On the Evolution of the United Kingdom Price Distributions

BM Chu[1] KP Huynh[2] DT Jacho-Chávez[3] O Kryvtsov[2]

[1]Carleton University

[2]Bank of Canada

[3]Emory University

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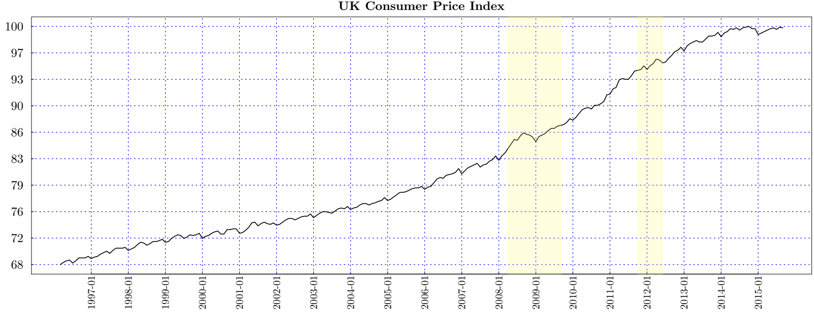
The views expressed are those of the authors and should not be attributed to the Bank of Canada.

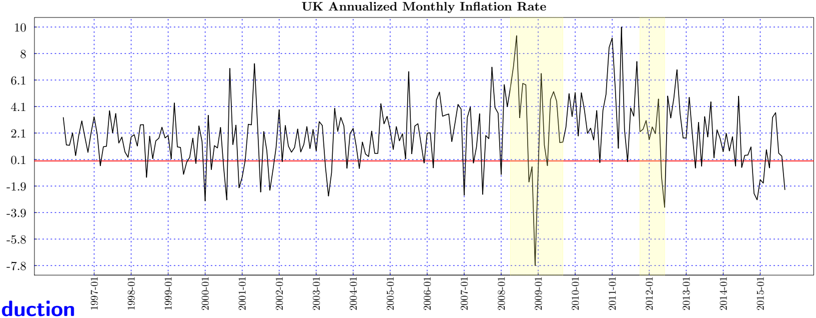
Contributions

- The paper provides a *distributional analysis* of a *publicly available monthly survey of prices* that a governmental statistical agency collects to construct the Consumer Price Index (CPI) for the United Kingdom.
- An adaptation of Kneip & Utikal's (2001, *JASA*) *Functional Principal Component Analysis* of density families is proposed utilizing the *Sampling Weights Kernel Density* estimator of Buskirk & Lohr (2007, *JSPI*) to take into account the complex survey nature of the data set.
- Develop an algorithm to conduct out-of-sample density forecasts.

forthcoming in the *<u>Annals of Applied Statistics</u>* *(accepted on 20 April 2018).*

UK Consumer Price Index & Inflation





JOURNAL

OF THE ROYAL STATISTICAL SOCIETY.

MARCH, 1924.

THE INTERRELATION AND DISTRIBUTION OF PRICES AND THEIR INCIDENCE UPON PRICE STABILIZATION.

By NORMAN CRUMP.

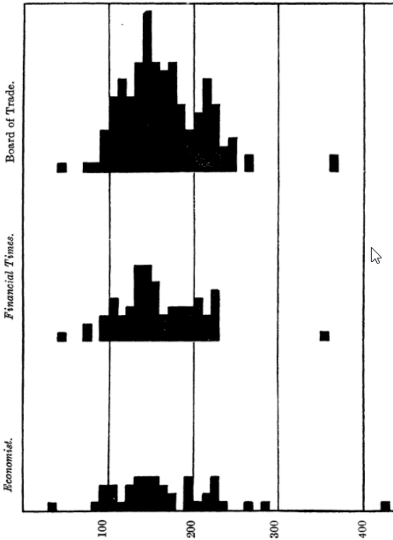
APPENDIX 5.—*Comparison of Four Index-Numbers.*

(1913 = 100.)

