \mathcal{X}_t \cdot \gamma + \varepsilon_t, \\ \widetilde{y_t^\circ} &= \widetilde{\mathcal{X}_t} \cdot \gamma + \varepsilon_t. \end{aligned}$$

Finally obtaining a multivariate linear regression with homoskedastic residuals and unitary diagonal covariance matrix:

$$\widetilde{y_t^\circ} = \widetilde{\mathcal{X}_t} \cdot \gamma + \varepsilon_t, \qquad \varepsilon_t \overset{iid}{\sim} \mathcal{MN}\left(\ \mathbf{0},\ I_n \ \right).$$

The transformed model can be stacked in columns:

$$\begin{bmatrix} \widetilde{y}_1^{\circ} \\ \vdots \\ \widetilde{y}_T^{\circ} \end{bmatrix} = \begin{bmatrix} \widetilde{\mathcal{X}}_1 \\ \vdots \\ \widetilde{\mathcal{X}}_T \end{bmatrix} \cdot \gamma + \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_T \end{bmatrix},$$

$$\underbrace{\widetilde{Y}^{\circ}}_{nT\times 1} = \underbrace{\widetilde{\mathcal{X}}}_{nT\times nq} \cdot \underbrace{\gamma}_{nq\times 1} + \underbrace{\varepsilon^{\circ}}_{nT\times 1}, \qquad \varepsilon^{\circ} \sim \mathcal{MN}\left(\mathbf{0},\ I_T \otimes I_n \right).$$

With the stacked version of the model, adopting the Normal conjugate prior for coefficients γ:

$$\gamma \sim \mathcal{MN}\left(\bar{\gamma},\ V_{\gamma} \right),$$

we can eventually draw from the posterior of γ:

$$\gamma \sim \mathcal{MN}\left(\widetilde{\gamma},\ \widetilde{V}_{\gamma} \right),$$

where

$$\widetilde{\gamma} = \widetilde{V}_{\gamma} \cdot \left(\widetilde{\mathcal{X}}' \cdot \widetilde{Y}^{\circ} + V_{\gamma}^{-1} \cdot \bar{\gamma} \right), \qquad \widetilde{V}_{\gamma} = \left(\widetilde{\mathcal{X}}' \cdot \widetilde{\mathcal{X}} + V_{\gamma}^{-1} \right)^{-1}.$$

A.2 Step 2: Draw loadings A

The second step of the GS aims at drawing the loadings contained in A. Recalling the following:

$$A \equiv \begin{bmatrix} A_1 & \dots & A_p \end{bmatrix}, \qquad x_t \equiv vec\left(x_t^{\bullet} \right), \qquad \underbrace{x_t^{\bullet}}_{n\times p} \equiv \begin{bmatrix} y_{t-1} & \dots & y_{t-p} \end{bmatrix},$$

$$Z_t \equiv \begin{bmatrix} B_0 y_{t-1} \\ \vdots \\ B_0 y_{t-p} \end{bmatrix} = \left(I_p \otimes B_0 \right) \cdot x_t = vec\left(B_0 \cdot x_t^{\bullet} \right),$$

the model can be restated as:

$$y_t - \mathcal{X}_t \cdot \gamma = \begin{bmatrix} A_1 & \dots & A_p \end{bmatrix} \begin{bmatrix} B_0 y_{t-1} \\ \vdots \\ B_0 y_{t-p} \end{bmatrix} + u_t,$$

$$y_t^\bullet = \underbrace{A}_{n\times rp} \cdot \underbrace{Z_t}_{rp\times 1} + u_t,$$

and can be stacked as:

$$\begin{bmatrix} y_1^{\bullet\prime} \\ y_2^{\bullet\prime} \\ \vdots \\ y_T^{\bullet\prime} \end{bmatrix} = \begin{bmatrix} Z_1' \\ Z_2' \\ \vdots \\ Z_T' \end{bmatrix} A' + \begin{bmatrix} u_1' \\ u_2' \\ \vdots \\ u_T' \end{bmatrix},$$

$$\underbrace{y^\bullet}_{T\times n} = \underbrace{Z}_{T\times rp} \cdot \underbrace{A'}_{rp\times n} + u.$$

Defining $a \equiv vec\,(A')$, and exploiting the Kronecker product's properties, this form can be vectorized and transformed in:

$$vec\,(y^\bullet) = vec\,(Z \cdot A' \cdot I_n) + vec\,(u)\,,$$

$$\underbrace{Y^\bullet}_{nT\times 1} = \underbrace{(I_n \otimes Z)}_{n\times nrp} \cdot \underbrace{a}_{nrp\times 1} + U,$$

where $\underbrace{U}_{nT\times 1}$ has the following distribution:

$$U \sim \mathcal{MN}\left(\mathbf{0}, \underbrace{V_u}_{n\times n} \right),$$

and

$$
V_u \equiv \begin{bmatrix}
\Omega_1^{(1,1)} & 0 & \cdots & 0 & \cdots & \cdots & \Omega_1^{(1,n)} & 0 & \cdots & 0 \\
0 & \Omega_2^{(1,1)} & \ddots & \vdots & \cdots & \cdots & 0 & \Omega_2^{(1,n)} & \ddots & \vdots \\
\vdots & \ddots & \ddots & 0 & \cdots & \cdots & \vdots & \ddots & \ddots & 0 \\
0 & \cdots & 0 & \Omega_T^{(1,1)} & \cdots & \cdots & 0 & \cdots & 0 & \Omega_T^{(1,n)} \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
\Omega_1^{(n,1)} & 0 & \cdots & 0 & \cdots & \cdots & \Omega_1^{(n,n)} & 0 & \cdots & 0 \\
0 & \Omega_2^{(n,1)} & \ddots & \vdots & \cdots & \cdots & 0 & \Omega_2^{(n,n)} & \ddots & \vdots \\
\vdots & \ddots & \ddots & 0 & \cdots & \cdots & \vdots & \ddots & \ddots & 0 \\
0 & \cdots & 0 & \Omega_T^{(n,1)} & \cdots & \cdots & 0 & \cdots & 0 & \Omega_T^{(nn)}
\end{bmatrix}
$$

$$
= \sum_{t=1}^{T} \left[\Omega_t \otimes (e_t \cdot e_t') \right].
$$

To use an informative prior on a we follow the approach by Gelman et al. (2014). The strategy incorporates the prior as observations. Considering a multivariate Normal prior with the following moments:

$$
a \sim \mathcal{MN}\left(\bar{a},\ V_a \right),
$$

it is possible to augment the model with nrp observations that express the prior information:

$$
\begin{bmatrix} Y^{\bullet} \\ \bar{a} \end{bmatrix} = \begin{bmatrix} I_n \otimes Z \\ I_{nrp} \end{bmatrix} a + \begin{bmatrix} U \\ U_a \end{bmatrix},
$$

$$
Y^{\diamond} = Z^{\diamond} a + U^{\diamond}, \quad U^{\diamond} \sim \mathcal{MN}\left(\mathbf{0}_{nT+nrp},\ V^{\diamond} \right),
$$

$$
V^{\diamond} = \begin{bmatrix} V_u & \mathbf{0}_{nT\times nrp} \\ \mathbf{0}_{nrp\times nT} & V_a \end{bmatrix}.
$$

A draw for a then comes from the following posterior:

$$
a \sim \mathcal{MN}\left(\widetilde{a},\ \left(Z^{\diamond\prime} V^{\diamond-1} Z^{\diamond}\right)^{-1} \right),
$$

$$
\widetilde{a} = \left(Z^{\diamond\prime} V^{\diamond-1} Z^{\diamond}\right)^{-1} Z^{\diamond\prime} V^{\diamond-1} Y^{\diamond}.
$$

In order to decrease the computational burden of this step throughout the sampling, the strategy proposed by Carriero et al. (2016a) is adopted, as generalized in Carriero et al. (2018): the triangular structure of the error is exploited, and coefficients are drawn equation by equation.

A.3 Step 3: Draw the factor weights elements in B_0

Given the restrictions and the nonlinear role of B_0, a Random walk Metropolis step on the kernel of the posterior of each element of B_0 is implemented, nested into the GS In order to do this, we first write the likelihood of the model. Given the reduced form VAR written as:

$$y_t = \mathcal{X}_t \cdot \gamma + A \cdot Z_t + u_t \ , \qquad u_t \overset{i}{\sim} \mathcal{MN}\left(\mathbf{0}, \ \Omega_t \right),$$

conditioning on all the elements, using the chain rule, we can write the likelihood kernel as:

$$f\left((y_t)_{t=1}^{T} \middle| \gamma, A, (\Omega_t)_{t=1}^{T}, \ B_0 \right) \propto \left(\prod_{t=1}^{T} |\Omega_t|^{-\frac{1}{2}} \right) \exp\left\{ -\frac{1}{2} \sum_{t=1}^{T} \widehat{y}_t' \cdot \Omega_t^{-1} \cdot \widehat{y}_t \right\},$$

where

$$\widehat{y}_t \equiv y_t - A \cdot Z_t - \mathcal{X}_t \cdot \gamma.$$

Now we consider the $r^* \equiv n - r$ scalar unrestricted elements of B_0, i.e. $(b_{0,j})_{j=1}^{r^*}$. Then, $\forall j \in \{1, \ldots, r^*\}$ we can define the set $b_{0,j-} \equiv (b_{0,s})_{s \neq j}$.

For a given prior $f(b_{0,j})$ on each element $b_{0,j}$, we can write the kernel of the conditional posterior of $b_{0,j}$ as:

$$f_{post}\left(b_{0,j} | \left(y_t, \Omega_t\right)_{t=1}^{T}, \ A, \ b_{0,j-} \right) \propto f\left((y_t)_{t=1}^{T} \middle| A, \ B_0, \ (\Omega_t)_{t=1}^{T} \right) \cdot f\left(b_{0,j}\right).$$

We are now ready to design the Metropolis step, separately for each j. Given the last step B_0^{i-1}, a random walk candidate is computed as:

$$b_{0,j}^{*} = b_{0,j}^{i-1} + c_j \cdot \eta_t,$$

where c_j is a scaling factor calibrated to have an acceptance rate of approximately 30%-35% and $\eta_t \overset{iid}{\sim} \mathcal{N}(0, v_j)$, with v_j being the variance of prior $f(b_{0,j})$. The candidate draw is accepted with probability:

$$\alpha_j = \min\left\{1, \frac{f_{post}\left(b^{*}_{0,j}\middle|\left(y_t, \Omega_t^{i-1}\right)_{t=1}^{T},\ A,\ b^{i-1}_{0,j-}\right)}{f_{post}\left(b^{i-1}_{0,j}\middle|\left(y_t, \Omega_t^{i-1}\right)_{t=1}^{T},\ A,\ b^{i-1}_{0,j-}\right)}\right\}.$$

When the candidate is accepted, then $b^{i}_{0,j-} = b^{*}_{0,j}$, otherwise $b^{i}_{0,j-} = b^{i-1}_{0,j-}$. Repeating this procedure $\forall j \in \{1, \ldots, r^*\}$, we build a draw B_0^i from the distribution of interest.

A.4 Step 4: draw the off-diagonal elements in G

To draw the off-diagonal elements, we restate the reduced form in the following way:

$$\begin{aligned} y_t &= \underbrace{\mathcal{X}_t}_{n\times nq} \cdot \underbrace{\gamma}_{nq\times 1} + \underbrace{A}_{n\times rp} \cdot \underbrace{Z_t}_{rp\times 1} + u_t, \\ y_t - \mathcal{X}_t \cdot \gamma - A \cdot Z_t &= G^{-1}\Sigma_t\varepsilon_t, \\ \widehat{y_t} &= G^{-1}\Sigma_t\varepsilon_t, \\ G \cdot \widehat{y_t} &= \Sigma_t\varepsilon_t. \end{aligned}$$

Removing ones from the diagonal of G, and bringing off diagonal elements on the right hand side, produces:

$$G = I_n + G^*.$$

This can be combined in the model to obtain:

$$\begin{aligned} \left(I_n + G^*\right)\ \widehat{y_t} &= \Sigma_t\ \varepsilon_t, \\ \widehat{y_t} &= -G^*\ \widehat{y_t} + \Sigma_t\ \varepsilon_t. \end{aligned}$$

Exploiting the Kronecker product's properties, we get:

$$-\ I_n \underbrace{G^*}_{n\times n} \underbrace{\widehat{y_t}}_{n\times 1} = -\underbrace{(I_n \otimes \widehat{y}_t')}_{n\times n^2} \underbrace{vec\left(G^{*\prime}\right)}_{n^2\times 1}$$

where $vec\left(G^{*\prime}\right)$ has zeros in positions $\left[(i-1)\,n + j\right]^{i\in\{1,\ldots,n\}}_{j\in\{1,\ldots,n\}}$. By removing the zeros, we obtain exactly the elements below the main diagonal of G gathered in the m-dimensional vector g. Removing the corresponding columns in $-\left(I_n \otimes \widehat{y}_t'\right)$ we construct the matrix W_t, which has the following form:

$$\underbrace{W_t}_{n \times m} = -1 \cdot \begin{bmatrix} 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\ \widehat{y}_{1,t} & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \vdots \\ 0 & \widehat{y}_{1,t} & \widehat{y}_{2,t} & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \vdots \\ 0 & 0 & 0 & \widehat{y}_{1,t} & \widehat{y}_{2,t} & \widehat{y}_{3,t} & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots & 0 \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & \widehat{y}_{1,t} & \widehat{y}_{2,t} & \widehat{y}_{3,t} & \cdots & \widehat{y}_{n-1,t} \end{bmatrix}.$$

We can then rewrite the model as:

$$\begin{aligned} \widehat{y}_t &= -G^* \widehat{y}_t + \Sigma_t \varepsilon_t, \\ \widehat{y}_t &= -\left(I_n \otimes \widehat{y}_t'\right) vec\left(G^{*\prime}\right) + \Sigma_t \varepsilon_t, \\ \widehat{y}_t &= W_t \; g + \varepsilon_t^*, \qquad \varepsilon_t^* \sim \mathcal{MN}\left(\; \mathbf{0}_{n \times 1}, \; \Sigma_t^2 \; \right). \end{aligned}$$

Next, we stack the model as:

$$\begin{bmatrix} \widehat{y}_1 \\ \widehat{y}_2 \\ \vdots \\ \widehat{y}_T \end{bmatrix} = \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_T \end{bmatrix} g + \begin{bmatrix} \varepsilon_1^* \\ \varepsilon_2^* \\ \vdots \\ \varepsilon_T^* \end{bmatrix},$$

$$\underbrace{\widehat{y}}_{nT \times 1} = \underbrace{W}_{Tn \times m} \cdot \underbrace{g}_{m \times 1} + \varepsilon^*, \qquad \varepsilon^* \sim \mathcal{MN}\left(\; \mathbf{0}_{nT \times 1}, \; \Sigma^2 \; \right) m$$

where Σ is the diagonal matrix containing all the stacked stochastic volatilities vectors in the main diagonal:

$$\Sigma = Diag\left(\begin{bmatrix} \sigma_1' & \sigma_2' & \dots & \sigma_T' \end{bmatrix}' \right).$$

We can then use a similar approach as the one implemented for a, following Gelman et al. (2014). Given the prior :

$$g \sim \mathcal{MN}\left(\; \bar{g}, \; V_g \; \right),$$

we augment the model with r observations that express the prior information:

$$\begin{bmatrix} \widehat{y} \\ \bar{g} \end{bmatrix} = \begin{bmatrix} W \\ I_m \end{bmatrix} g + \begin{bmatrix} \varepsilon^* \\ \varepsilon_g \end{bmatrix},$$

$$\widehat{Y}^\diamond = W^\diamond g + \varepsilon^\diamond, \quad \varepsilon^\diamond \sim \mathcal{MN}\left(\mathbf{0}_{nT+m},\ V_\varepsilon^\diamond \right),$$

$$V_\varepsilon^\diamond = \begin{bmatrix} \Sigma^2 & \mathbf{0}_{nT\times m} \\ \mathbf{0}_{m\times nT} & V_g \end{bmatrix}.$$

A draw for g is finally obtained through the following posterior:

$$g \sim \mathcal{MN}\left(\widetilde{g},\ \left(W^{\diamond\prime} V_\varepsilon^{\diamond -1} W^\diamond\right)^{-1} \right),$$

$$\widetilde{g} = \left(W^{\diamond\prime} V_\varepsilon^{\diamond -1} W^\diamond\right)^{-1} W^{\diamond\prime} V_\varepsilon^{\diamond -1} \widehat{Y}^\diamond$$

A.5 Step 5-6: Draw the indexes of the mixture in S and then a history of volatilities $(\sigma_t)_{t=1}^T$

An important final step concerns the draw of stochastic volatilities. However, before drawing the (unobservable) stochastic volatilities is necessary to draw the matrix S containing the indexes of Normal components of the mixture, as suggested by Del Negro and Primiceri (2015).

To start building the necessary form, recall the model formulation used previously and transform it as:

$$y_t = \mathcal{X}_t \cdot \gamma + A \cdot Z_t + G^{-1}\Sigma_t \varepsilon_t,$$

$$\underbrace{G\left(y_t - \mathcal{X}_t \cdot \gamma - A \cdot Z_t\right)}_{\widetilde{y}_t} = \Sigma_t \varepsilon_t,$$

$$\widetilde{y}_t = \Sigma_t \varepsilon_t.$$

Having this formulation, we adopt the same procedure as in the MAI-SV Gibbs Sampler illustrated in the Appendix of Carriero et al. (2018), which implements the Omori et al. (2007) procedure to approximate the $\log \chi_1^2$ innovations as mixture of Normal components.

A.6 Step 7: Draw a covariance matrix Q_σ

Conditioning on the new $(\sigma_t^i)_{t=0}^T$, we can draw the covariance matrix Q_σ. Indeed, recall that:

$$\log \sigma_t = \log \sigma_{t-1} + \nu_{\sigma,t}, \qquad \nu_{\sigma,t} \overset{iid}{\sim} \mathcal{MN}\left(\mathbf{0},\ \underbrace{Q_\sigma}_{n\times n}\right).$$

But then, having a complete history of the sigmas, given the random walk law of motion, is equivalent to having a complete histories of innovations $\nu_{\sigma,t}$. Stacking the $\nu_{\sigma,t}$ across time, we get:

$$\underbrace{\nu_\sigma^*}_{n\times T} = \begin{bmatrix} \nu_{\sigma,1} & \nu_{\sigma,2} & \dots & \nu_{\sigma,T} \end{bmatrix},$$

and we can easily compute the innovations sum of squares matrix:

$$\underbrace{S_\sigma}_{n\times n} = \underbrace{\nu_\sigma^*}_{n\times T}\underbrace{\nu_\sigma^{*\prime}}_{T\times n}.$$

If the prior on the matrix Q_σ is a $n \times n$ Inverse Wishart with scale matrix $\bar{Q}_\sigma$ and degrees of freedom $\tau_{\sigma,0}$:

$$Q_\sigma \sim \mathcal{IW}_n\left(\bar{Q}_\sigma,\ \tau_{\sigma,0}\right),$$

then the posterior is conjugate and given by:

$$Q_\sigma \big| \left(\sigma_t^i\right)_{t=0}^T \sim \mathcal{IW}_n\left(S_\sigma + \bar{Q}_\sigma,\ \tau_{\sigma,0} + T\right).$$

B Forecasting Evaluation Tables

The following tables contain the Root Mean Squares Errors, Predictive Log Scores and Continous Rank Probability Scores relative to the forecasting evaluation section.

Table 1a: Root Mean Squared Forecast Errors (RMSE for MAI-AR-SV, RMSE ratios in all others)

USA

	MAI-AR-SV	MAI-AR	AR	AR-SV	BVAR	BVAR-SV
$h = 1$	0.254	1.017***	1.024*	1.026	0.972***	0.976
$h = 2$	0.417	1.029***	1.032	1.045**	0.933**	0.923*
$h = 3$	0.525	1.035***	1.061	1.073**	0.933	0.918
$h = 4$	0.602	1.046***	1.088	1.105**	0.947	0.922
$h = 5$	0.583	1.053***	1.127	1.148**	0.987	0.952
$h = 6$	0.553	1.048**	1.183	1.199**	1.046	1.004
$h = 7$	0.543	1.040**	1.242	1.255**	1.087	1.038
$h = 8$	0.538	1.023**	1.283	1.296**	1.096	1.054

Australia

	MAI-AR-SV	MAI-AR	AR	AR-SV	BVAR	BVAR-SV
$h = 1$	0.203	0.983***	1.022***	1.020	1.015***	1.023
$h = 2$	0.309	0.976***	1.042***	1.046	1.035*	1.069
$h = 3$	0.401	0.971***	1.060***	1.067	1.081*	1.137*
$h = 4$	0.485	0.974***	1.065***	1.078	1.091	1.162**
$h = 5$	0.518	0.969***	1.087***	1.101	1.120*	1.198**
$h = 6$	0.547	0.966***	1.105***	1.119*	1.132*	1.209**
$h = 7$	0.562	0.963***	1.125***	1.137*	1.134*	1.195**
$h = 8$	0.576	0.961***	1.149***	1.159*	1.133*	1.169**

Austria

	MAI-AR-SV	MAI-AR	AR	AR-SV	BVAR	BVAR-SV
$h = 1$	0.209	1.069***	1.024***	1.001	0.968***	0.948
$h = 2$	0.343	1.069***	1.020***	0.995	0.974***	0.919
$h = 3$	0.450	1.047***	1.022***	1.014	0.980*	0.920
$h = 4$	0.535	1.049***	1.027***	1.029	1.001	0.951
$h = 5$	0.575	1.062***	1.056***	1.055	1.033	0.979
$h = 6$	0.595	1.078***	1.099***	1.094	1.066	1.009
$h = 7$	0.618	1.094***	1.129***	1.125	1.097	1.039
$h = 8$	0.637	1.107***	1.148**	1.143	1.117	1.057