	Board of Trade.	*Financial Times.*	*Economist.*	Statistique Générale de la France.
Upper quartile .... (Q_1)	194	200	195	479
Median .... (M)	160	152	156	386
Lower quartile .. . . (Q_2)	132	130	129	329
Arithmetic mean . .. . (A)	164·7	158·6	166·3	421·2
Geometric mean .... . . (G)	157·9	150·9	154·8	390·7
Standard deviation . . (σ)	48·2	47·4	64·6	168·5
Coefficient of variation . (V)	0·293	0·299	0·388	0·400
Angle of deviation .... (α)	17° 4′	17° 24′	22° 50′	23° 36′

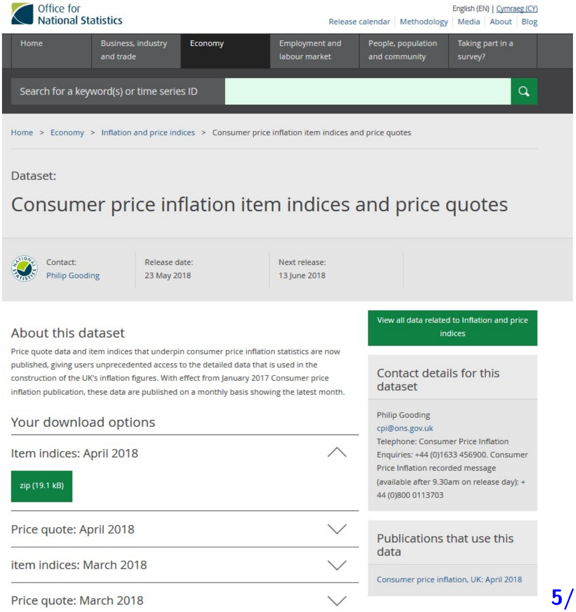
Note.—Period covered: British figures, average prices for first half of 1923; French figures, end of September, 1923.

DISTRIBUTION OF BRITISH INDEX-NUMBERS.



Office for National Statistics (ONS)

- Data has been placed on the ONS website.
- Download the data and documentation.
- Thanks to the ONS for their assistance in this project.



UK CPI

- Monthly collection, except for some services and seasonal items.
- 03/1996-09/2015 or 235 months.
- 110,000 units per month.
- 26 million observations.
- Stratified sampling by:

  (1) shop type,

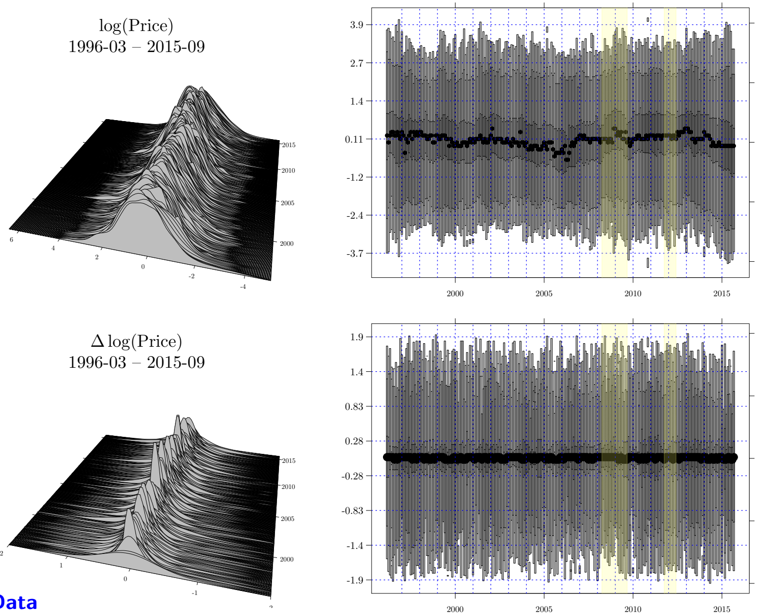
  (2) region,

  (3) shop type $\times$ region.

  Shop type: multiple and independents (less than 10 outlets).

Figure 1: UK Regions

Figure 2: log *Price* - Δ log *Price*, Demeaned & Standardized



Functional Principal Components

Karhunen-Loéve decomposition by Kneip and Utikal (JASA, 2001):

$$f_t = f_\mu + \sum_{j=1}^{J} \theta_{t;j} g_j,$$

$f_\mu = \sum_{t=1}^{T} f_t / T$ is the common mean.

$\theta_{t;j}$ are the j-th components at time t.