Belgium

	MAI-AR-SV	MAI-AR	AR	AR-SV	BVAR	BVAR-SV
$h = 1$	0.200	0.972***	1.020***	1.054***	1.010	1.040
$h = 2$	0.331	0.980***	1.042***	1.091**	0.982	0.973
$h = 3$	0.438	0.990***	1.045***	1.078*	0.959	0.927
$h = 4$	0.544	1.012***	1.034**	1.058	0.932	0.897
$h = 5$	0.594	1.027***	1.033*	1.057*	0.925	0.891
$h = 6$	0.609	1.037***	1.039	1.066*	0.942	0.900
$h = 7$	0.616	1.041***	1.053	1.078	0.955	0.902
$h = 8$	0.611	1.041***	1.077	1.103	0.960	0.898

Canada

	MAI-AR-SV	MAI-AR	AR	AR-SV	BVAR	BVAR-SV
$h = 1$	0.227	1.000***	1.040	1.036*	0.975***	0.988
$h = 2$	0.363	0.997***	1.047	1.040*	0.950***	0.971
$h = 3$	0.456	0.993***	1.053	1.043	0.941***	0.964*
$h = 4$	0.530	1.006***	1.048	1.041	0.936***	0.963**
$h = 5$	0.551	1.002***	1.017	1.011	0.929**	0.978**
$h = 6$	0.560	0.989***	0.989	0.983	0.921**	0.992**
$h = 7$	0.574	0.986***	0.986	0.980	0.920*	0.999
$h = 8$	0.588	0.984***	0.995	0.987	0.925	1.008

Finland

	MAI-AR-SV	MAI-AR	AR	AR-SV	BVAR	BVAR-SV
$h = 1$	0.124	0.991***	1.041***	1.022	1.098***	1.147***
$h = 2$	0.233	1.006***	0.991***	0.963*	1.055**	1.108
$h = 3$	0.335	1.019***	0.973***	0.931**	1.013	1.073
$h = 4$	0.436	1.026***	0.966***	0.922**	0.985	1.046
$h = 5$	0.513	1.042***	0.961***	0.907*	0.990	1.041
$h = 6$	0.567	1.049***	0.973**	0.908	1.007	1.044
$h = 7$	0.606	1.056***	0.999*	0.925	1.032	1.047
$h = 8$	0.634	1.066***	1.026*	0.944	1.049	1.044

France

	MAI-AR-SV	MAI-AR	AR	AR-SV	BVAR	BVAR-SV
$h = 1$	0.108	1.034***	0.972***	0.974**	1.072***	1.078
$h = 2$	0.180	1.080***	0.964***	0.956	1.089	1.091
$h = 3$	0.234	1.114***	0.991***	0.947	1.125	1.125
$h = 4$	0.292	1.144***	1.003***	0.955	1.113	1.125
$h = 5$	0.323	1.172***	1.016**	0.946	1.131	1.143
$h = 6$	0.341	1.196***	1.036**	0.937	1.169	1.173
$h = 7$	0.360	1.212***	1.050**	0.926	1.182	1.175**
$h = 8$	0.382	1.22***	1.059*	0.914	1.173	1.158**

Germany

	MAI-AR-SV	MAI-AR	AR	AR-SV	BVAR	BVAR-SV
$h = 1$	0.274	0.996***	0.982	0.984	1.026	1.035
$h = 2$	0.418	1.003***	0.978	0.991	1.063	1.051
$h = 3$	0.505	1.022***	1.018	1.030	1.136	1.104
$h = 4$	0.594	1.035***	1.042	1.057	1.152	1.124
$h = 5$	0.624	1.040***	1.063	1.078	1.186	1.171
$h = 6$	0.655	1.050***	1.084	1.102	1.202	1.188
$h = 7$	0.689	1.061***	1.101	1.120	1.210	1.186
$h = 8$	0.717	1.072***	1.111	1.132	1.224	1.188

Greece

	MAI-AR-SV	MAI-AR	AR	AR-SV	BVAR	BVAR-SV
$h = 1$	0.111	1.044***	1.011***	0.996	1.138***	1.069***
$h = 2$	0.193	0.985***	1.027***	0.984	1.161***	1.097***
$h = 3$	0.256	0.999***	1.057***	0.985	1.169***	1.082***
$h = 4$	0.297	0.999***	1.087***	0.988	1.193***	1.083**
$h = 5$	0.306	1.004***	1.151***	1.005	1.239***	1.110**
$h = 6$	0.313	1.023***	1.239***	1.045	1.277***	1.151**
$h = 7$	0.312	1.065***	1.359***	1.116	1.320***	1.190**
$h = 8$	0.311	1.124***	1.490***	1.206	1.336***	1.205*

Italy

	MAI-AR-SV	MAI-AR	AR	AR-SV	BVAR	BVAR-SV
$h = 1$	0.065	1.135***	1.032***	0.960***	1.172***	1.126***
$h = 2$	0.128	1.074***	1.024***	0.946**	1.146***	1.155**
$h = 3$	0.187	1.057***	1.040***	0.951*	1.116*	1.190
$h = 4$	0.250	1.057***	1.023***	0.937	1.080	1.190
$h = 5$	0.296	1.059***	1.030***	0.933	1.061	1.191
$h = 6$	0.331	1.058***	1.054***	0.939	1.049	1.185
$h = 7$	0.363	1.064***	1.071***	0.940	1.027	1.160
$h = 8$	0.387	1.066***	1.101***	0.956	0.998	1.137

Statistically significant differences according to the Diebold-Mariano t-statistic are indicated by asterisks, where

*,** and ** correspond respectively to 10%,5% and 1% significance levels

Table 1b: Root Mean Squared Forecast Errors (RMSE for MAI-AR-SV, RMSE ratios in all others)

Japan

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h = 1$	0.142	1.131***	1.092***	1.018	1.143***	1.119***
$h = 2$	0.230	1.180***	1.113***	0.969*	1.218***	1.123*
$h = 3$	0.313	1.230***	1.131***	0.927**	1.274***	1.134
$h = 4$	0.400	1.253***	1.159***	0.908**	1.273***	1.107
$h = 5$	0.464	1.289***	1.178***	0.867***	1.278***	1.085
$h = 6$	0.517	1.312***	1.21***	0.851***	1.262***	1.063
$h = 7$	0.563	1.322***	1.243***	0.848***	1.234***	1.033
$h = 8$	0.602	1.330***	1.264***	0.842***	1.208***	1.016

Luxembourg

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h = 1$	0.194	0.998***	1.013***	1.024	1.020	1.042**
$h = 2$	0.307	1.015***	1.014***	1.030	1.027	1.042
$h = 3$	0.377	1.027***	1.029***	1.042	1.038	1.048
$h = 4$	0.448	1.033***	1.046**	1.059	1.021	1.029
$h = 5$	0.473	1.041***	1.066**	1.083	1.027	1.033
$h = 6$	0.474	1.038***	1.101**	1.118	1.046	1.046
$h = 7$	0.485	1.030***	1.140**	1.155*	1.069	1.062
$h = 8$	0.492	1.024***	1.168***	1.182*	1.096	1.086

Netherlands

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h = 1$	0.156	0.980***	1.030***	1.012	1.157***	1.085**
$h = 2$	0.231	0.991***	1.032***	1.007	1.231***	1.168**
$h = 3$	0.283	1.000***	1.065***	1.040	1.267***	1.234*
$h = 4$	0.359	1.009***	1.033***	1.025	1.200*	1.201
$h = 5$	0.400	1.032***	1.025***	1.006	1.168	1.186
$h = 6$	0.436	1.051***	1.033**	1.005	1.130	1.158
$h = 7$	0.480	1.068***	1.019**	0.996	1.090	1.110
$h = 8$	0.510	1.083***	1.015*	0.987	1.072	1.075

New Zealand

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h = 1$	0.153	0.962***	0.966***	0.959	1.101***	1.082*
$h = 2$	0.259	0.935***	0.928***	0.873	1.140**	1.134
$h = 3$	0.354	0.919***	0.925**	0.813	1.133*	1.141
$h = 4$	0.437	0.922***	0.942**	0.811	1.102**	1.120
$h = 5$	0.491	0.921***	0.948**	0.782	1.064**	1.085**
$h = 6$	0.532	0.921***	0.964**	0.765	1.036*	1.045*
$h = 7$	0.561	0.931***	0.986**	0.760	1.008	0.998
$h = 8$	0.575	0.947***	1.021**	0.768	0.984	0.951

Norway

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h = 1$	0.248	1.004***	1.028***	0.999	0.965	0.961
$h = 2$	0.350	1.090***	1.086***	1.012	0.957	0.940*
$h = 3$	0.443	1.121***	1.121***	1.014	0.936	0.929*
$h = 4$	0.538	1.134***	1.145***	1.020	0.917	0.905*
$h = 5$	0.578	1.162***	1.171***	1.001	0.938	0.903
$h = 6$	0.601	1.165***	1.224***	1.014	0.959	0.901
$h = 7$	0.614	1.157***	1.282***	1.036	0.973	0.898
$h = 8$	0.630	1.140***	1.327***	1.063	0.972	0.889

Portugal

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h = 1$	0.079	1.500***	1.375***	1.013***	1.361***	1.085***
$h = 2$	0.136	1.341***	1.384***	1.014***	1.287***	1.064***
$h = 3$	0.182	1.261***	1.446***	1.023*	1.274***	1.094***
$h = 4$	0.227	1.148***	1.504***	1.035	1.276***	1.112***
$h = 5$	0.257	1.080***	1.580***	1.048	1.267***	1.099***
$h = 6$	0.286	1.032***	1.637***	1.058	1.220***	1.076***
$h = 7$	0.312	0.995***	1.690***	1.079	1.154***	1.050**
$h = 8$	0.335	0.998***	1.729***	1.093	1.094***	1.032**

Spain

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h = 1$	0.116	0.982***	1.012***	0.984**	1.065***	0.989
$h = 2$	0.204	0.983***	0.987***	0.936**	1.028***	0.999
$h = 3$	0.277	0.994***	1.001***	0.922**	1.006	1.011
$h = 4$	0.341	1.017***	1.013***	0.911*	0.979	1.012
$h = 5$	0.385	1.026***	1.027***	0.894*	0.968	1.018
$h = 6$	0.421	1.026***	1.048***	0.882**	0.955	1.002
$h = 7$	0.457	1.028***	1.063***	0.875*	0.934	0.977
$h = 8$	0.488	1.033***	1.081***	0.874*	0.916	0.968

Sweden

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h = 1$	0.235	1.024***	1.038***	1.033	1.047	1.093*
$h = 2$	0.383	1.044***	1.053**	1.052	1.056	1.070
$h = 3$	0.499	1.050***	1.089**	1.077	1.075	1.055
$h = 4$	0.609	1.059***	1.108**	1.093	1.031	0.992
$h = 5$	0.668	1.069***	1.126**	1.105	1.014	0.966
$h = 6$	0.698	1.081***	1.170***	1.138	1.012	0.956
$h = 7$	0.721	1.097***	1.215***	1.172	1.007	0.942
$h = 8$	0.740	1.109***	1.257***	1.206	1.01	0.938

Switzerland

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h = 1$	0.194	1.085***	1.017***	0.988***	1.155***	1.165***
$h = 2$	0.355	1.081***	0.988**	0.956***	1.143*	1.157*
$h = 3$	0.496	1.083***	0.976	0.941***	1.125	1.128
$h = 4$	0.609	1.097***	0.974	0.937**	1.111	1.098
$h = 5$	0.673	1.121***	0.985	0.941*	1.114	1.071
$h = 6$	0.705	1.152***	1.012	0.952*	1.137	1.052
$h = 7$	0.733	1.179***	1.041	0.968	1.166	1.037
$h = 8$	0.762	1.198***	1.067	0.982	1.199	1.030

United Kingdom

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h = 1$	0.114	1.054***	1.008***	0.989***	1.201***	1.112
$h = 2$	0.184	1.062***	1.044***	1.003***	1.245**	1.138
$h = 3$	0.245	1.069***	1.057***	0.980**	1.237	1.154
$h = 4$	0.316	1.073***	1.035***	0.939**	1.173	1.132
$h = 5$	0.359	1.077***	1.043***	0.921*	1.122	1.104
$h = 6$	0.396	1.077***	1.056***	0.911*	1.088	1.080
$h = 7$	0.433	1.074***	1.073**	0.918*	1.063	1.054
$h = 8$	0.459	1.072***	1.107**	0.947*	1.047	1.035

Statistically significant differences according to the Diebold-Mariano t-statistic are indicated by asterisks, where

*,** and ** correspond respectively to 10%,5% and 1% significance levels

Table 2a: Average Log Predictive Scores (scores for MAI-AR-SV, score differences in all others)

USA

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.166	−0.292***	−1.263	−0.032	−1.506	−0.022
$h=2$	−0.343	−0.277***	−1.161	−0.063*	−0.971	−0.016
$h=3$	−0.600	−0.267***	−0.532	−0.094*	−0.410	−0.044
$h=4$	−0.805	−0.241***	−0.229	−0.130**	−0.110	−0.048
$h=5$	−0.842	−0.276***	−0.180*	−0.154**	−0.053	−0.056
$h=6$	−0.867	−0.308***	−0.162**	−0.180***	−0.052	−0.073
$h=7$	−0.892	−0.337***	−0.180**	−0.208***	−0.067	−0.086
$h=8$	−0.927	−0.343***	−0.182**	−0.223***	−0.064	−0.087

Australia

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.206	−0.334***	−0.139**	−0.044	−0.069	0.025
$h=2$	−0.291	−0.197**	−0.052	−0.050	0.012	−0.024
$h=3$	−0.552	−0.163*	−0.053	−0.083	−0.013	−0.080
$h=4$	−0.722	−0.169	−0.075	−0.104	−0.026	−0.112
$h=5$	−0.770	−0.215*	−0.136	−0.139	−0.067	−0.139**
$h=6$	−0.809	−0.238**	−0.183*	−0.164*	−0.091	−0.150*
$h=7$	−0.833	−0.260**	−0.216**	−0.182*	−0.105	−0.134*
$h=8$	−0.871	−0.256**	−0.233**	−0.204*	−0.105	−0.114

Austria

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.126	−0.516***	−0.265***	0.004	−0.146*	0.052
$h=2$	−0.377	−0.386***	−0.172*	−0.007	−0.044	0.102
$h=3$	−0.696	−0.241*	−0.078	0.022	0.047	0.143
$h=4$	−0.863	−0.195	−0.064	−0.040	0.045	0.088
$h=5$	−0.919	−0.213**	−0.102	−0.081	0.002	0.045
$h=6$	−0.957	−0.227**	−0.132	−0.128	−0.023	0.021
$h=7$	−1.016	−0.212*	−0.131	−0.170*	−0.032	0.000
$h=8$	−1.059	−0.206	−0.129	−0.204**	−0.039	−0.014

Belgium

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.239	−0.283***	−0.110**	−0.040	−0.074	−0.058
$h=2$	−0.191	−0.307***	−0.189***	−0.101***	−0.151*	−0.535
$h=3$	−0.453	−0.288**	−0.247**	−0.111*	−0.376*	−0.650
$h=4$	−0.723	−0.226*	−0.191*	−0.106	−0.232	−0.694
$h=5$	−0.822	−0.240**	−0.151*	−0.125*	−0.051	0.050
$h=6$	−0.863	−0.267**	−0.154*	−0.152**	−0.038	0.060
$h=7$	−0.912	−0.270***	−0.140*	−0.172***	0.007	0.077
$h=8$	−0.942	−0.273***	−0.141*	−0.195***	0.046	0.096

Canada

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.075	−0.119**	−0.064	−0.086**	−0.032	−0.003
$h=2$	−0.352	−0.130**	−0.127	−0.109**	−0.147	−0.004
$h=3$	−0.556	−0.160**	−0.127	−0.137*	−0.070	−0.020
$h=4$	−0.705	−0.173**	−0.126	−0.122	−0.066	−0.005
$h=5$	−0.758	−0.193**	−0.096	−0.109	0.030	0.019
$h=6$	−0.789	−0.211**	−0.086	−0.104	0.063	0.036
$h=7$	−0.826	−0.227**	−0.095	−0.124	0.069	0.047
$h=8$	−0.862	−0.232**	−0.105	−0.149*	0.067	0.034

Finland

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.651	−0.550***	−0.271***	−0.022	−0.199***	−0.108**
$h=2$	0.055	−0.478***	−0.204***	−0.049	−0.111	−0.114
$h=3$	−0.301	−0.402***	−0.156*	0.014	−0.045	−0.091
$h=4$	−0.581	−0.307**	−0.093	−0.018	0.020	−0.042
$h=5$	−0.759	−0.259*	−0.056	0.018	0.048	−0.010
$h=6$	−0.850	−0.249*	−0.068	−0.007	0.040	−0.004
$h=7$	−0.924	−0.235*	−0.074	−0.031	0.032	0.014
$h=8$	−0.978	−0.232	−0.082	−0.056	0.020	0.022

France

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.806	−0.429***	−0.153***	0.028	−0.119**	−0.072*
$h=2$	0.340	−0.519***	−0.190***	−0.003	−0.149*	−0.088
$h=3$	0.078	−0.550***	−0.219***	−0.032	−0.172	−0.129
$h=4$	−0.163	−0.488***	−0.182*	−0.040	−0.142	−0.124
$h=5$	−0.263	−0.502***	−0.207**	−0.043	−0.175	−0.136
$h=6$	−0.333	−0.514***	−0.223**	−0.046	−0.206*	−0.163*
$h=7$	−0.410	−0.503***	−0.221**	−0.043	−0.211*	−0.167**
$h=8$	−0.479	−0.498***	−0.216**	−0.039	−0.211*	−0.171**

Germany

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	−0.072	−0.178***	−0.035	−0.014	−0.062	−0.052
$h=2$	−0.504	−0.172**	−0.023	−0.037	−0.073	−0.055
$h=3$	−0.706	−0.188**	−0.054	−0.081	−0.127	−0.099
$h=4$	−0.904	−0.162*	−0.047	−0.103**	−0.092	−0.105
$h=5$	−0.994	−0.165*	−0.039	−0.127**	−0.079	−0.092
$h=6$	−1.068	−0.162*	−0.042	−0.139***	−0.071	−0.091
$h=7$	−1.140	−0.151*	−0.036	−0.153***	−0.064	−0.073
$h=8$	−1.194	−0.141*	−0.031	−0.155***	−0.059	−0.067

Greece

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.839	−0.761***	−0.428***	−0.002	−0.369***	−0.104***
$h=2$	0.314	−0.752***	−0.414***	−0.003	−0.339***	−0.109**
$h=3$	0.035	−0.777***	−0.423***	−0.022	−0.340***	−0.122*
$h=4$	−0.139	−0.768***	−0.424***	−0.039	−0.342***	−0.131*
$h=5$	−0.216	−0.793***	−0.458***	−0.047	−0.372***	−0.145**
$h=6$	−0.269	−0.806***	−0.491***	−0.056	−0.391***	−0.170**
$h=7$	−0.306	−0.821***	−0.527***	−0.067	−0.405***	−0.187***
$h=8$	−0.350	−0.820***	−0.545***	−0.081	−0.399***	−0.198***

Italy

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	1.262	−0.956***	−0.616***	0.101***	−0.431***	−0.146***
$h=2$	0.638	−0.843***	−0.522***	0.063	−0.283***	−0.142***
$h=3$	0.254	−0.752***	−0.451***	0.043	−0.179	−0.139*
$h=4$	−0.026	−0.644***	−0.375**	0.039	−0.117	−0.125
$h=5$	−0.190	−0.596***	−0.352**	0.030	−0.111	−0.127
$h=6$	−0.298	−0.574***	−0.350**	0.029	−0.128	−0.131
$h=7$	−0.394	−0.547***	−0.338**	0.029	−0.128	−0.126
$h=8$	−0.468	−0.529***	−0.338**	0.012	−0.124	−0.126

Statistically significant differences according to the Diebold-Mariano t-statistic are indicated by asterisks, where

*,** and ** correspond respectively to 10%,5% and 1% significance levels

Table 2b: Average Log Predictive Scores (scores for MAI-AR-SV, score differences in all others)

Japan

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.505	−0.666***	−0.356***	0.000	−0.251***	−0.094**
$h=2$	0.033	−0.609***	−0.331***	0.051	−0.243***	−0.091
$h=3$	−0.270	−0.553***	−0.306***	0.093*	−0.241**	−0.082
$h=4$	−0.537	−0.475***	−0.260**	0.128**	−0.201	−0.040
$h=5$	−0.671	−0.474***	−0.274**	0.151**	−0.214	−0.037
$h=6$	−0.788	−0.453***	−0.267**	0.167**	−0.197	−0.016
$h=7$	−0.870	−0.445***	−0.276**	0.172**	−0.190	−0.007
$h=8$	−0.946	−0.426***	−0.270**	0.181**	−0.167	0.008

Luxembourg

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.245	−0.302***	−0.094**	−0.075	−0.079**	−0.027
$h=2$	−0.199	−0.304***	−0.120***	−0.072*	−0.174	−0.038
$h=3$	−0.426	−0.307***	−0.130**	−0.070	−0.19	−0.055
$h=4$	−0.619	−0.282***	−0.127	−0.105*	−0.048	−0.054
$h=5$	−0.687	−0.312***	−0.149*	−0.117**	−0.042	−0.060
$h=6$	−0.718	−0.332***	−0.182**	−0.133**	−0.038	−0.054
$h=7$	−0.766	−0.332***	−0.193***	−0.145**	−0.027	−0.058
$h=8$	−0.803	−0.333***	−0.207***	−0.148**	−0.025	−0.076