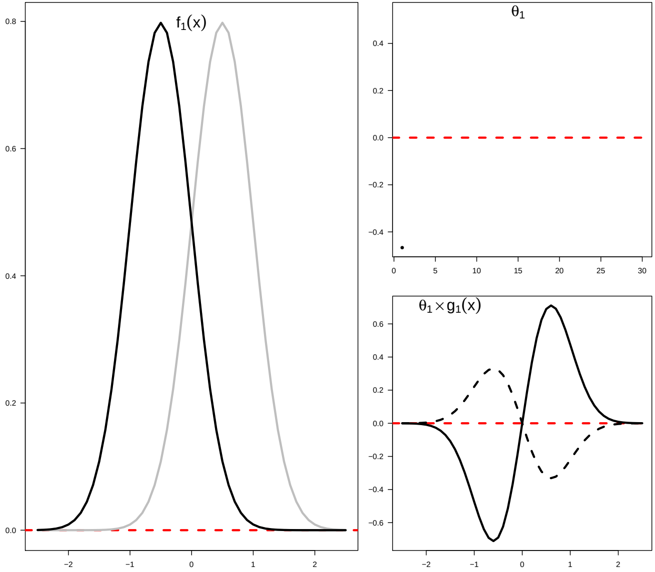
g_j is the time-invariant profile for the j-th component.

Singular-value decomposition to $[M_{ts}]_{T \times T} =< f_t - f_\mu, f_s - f_\mu >$:

$$\theta_{tr} = \lambda_r^{1/2} p_{tr}, \qquad g_r = \sum_{t=1}^{T} \theta_{t;r} f_t / \sum_{t=1}^{T} \theta_{t;r}^2.$$

$$\sum_{t=1}^{T} \theta_{t,j} = 0, \ \sum_{t=1}^{T} \theta_{t,j}\theta_{t,l} = 0 \text{ if } j \neq l, \ \sum_{t=1}^{T} \theta_{t,j}^2 = \lambda_j, \ j = 1, \ldots, J.$$

$$f_t = f_\mu + \theta_{t;1} g_1,$$



FPCA with Complex Survey Data

Buskirk & Lohr (2007): Sample-Weighted Kernel Density

Finite population of size N is divided into L distinct strata with respective sizes $N_1, \ldots, N_L$, i.e. $N = \sum_{k=1}^{L} N_k$.

The strata density function, $f(x) = \sum_{k=1}^{L} W_k f_k(x)$ where $W_k = N_k/N$, $k = 1, \ldots, L$, can be estimated by

$$\widehat{f}_{S_{N_n}}(x) = \frac{1}{wh} \sum_{i \in S_{N_n}} K\left(\frac{x - x_i}{h}\right) w_i$$

where S_{N_n} is any sample of size n taken from this strata density, $w_i = \pi_i^{-1}$ with $\pi_i = \Pr_D\{i \in S_{N_n}\}$ and $w = \sum_{i \in S_{N_n}} w_i$. For each i, w_i is called the unit's *sampling weight*.

FPCA with Complex Survey Data

Allow for weak-dependence in longitudinal data: T stratified samples, $\{\{(X_{it}, w_{it})^\top\}_{i=1}^{n_t}\}_{t=1}^{T} \Rightarrow \widetilde{M}_{ts} =< \widehat{f}_{S_t,h} - \widehat{f}_{S,\mu}, \widehat{f}_{S_s,h} - \widehat{f}_{S,\mu} >$,
where $\widehat{f}_{S,\mu} = T^{-1} \sum_{t=1}^{T} \widehat{f}_{S_t,h}$, and $\langle \xi_1, \xi_2 \rangle = \int \xi_1(x)\xi_2(x)\varpi(x)dx$, for some ϖ continuous, uniformly bounded weight function.

Let $\widehat{M}_{ts}$ be the *biased corrected* version of $\widetilde{M}_{ts}$ with $svd(\widetilde{M}_{ts}) \Rightarrow \widehat{\lambda}_r$ and $\widehat{p}_r$ of $\widehat{M}$:

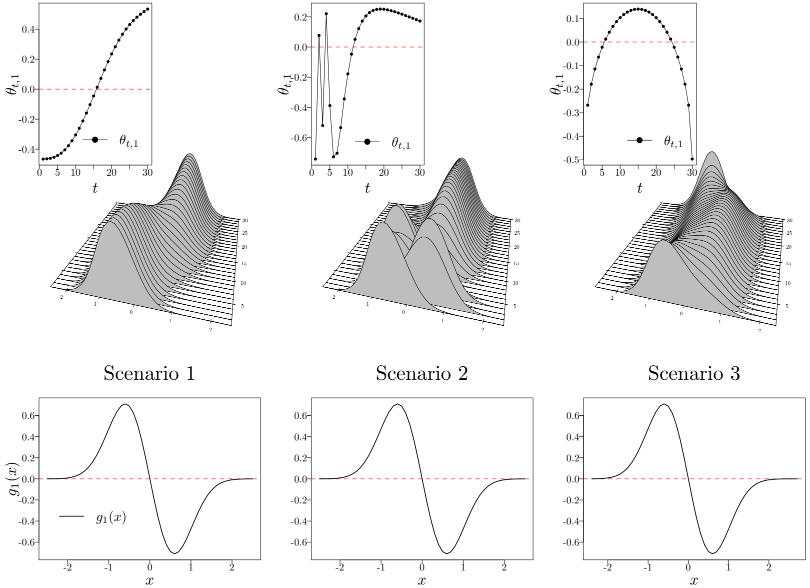
$$\widehat{\theta}_{t,r} = \widehat{\lambda}_r^{1/2} \widehat{p}_{t,r}, \quad \widehat{g}_r = \frac{\sum_{t=1}^{T} \widehat{\theta}_{t,r} \widehat{f}_{S_t,h}}{\sum_{t=1}^{T} \widehat{\theta}_{t,r}^2}. \tag{1}$$

$$\sum_{t=1}^{T} \theta_{t,j} = 0, \ \sum_{t=1}^{T} \theta_{t,j}\theta_{t,l} = 0 \text{ if } j \neq l, \ \sum_{t=1}^{T} \theta_{t,j}^2 = \lambda_j, \ j = 1, \ldots, J, \tag{2}$$

Computational Implementation

1. Cross-validate 235 bandwidths for each cross-sectional density.
   24 hours with 474 cores on EDITH using RSQLite and npRmpi.
2. Numerical integrate $[\widehat{M}_{ts}]_{T\times T} = < \widehat{f}_t - \widehat{f}_\mu, \widehat{f}_s - \widehat{f}_\mu >$
   $T \times (T-1)/2$ or 27,495 integrals.
   1 hour with 474 cores on EDITH using RSQLite and RSnow.
3. Singular value decomposition of $[\widehat{M}_{ts}]_{T\times T}$ to retrieve T-eigenvalues/vectors.
4. Scree plot (rank-order eigenvalues) and compute $\widehat{\theta}_{t;j}$ and $\widehat{g}_j$.
   The first four components of $\log Price$ and $\Delta \log Price$ account for 5% and 7%, respectively. Compared to the rule-of-thumb of $1/T$ for significance.