Netherlands

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.388	−0.623***	−0.347***	0.020	−0.237***	−0.064
$h=2$	−0.002	−0.625***	−0.342***	−0.006	−0.219***	−0.106*
$h=3$	−0.213	−0.604***	−0.338***	−0.037	−0.211**	−0.130*
$h=4$	−0.441	−0.508***	−0.261**	−0.030	−0.151	−0.108
$h=5$	−0.540	−0.500***	−0.255**	−0.032	−0.141	−0.111
$h=6$	−0.634	−0.479***	−0.232**	−0.022	−0.123	−0.109
$h=7$	−0.729	−0.451***	−0.197*	−0.016	−0.108	−0.102
$h=8$	−0.792	−0.438***	−0.176	−0.006	−0.099	−0.105

New Zealand

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.501	−0.511***	−0.227***	0.009	−0.198***	−0.056*
$h=2$	0.008	−0.512***	−0.226***	−0.008	−0.195***	−0.087*
$h=3$	−0.249	−0.524***	−0.264***	−0.015	−0.214***	−0.112*
$h=4$	−0.442	−0.481***	−0.259**	−0.014	−0.189**	−0.109*
$h=5$	−0.540	−0.465***	−0.275*	0.012	−0.171*	−0.083
$h=6$	−0.591	−0.463***	−0.308*	0.024	−0.186	−0.079
$h=7$	−0.645	−0.450***	−0.319*	0.037	−0.183	−0.062
$h=8$	−0.691	−0.436***	−0.323*	0.041	−0.167	−0.048

Norway

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.040	−0.350***	−0.162***	0.000	−0.064	0.003
$h=2$	−0.361	−0.311***	−0.152**	−0.019	0.001	0.032
$h=3$	−0.624	−0.265***	−0.144	−0.047	0.068	0.058
$h=4$	−0.836	−0.210**	−0.129	−0.065	0.136	0.121
$h=5$	−0.940	−0.184	−0.128	−0.052	0.152	0.165
$h=6$	−0.984	−0.188	−0.167	−0.070	0.144	0.198*
$h=7$	−0.995	−0.204	−0.218	−0.095	0.115	0.201*
$h=8$	−1.022	−0.204	−0.249*	−0.120	0.110	0.201*

Portugal

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.998	−1.281***	−0.973***	0.094***	−0.848***	−0.194***
$h=2$	0.525	−1.049***	−0.818***	0.045	−0.673***	−0.193***
$h=3$	0.258	−0.949***	−0.765***	−0.010	−0.581***	−0.210***
$h=4$	0.051	−0.870***	−0.737***	−0.044	−0.497***	−0.208**
$h=5$	−0.067	−0.803***	−0.738***	−0.064	−0.464***	−0.212*
$h=6$	−0.155	−0.753***	−0.744***	−0.092	−0.429***	−0.217*
$h=7$	−0.232	−0.698***	−0.750***	−0.106	−0.390**	−0.213
$h=8$	−0.304	−0.650***	−0.738***	−0.107	−0.345**	−0.202

Spain

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.734	−0.670***	−0.393***	0.029	−0.236***	−0.040
$h=2$	0.189	−0.520***	−0.286**	0.071	−0.136	−0.050
$h=3$	−0.196	−0.376	−0.173	0.159	−0.005	0.022
$h=4$	−0.429	−0.310	−0.127	0.152	0.052	0.019
$h=5$	−0.485	−0.361**	−0.201	0.089	−0.001	−0.036
$h=6$	−0.577	−0.340*	−0.205	0.094	0.002	−0.033
$h=7$	−0.653	−0.328**	−0.212	0.065	0.011	−0.020
$h=8$	−0.729	−0.308**	−0.204	0.058	0.027	−0.005

Sweden

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.108	−0.320***	−0.146**	−0.046	−0.102	−0.036
$h=2$	−0.425	−0.184*	−0.092	−0.140	−0.086	−0.012
$h=3$	−0.784	−0.052	−0.025	−0.567	−0.047	0.063
$h=4$	−0.981	−0.044	−0.052	−0.420	−0.022	0.073
$h=5$	−1.081	−0.040	−0.060	−0.095	0.020	0.105
$h=6$	−1.098	−0.083	−0.134	−0.157	0.010	0.089
$h=7$	−1.103	−0.125	−0.208	−0.209	−0.006	0.071
$h=8$	−1.118	−0.145	−0.256*	−0.272**	−0.014	0.047

Switzerland

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.199	−0.374***	−0.103***	0.020	−0.135***	−0.111***
$h=2$	−0.362	−0.297***	−0.072	0.021	−0.148**	−0.134
$h=3$	−0.715	−0.215**	−0.021	0.000	−0.118	−0.104
$h=4$	−0.935	−0.179**	0.008	−0.001	−0.094	−0.078
$h=5$	−1.048	−0.183**	0.007	−0.010	−0.076	−0.036
$h=6$	−1.102	−0.201**	−0.019	−0.016	−0.088	−0.009
$h=7$	−1.153	−0.213***	−0.029	−0.013	−0.102	0.009
$h=8$	−1.197	−0.215**	−0.050	−0.027	−0.126	0.007

United Kingdom

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.738	−0.681***	−0.351***	0.106*	−0.284***	−0.082***
$h=2$	0.227	−0.663***	−0.356***	0.108	−0.245***	−0.074*
$h=3$	−0.061	−0.645***	−0.364***	0.141	−0.229***	−0.091*
$h=4$	−0.287	−0.579***	−0.330***	0.176	−0.202**	−0.096
$h=5$	−0.414	−0.553***	−0.319**	0.191	−0.187**	−0.101
$h=6$	−0.500	−0.536***	−0.321**	0.182	−0.188**	−0.110
$h=7$	−0.580	−0.520***	−0.316**	0.163	−0.189**	−0.108
$h=8$	−0.636	−0.511***	−0.324***	0.107	−0.204*	−0.121

Statistically significant differences according to the Diebold-Mariano t-statistic are indicated by asterisks, where

*,** and ** correspond respectively to 10%,5% and 1% significance levels

Table 3a: Average Continuous Rank Probability Scores (CRPS for MAI-AR-SV, CRPS ratios in all others)

USA

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.123	1.098***	1.028	1.02	0.993	1.000
$h=2$	0.206	1.115***	1.060*	1.042**	0.987	0.966
$h=3$	0.264	1.126***	1.097**	1.071**	0.998	0.970
$h=4$	0.315	1.126***	1.109**	1.106**	0.993	0.966
$h=5$	0.318	1.155***	1.125**	1.141***	1.006	0.983
$h=6$	0.316	1.172***	1.153***	1.185***	1.028	1.019
$h=7$	0.319	1.194***	1.195***	1.227***	1.056	1.055
$h=8$	0.327	1.197**	1.218***	1.249***	1.052	1.063

Australia

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.107	1.191***	1.081**	1.045	1.033	1.004
$h=2$	0.176	1.086**	1.041	1.040	1.008	1.039
$h=3$	0.231	1.051	1.055	1.071	1.043	1.107
$h=4$	0.277	1.056	1.063	1.081	1.063	1.141*
$h=5$	0.293	1.075	1.103	1.117	1.095	1.175**
$h=6$	0.306	1.090	1.137	1.143	1.107	1.184**
$h=7$	0.312	1.107	1.172*	1.165	1.109	1.165*
$h=8$	0.320	1.112	1.196*	1.186	1.107	1.141

Austria

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.114	1.404***	1.172***	1.006	1.065	0.951
$h=2$	0.191	1.268***	1.102*	1.002	1.001	0.918
$h=3$	0.248	1.192***	1.087	1.031	0.998	0.929
$h=4$	0.301	1.138***	1.066	1.053	1.001	0.963
$h=5$	0.327	1.142***	1.092	1.081	1.027	0.987
$h=6$	0.343	1.148***	1.124	1.121*	1.052	1.008
$h=7$	0.361	1.149***	1.142	1.158**	1.074	1.030
$h=8$	0.375	1.153**	1.152	1.179**	1.088	1.043

Belgium

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.107	1.128***	1.053**	1.047	1.050	1.060
$h=2$	0.169	1.139***	1.099***	1.096**	1.055	1.029
$h=3$	0.215	1.143***	1.117**	1.103**	1.042	0.989
$h=4$	0.274	1.122***	1.087*	1.089*	0.999	0.944
$h=5$	0.309	1.127***	1.083	1.099*	0.975	0.921
$h=6$	0.325	1.144***	1.095	1.121**	0.973	0.913
$h=7$	0.339	1.147***	1.101	1.135**	0.972	0.910
$h=8$	0.346	1.150***	1.117*	1.158***	0.959	0.900

Canada

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.126	1.044**	1.039	1.049*	0.983	0.996
$h=2$	0.196	1.048*	1.060	1.060	0.969	0.984
$h=3$	0.246	1.053*	1.073	1.073	0.976	0.993
$h=4$	0.285	1.065**	1.060	1.063	0.971	0.990
$h=5$	0.297	1.081**	1.039	1.044	0.968	1.001
$h=6$	0.305	1.087**	1.029	1.034	0.945	0.994
$h=7$	0.314	1.093**	1.034	1.048	0.931	0.983
$h=8$	0.324	1.102*	1.046	1.063	0.922	0.981

Finland

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.069	1.368***	1.155***	1.014	1.141***	1.133**
$h=2$	0.128	1.295***	1.091*	0.970	1.092	1.129
$h=3$	0.184	1.232***	1.052	0.948	1.046	1.107
$h=4$	0.241	1.172***	1.027	0.945	1.003	1.068
$h=5$	0.286	1.149**	1.011	0.934	0.987	1.044
$h=6$	0.319	1.138**	1.017	0.942	0.981	1.026
$h=7$	0.345	1.129*	1.035	0.966	0.984	1.009
$h=8$	0.365	1.13*	1.053	0.986	0.989	0.997

France

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.059	1.271***	1.053*	0.966	1.108**	1.090*
$h=2$	0.095	1.382***	1.083*	0.984	1.146	1.120
$h=3$	0.122	1.436***	1.124*	0.998	1.199	1.180
$h=4$	0.156	1.392***	1.105	1.007	1.172	1.174
$h=5$	0.174	1.407***	1.121	1.002	1.191	1.185
$h=6$	0.188	1.421***	1.137	0.995	1.214	1.201*
$h=7$	0.202	1.420***	1.140	0.989	1.215	1.193**
$h=8$	0.217	1.416***	1.144	0.980	1.200	1.175**

Germany

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.147	1.072***	0.989	0.992	1.028	1.053
$h=2$	0.225	1.079**	0.981	0.995	1.050	1.055
$h=3$	0.274	1.093**	1.027	1.042	1.102	1.097
$h=4$	0.329	1.089**	1.046	1.070	1.100	1.101
$h=5$	0.354	1.100**	1.059	1.096*	1.107	1.116
$h=6$	0.379	1.099**	1.065	1.114*	1.122	1.124
$h=7$	0.405	1.1**	1.073	1.128**	1.132	1.12
$h=8$	0.427	1.098**	1.077	1.134**	1.142	1.115

Greece

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.059	1.597***	1.252***	1.003	1.270***	1.085**
$h=2$	0.101	1.563***	1.250***	1.000	1.267***	1.107*
$h=3$	0.133	1.616***	1.286***	1.016	1.286***	1.120*
$h=4$	0.158	1.615***	1.300**	1.025	1.305***	1.124*
$h=5$	0.167	1.667***	1.366**	1.043	1.351***	1.150*
$h=6$	0.174	1.711***	1.440***	1.076	1.385***	1.184*
$h=7$	0.178	1.761***	1.533***	1.117	1.415***	1.209*
$h=8$	0.182	1.800***	1.609***	1.164	1.408***	1.215*

Italy

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h=1$	0.036	2.026***	1.509***	0.948	1.370***	1.158***
$h=2$	0.068	1.797***	1.388***	0.959	1.238*	1.174**
$h=3$	0.101	1.651***	1.321***	0.968	1.150	1.193
$h=4$	0.136	1.502***	1.243**	0.957	1.089	1.182
$h=5$	0.163	1.442***	1.221	0.954	1.074	1.178
$h=6$	0.183	1.408***	1.224	0.960	1.078	1.173
$h=7$	0.201	1.384***	1.225	0.964	1.07	1.155
$h=8$	0.216	1.376***	1.239	0.983	1.054	1.14

Statistically significant differences according to the Diebold-Mariano t-statistic are indicated by asterisks, where

*,** and ** correspond respectively to 10%,5% and 1% significance levels

Table 3b: Average Continuous Rank Probability Scores (CRPS for MAI-AR-SV, CRPS ratios in all others)

Japan

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h = 1$	0.079	1.558***	1.253***	1.021	1.209***	1.117***
$h = 2$	0.129	1.502***	1.236***	0.962	1.234**	1.117
$h = 3$	0.177	1.456***	1.219***	0.911*	1.252*	1.115
$h = 4$	0.226	1.423***	1.227***	0.889*	1.245	1.083
$h = 5$	0.264	1.418***	1.235***	0.851**	1.247	1.053
$h = 6$	0.299	1.406***	1.244***	0.835**	1.223	1.025
$h = 7$	0.328	1.397***	1.266***	0.831**	1.199	1.000
$h = 8$	0.353	1.384***	1.276***	0.826**	1.169	0.985

Luxembourg

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h = 1$	0.103	1.171***	1.054**	1.031	1.036	1.030
$h = 2$	0.164	1.168***	1.051	1.043	1.047	1.047
$h = 3$	0.201	1.191***	1.076	1.061	1.060	1.059
$h = 4$	0.244	1.177***	1.089	1.087	1.034	1.037
$h = 5$	0.263	1.190***	1.107	1.106*	1.035	1.036
$h = 6$	0.269	1.206***	1.150**	1.141**	1.038	1.034
$h = 7$	0.280	1.201***	1.179**	1.171**	1.053	1.051
$h = 8$	0.289	1.197***	1.196**	1.185**	1.066	1.066

Netherlands

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h = 1$	0.087	1.483***	1.233***	1.006	1.230***	1.096*
$h = 2$	0.130	1.484***	1.224***	1.017	1.250**	1.151*
$h = 3$	0.160	1.465***	1.239***	1.053	1.271*	1.212*
$h = 4$	0.199	1.379***	1.184**	1.054	1.208	1.189
$h = 5$	0.224	1.361***	1.163	1.038	1.172	1.169
$h = 6$	0.246	1.347***	1.152	1.029	1.133	1.147
$h = 7$	0.275	1.311***	1.112	1.018	1.09	1.112
$h = 8$	0.295	1.303***	1.089	1.005	1.071	1.090

New Zealand

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h = 1$	0.082	1.313***	1.085**	0.969	1.152***	1.067
$h = 2$	0.139	1.276***	1.061	0.933	1.170**	1.114
$h = 3$	0.186	1.265***	1.080	0.904	1.179*	1.137
$h = 4$	0.229	1.213***	1.081	0.899	1.135	1.120
$h = 5$	0.256	1.193**	1.093	0.872	1.105	1.089
$h = 6$	0.274	1.181*	1.118	0.849	1.089	1.061
$h = 7$	0.289	1.180*	1.144	0.841	1.068	1.020
$h = 8$	0.298	1.187*	1.174	0.844	1.052	0.983

Norway

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h = 1$	0.132	1.189***	1.084***	0.996	1.007	0.979
$h = 2$	0.194	1.206***	1.115**	1.010	0.978	0.959
$h = 3$	0.250	1.194***	1.137**	1.018	0.940	0.935
$h = 4$	0.303	1.183***	1.151*	1.036	0.908	0.903
$h = 5$	0.333	1.184***	1.171*	1.024	0.913	0.881
$h = 6$	0.349	1.186***	1.225**	1.045	0.919	0.863
$h = 7$	0.356	1.184***	1.291**	1.071	0.926	0.848
$h = 8$	0.368	1.168**	1.331***	1.098	0.919	0.836

Portugal

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h = 1$	0.046	2.787***	2.130***	0.971	1.922***	1.158***
$h = 2$	0.076	2.214***	1.864***	1.004	1.661***	1.150**
$h = 3$	0.101	2.006***	1.821***	1.037	1.551***	1.170*
$h = 4$	0.124	1.857***	1.821***	1.069	1.482***	1.179
$h = 5$	0.140	1.739***	1.870***	1.095	1.447***	1.181
$h = 6$	0.155	1.639***	1.902***	1.118	1.394***	1.175
$h = 7$	0.170	1.547***	1.923***	1.134	1.323**	1.158
$h = 8$	0.184	1.475***	1.923***	1.137	1.247	1.135

Spain

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h = 1$	0.063	1.514***	1.248***	0.981	1.171***	1.027
$h = 2$	0.109	1.358***	1.163***	0.941	1.102	1.036
$h = 3$	0.148	1.298***	1.147**	0.932	1.069	1.041
$h = 4$	0.186	1.251***	1.127	0.923	1.018	1.033
$h = 5$	0.212	1.234***	1.132	0.909	1.001	1.033
$h = 6$	0.236	1.208***	1.138	0.901	0.980	1.014
$h = 7$	0.260	1.180**	1.137	0.900	0.951	0.985
$h = 8$	0.280	1.17**	1.142	0.905	0.931	0.974

Sweden

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h = 1$	0.124	1.182***	1.067	1.016	1.064	1.070
$h = 2$	0.209	1.113***	1.065	1.044	1.069	1.041
$h = 3$	0.275	1.093*	1.099	1.071	1.095	1.032
$h = 4$	0.342	1.083	1.112	1.082	1.047	0.977
$h = 5$	0.381	1.082	1.125	1.090	1.024	0.947
$h = 6$	0.399	1.097	1.179*	1.131	1.023	0.941
$h = 7$	0.410	1.120	1.246**	1.186*	1.026	0.936
$h = 8$	0.420	1.137*	1.305***	1.240***	1.033	0.944

Switzerland

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h = 1$	0.108	1.256***	1.057**	0.985	1.151***	1.145***
$h = 2$	0.194	1.192***	1.017	0.960	1.149*	1.151
$h = 3$	0.274	1.151***	0.996	0.954	1.133	1.128
$h = 4$	0.343	1.138***	0.984	0.953	1.103	1.088
$h = 5$	0.384	1.153***	0.994	0.957	1.096	1.050
$h = 6$	0.406	1.175***	1.019	0.969	1.115	1.021
$h = 7$	0.426	1.199***	1.047	0.987	1.144	1.001
$h = 8$	0.446	1.215***	1.074	1.003	1.179	0.995

United Kingdom

	MAI-AR-SV	*MAI-AR*	*AR*	*AR-SV*	*BVAR*	*BVAR-SV*
$h = 1$	0.063	1.533***	1.195***	0.936	1.238***	1.086*
$h = 2$	0.104	1.531***	1.224**	0.952	1.243**	1.100
$h = 3$	0.138	1.517***	1.245**	0.933	1.241**	1.130*
$h = 4$	0.176	1.436***	1.205	0.903	1.195*	1.123
$h = 5$	0.202	1.406***	1.195	0.883	1.158	1.105
$h = 6$	0.222	1.382***	1.197	0.878	1.135	1.087
$h = 7$	0.241	1.368***	1.214	0.898	1.126	1.074
$h = 8$	0.255	1.361***	1.241	0.940	1.126	1.068

Statistically significant differences according to the Diebold-Mariano t-statistic are indicated by asterisks, where

*,** and ** correspond respectively to 10%,5% and 1% significance levels

C Additional Figures

C.1 Core Inflation, Data and Decompositions

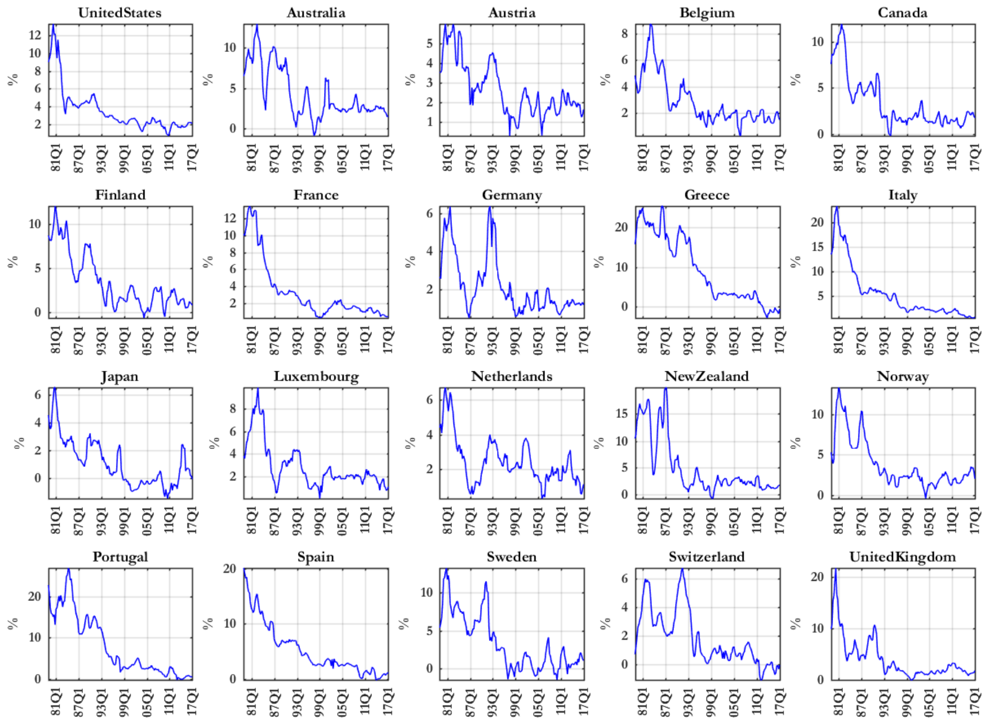


Figure 20: Non-Food & non-Energy inflation rates (year on year growth rates in quarterly CPIs)