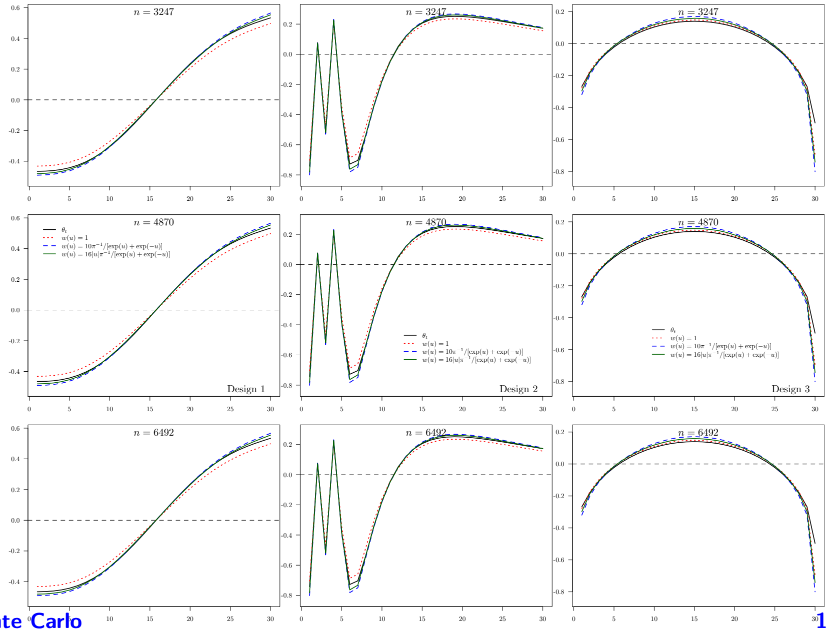
Monte Carlo Designs



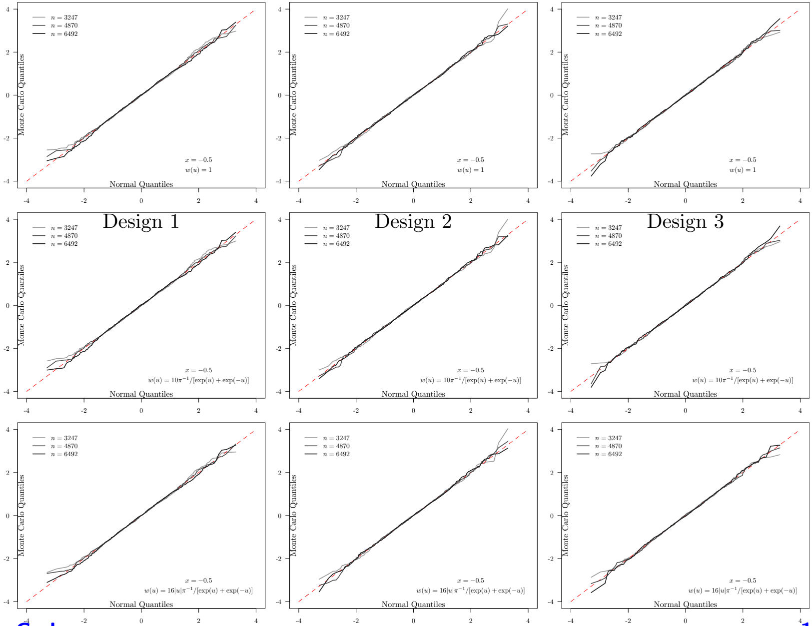
Monte Carlo Details

1. For each $t = 1, \ldots, 30$, we set $N = 20,000$ distributed over $L = 6$ sub-populations, $N_1 = 2,223$, $N_2 = 4,445$, $N_3 = 6,666$, $N_4 = 2,219$, $N_5 = 2,223$, and $N_6 = 2,224$.
2. We draw samples $n_1 = c \times 667$, $n_2 = c \times 712$, $n_3 = c \times 1,000$, $n_4 = c \times 111$, $n_5 = c \times 556$, $n_6 = c \times 201$ ($n = c \times 3247$) from each f_t in the designs for $c \in \{1, 1.5, 2\}$ along with their inclusion probabilities.
3. In each of $1,000$ Monte Carlo replications, Silverman's rule-of-thumb bandwidths and second-order gaussian kernels are used when calculating the 435 numerical integrals.
4. Three different weighting functions were used: $w(u) = 1$; $w(u) = 10\pi^{-1}/[\exp(u) + \exp(-u)]$; and $w(u) = 16|u|\pi^{-1}/[\exp(u) + \exp(-u)]$.

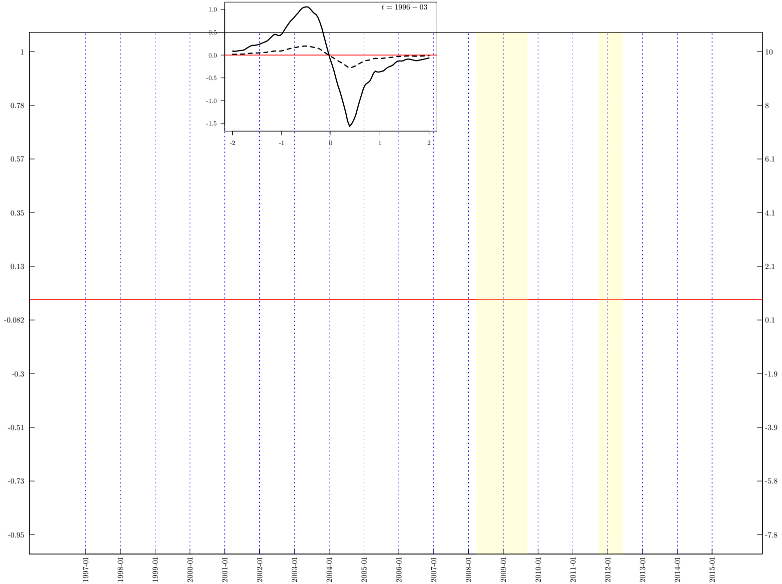
Monte Carlo: θ_t Performance



Monte Carlo: $g(x)$ Performance



$\Delta \log \mathit{Price}$: $\widehat{\theta}_{t,1} \times \widehat{g}_1$, $\widehat{\theta}_{t,1}$, and UK Monthly inflation rate (blue)



Density Forecasting with FPCA

$$f_t = f_\mu + \sum_{j=1}^{J} \theta_{t;j} g_j,$$

Step 1. Using the first T^* estimated densities, $\{\widehat{f}_t\}_{t=1}^{T^*}$, calculate $\{\{\widehat{\theta}^*_{t,r}\}_{r=1}^{L^*}\}_{t=1}^{T^*}$ and $\{\widehat{g}^*_r\}_{r=1}^{L^*}$, where $L^* \le T^*$ represents the number of the first non-zero eigenvalues of the $T^* \times T^*$-matrix $\widehat{\mathcal{M}}^*$. Also set $\widehat{f}^*_\mu = (1/T^*) \sum_{t=1}^{T^*} \widehat{f}_t$.

Step 2. Exploiting the orthonormal features of $\{\{\widehat{\theta}^*_{t,r}\}_{r=1}^{L^*}\}_{t=1}^{T^*}$, we utilize the algorithm in Hyndman & Khandakar (2008) to automatically identify the best-fitted ARMA model for each generated series, $\{\widehat{\theta}^*_{t,r}\}_{t=1}^{T^*}$, $r = 1, \dots, L^*$, and then proceed to obtain an automatic forecast for period $T^* + \ell$, i.e., $\{\widehat{\widehat{\theta}}_{T^*+\ell|T^*,r}\}_{r=1}^{L^*}$.

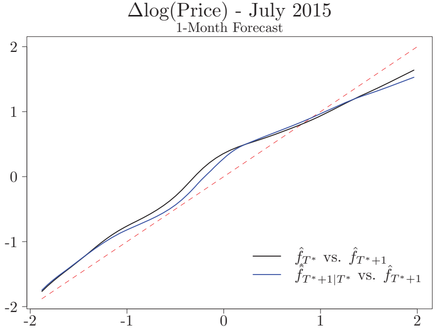
Density Forecasting with FPCA (cont.)

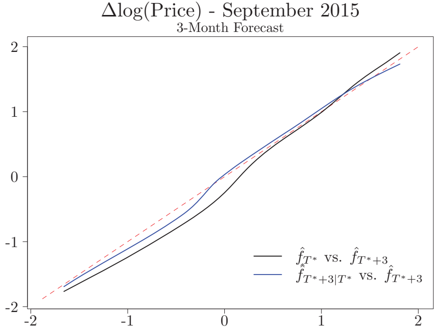
Step 3. Set

$$\widehat{J} = \arg \min_{l \in \{1,\dots,L^*\}} \int \left(\widehat{f}_{T^*+\ell}(x) - \widehat{f}^*_\mu(x) - \sum_{r=1}^{l} \hat{\hat{\theta}}_{T^*+\ell|T^*,r} \widehat{g}^*_r(x) \right)^2 dx,$$

$$\hat{\hat{f}}_{T^*+\ell|T^*} = \widehat{f}^*_\mu + \sum_{r=1}^{\widehat{J}} \hat{\hat{\theta}}_{T^*+\ell|T^*,r} \widehat{g}^*_r. \tag{3}$$

QQ Plots Forecast for $\Delta \log Price$





$\widehat{J}$=4 with $\hat{\hat{\theta}}_{t,1} \sim SARMA(0,0)(2,0)_{12}$, $\hat{\hat{\theta}}_{t,2} \sim SARMA(1,1)(0,0)_{12}$, $\hat{\hat{\theta}}_{t,3} \sim SARMA(0,2)(1,1)_{12}$, and $\hat{\hat{\theta}}_{t,4} \sim SARMA(1,1)(0,0)_{12}$.

Summary

- Develop methodology to account for sampling weights in nonparametric estimation of FPCA.
- Demonstrate the efficacy of FPCA to visualize the dynamics of cross-sectional distributions.
- Application to UK Consumer Price Distributions via ONS.
- Conduct an out-of-sample forecasting exercise.

Thanks/Merci/Danke!