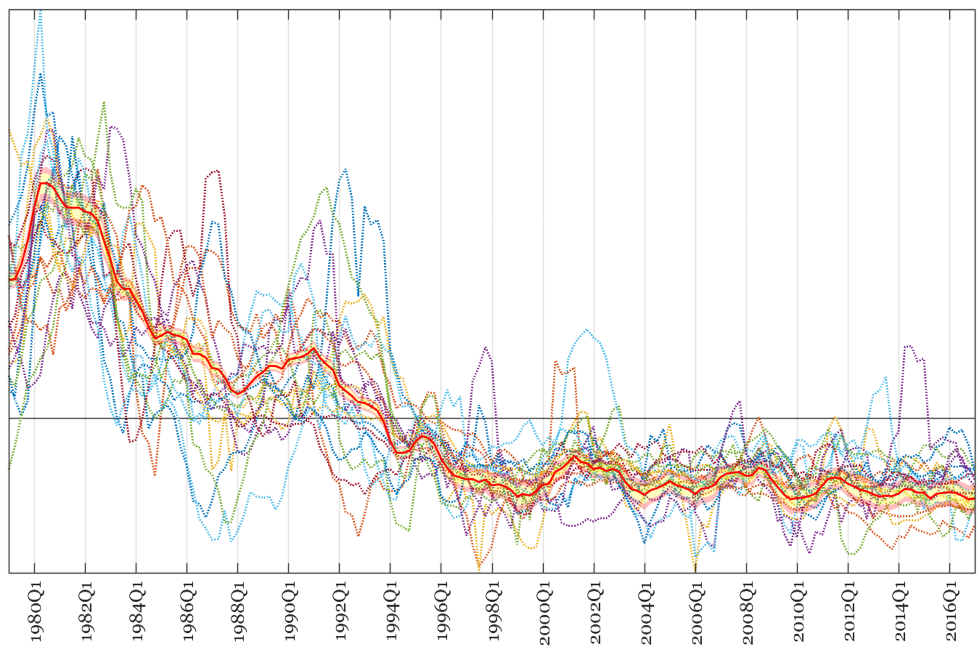


Figure 21: MAI-AR-SV estimated common factor (with posterior bands) Vs Data. Core inflation

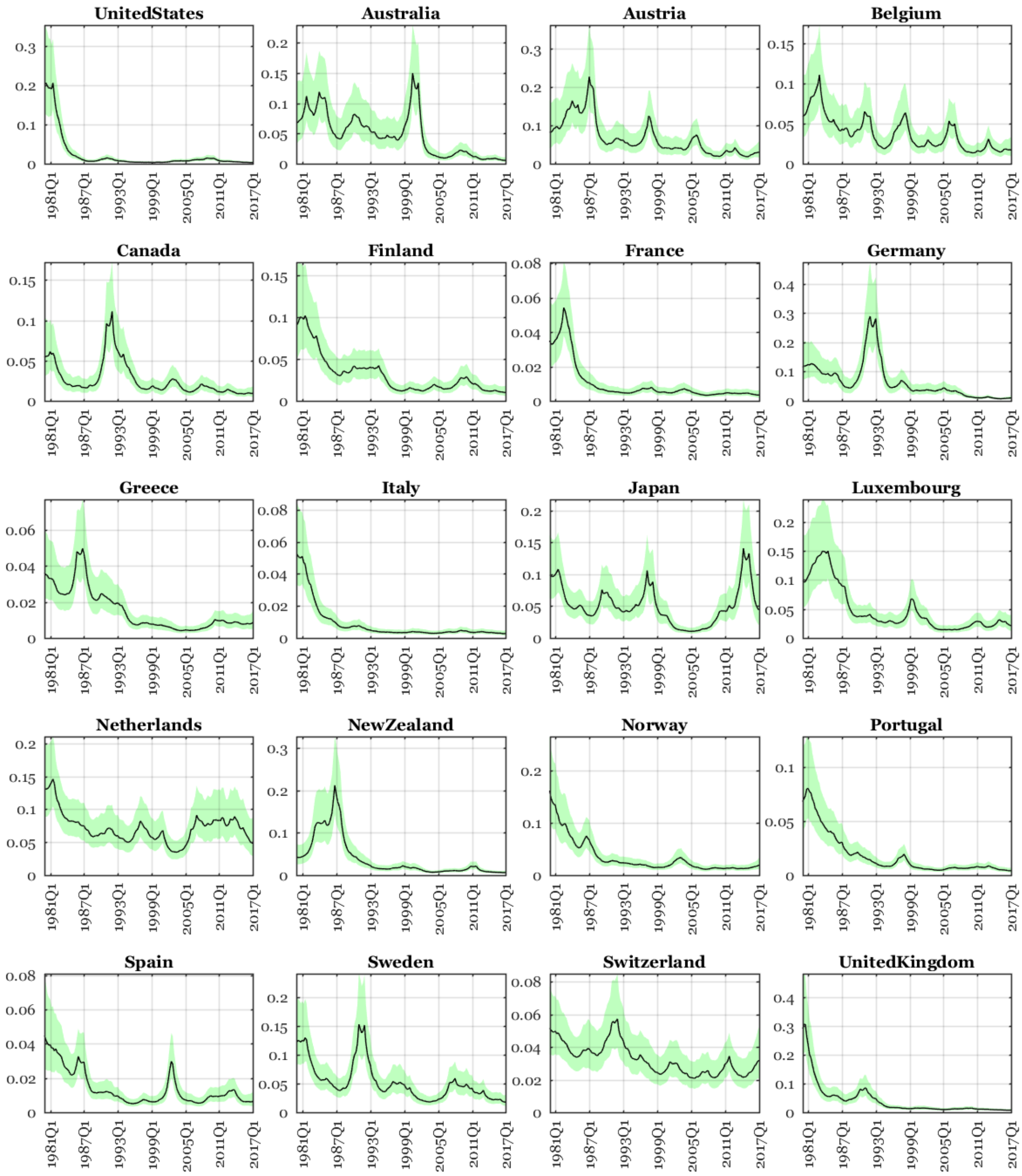


Figure 22: MAI-AR-SV, Residuals' Volatility, posterior bands. Core inflation

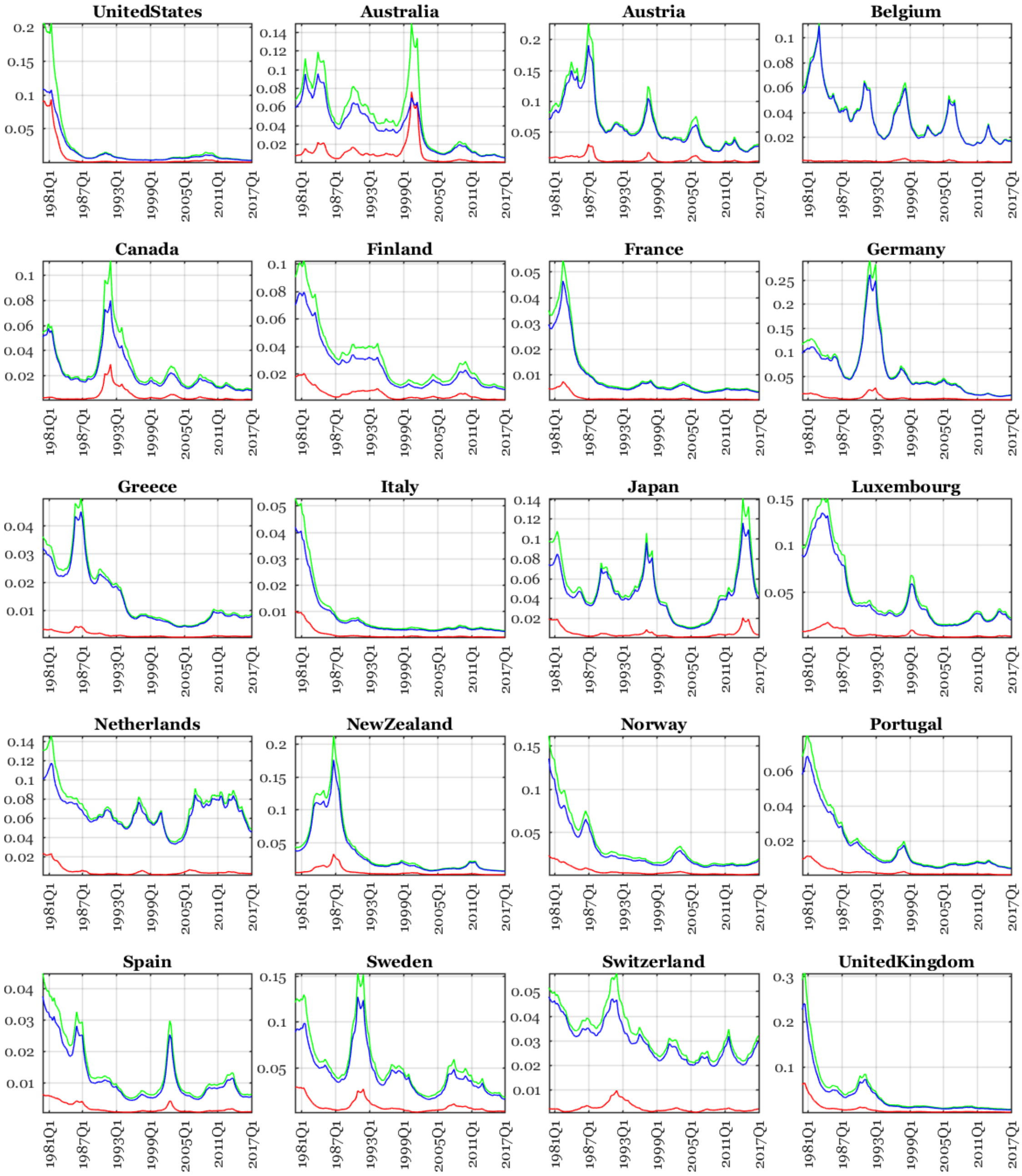


Figure 23: MAI-AR-SV, Residuals' Volatility, TV decomposition, Common (red), Idio (green), total (blue). Core inflation

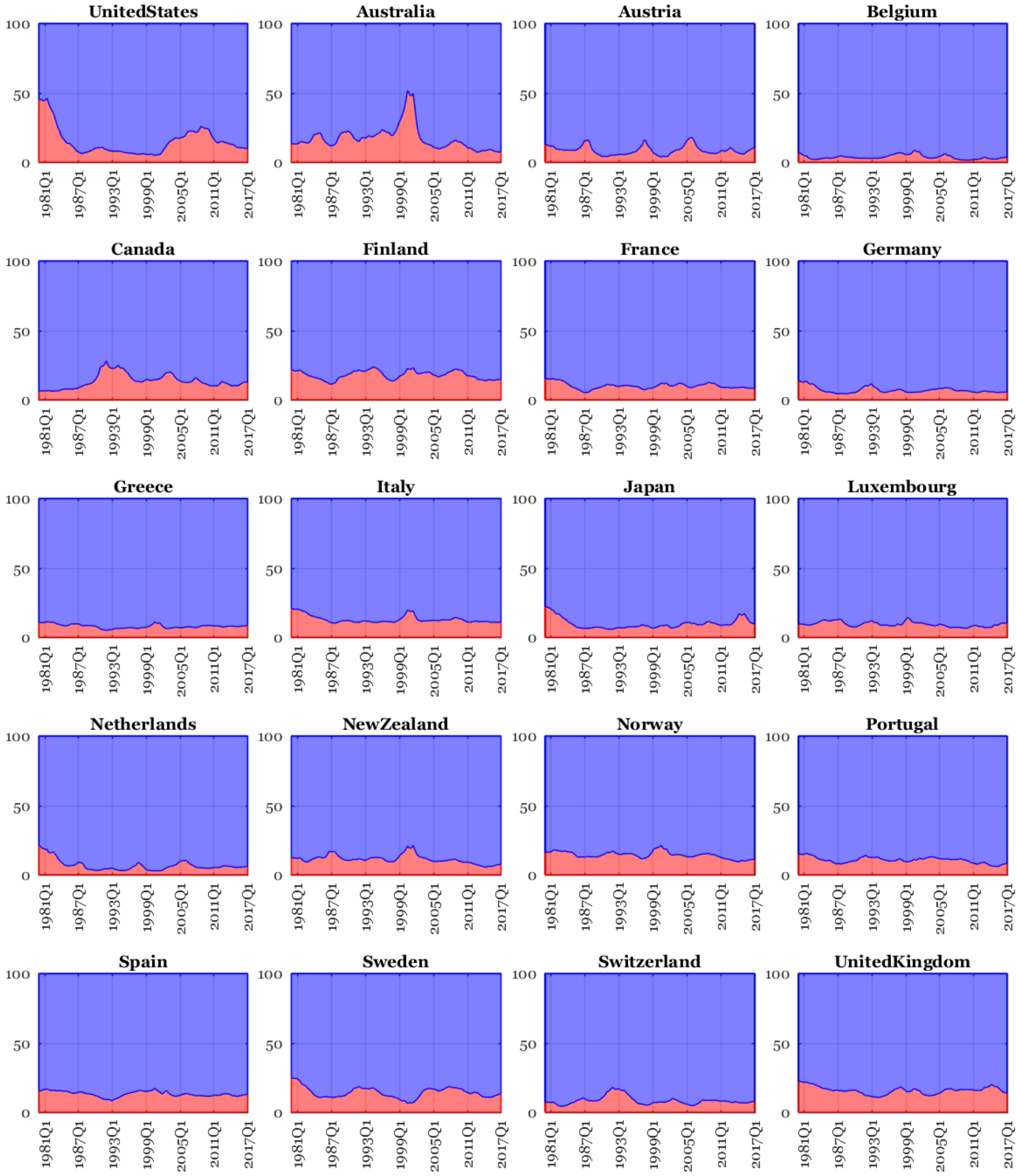


Figure 24: MAI-AR-SV, Residuals' Volatility, TV decomposition shares (%), Common (red), Idio (blue). Core inflation

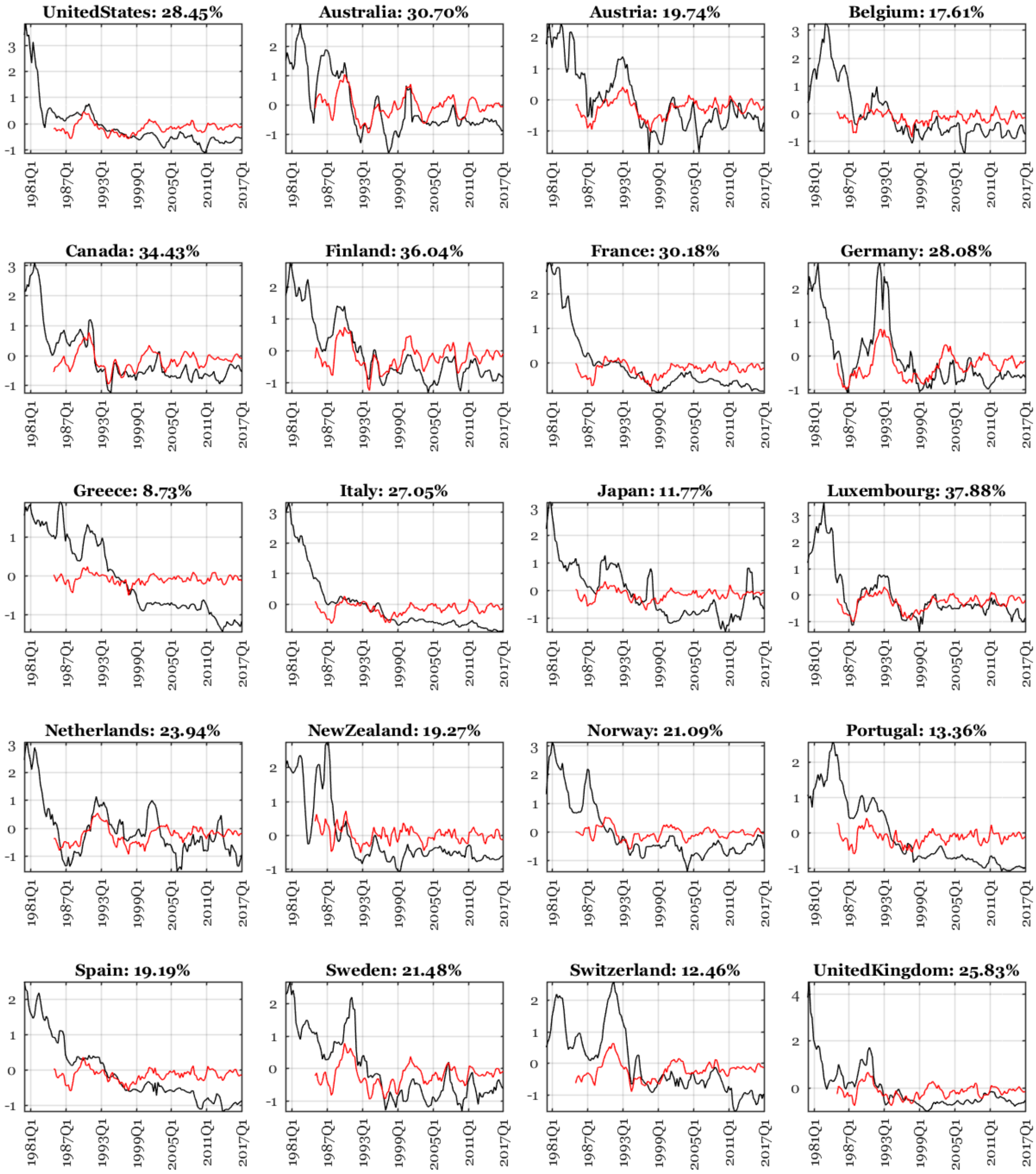


Figure 25: MAI-AR-SV, Actual series and Common component (red). Core inflation

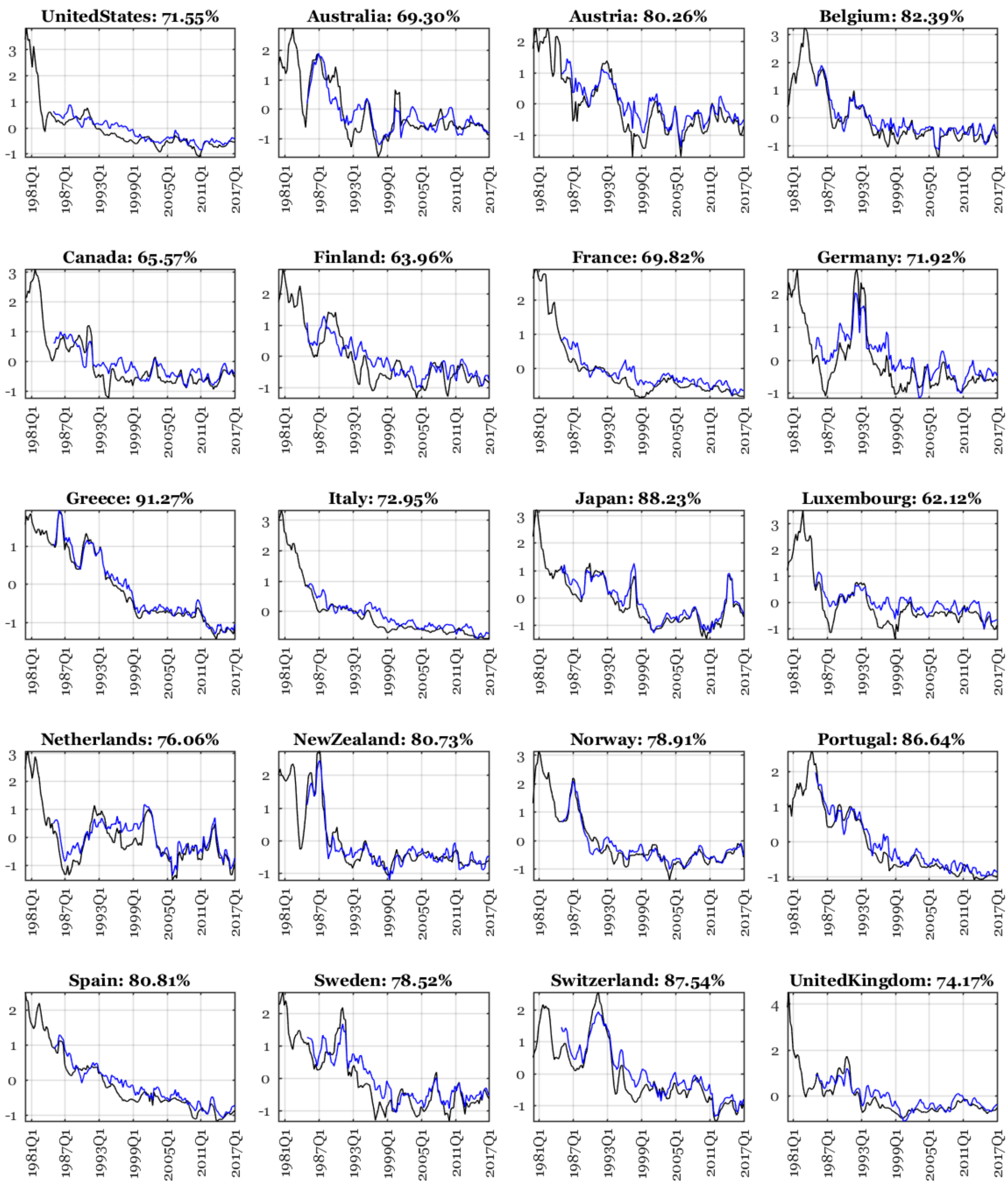


Figure 26: MAI-AR-SV, Actual series and Idiosyncratic component (blue). Core inflation