Relationships in the Interbank Market*

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Abstract

The market for central bank reserves is mainly over-the-counter and exhibits a core-periphery network structure. This paper develops a model of relationship lending in the unsecured interbank market. Banks choose to build relationships in order to insure against liquidity shocks and to economize on the cost to trade in the interbank market. Relationships can explain some anomalies in the level of interest rates – e.g., the fact that banks sometimes trade below the central bank's deposit rate, as we find using data from the ECB. The model also helps understand how monetary policy affects the network structure of the interbank market and its functioning.

Keywords: relationships, interbank market, core-periphery, networks, corridor system.

1 Introduction

Major central banks implement monetary policy by targeting the overnight rate in the unsecured segment of the interbank market for reserves – the very short term rate of the yield curve. The textbook principles of monetary policy implementation are intuitive: Each bank holds a reserve account at the central bank. Over the course of a normal business day, this account is subject to shocks depending on the banks' payment outflows (negative shock) and inflows (positive shock) driven by business activities. Banks seek to manage their account balance to satisfy some reserve requirements.[1] By changing the

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[1]The minimum reserve requirement can be either positive (e.g., in the U.S.) or zero (e.g., in Canada).

supply of reserves, a central bank influences the interest rate at which banks borrow or lend reserves in the interbank market, and as a consequence the marginal cost of making loans to businesses and individuals. In recent years, many major central banks have refined this system by offering two facilities: In addition to auctioning reserves, the central bank stands ready to lend reserves at a penalty rate – the lending rate – if banks end up short of reserves. Symmetrically, banks can earn an interest at the deposit rate if they end up holding reserves in excess of the requirement. As a consequence the interbank rate should stay within the bands of the corridor defined by the lending and the deposit rates. This is known as the "corridor system" for monetary policy implementation.

The reality is more complex than this basic narrative. Bowman, Gagnon, and Leahy (2010) and others report that in many jurisdictions with large excess reserves, banks have been trading below the deposit rate (supposedly the floor of the corridor).[2] It is also well known that banks sometimes trade above the lending rate (supposedly the ceiling of the corridor). This is a challenge to the basic intuition that simple arbitrage would maintain the rates within the bands of the corridor. At a time when central banks are thinking of exiting quantitative easing policies, we may wonder whether these apparent details are symptomatic of a dysfunctional interbank market that will hamper exit, or if they are "natural" phenomena with little relevance for the conduct of monetary policy during the exit stage.

Contrary to folk belief, the interbank market is very far from being the epitome of the Walrasian market. For example, every year the ECB money market survey (e.g. ECB, 2013) shows that the majority of the transactions in the European (unsecured) interbank

[2] Bowman, Gagnon and Leahy (2010) review the experience of eight major central banks and report that the deposit rates on reserve do not always provide a lower bound for short-term market rates. In particular, the (weighted) average of overnight market rates for reserve balances sometimes stayed below the deposit rate during the recent financial crisis, when reserve balances were abundant and the central bank moved its overnight target towards the deposit rate on reserves. In some countries, a potential explanation for this puzzling observation is that some participants in the money market cannot earn interest on their deposits at the central bank (e.g. GSEs in the U.S.). In other cases, such as Japan, there is no clear institutional feature that can explain why the average overnight rate stayed below the floor. For example, according to the above study, in Japan, "[t]he uncollateralized overnight call rate is similar to the federal funds rate, in that it is a daily weighted average of transactions in the uncollateralized overnight market. Participants in the market include domestic city, regional, and trust banks, foreign banks, securities companies, and other firms dealing in Japanese money markets. All of the economically important participants active in the call market are also eligible to participate in the deposit facility...the overnight call rate has occasionally fallen below the Bank of Japan's deposit rate in the period since November 2008, but never by more than 2 basis points." Similarly, in Canada, there are anecdotal evidence that some banks occasionally lent and borrowed at rates below the floor during the crisis period, even though they had direct access to central bank deposit facilities. Concerning the explanation for these negative spreads in Canada, some practitioners argued that banks lent below the floor due to concerns about their reputation and relationship with other banks in the market (see Lascelles, 2009).

market is made over-the-counter (OTC). Many banks maintain long term relationships with their partners. Smaller banks tend to trade with just a few other banks, if not only one, or they directly access the central bank facilities without even trading with another bank. We review the evidence below, but these facts are now well accepted.

In this paper we analyze the effects of long-term trading relationships and monetary policy on interbank trading volume and rates, the network structure of the interbank market and its functioning. We show that modeling relationships matters for both individual and aggregate demand for liquidity, and can explain why banks trade below the deposit rate or above the lending rate. Our analysis also sheds light on the effect of monetary policy on the network structure of the interbank market. In particular we show that the accommodative monetary policy stance can lower the value of building and maintaining relationships, so that in steady state, no or few relationships exist. Some central bankers have been pointing out this phenomenon and it arises endogenously from our model.

We model the corridor system for monetary policy implementation under zero-reserve requirements.[3] At the beginning of each day, banks face a shock to their reserves holdings. Facing the reserve requirement, banks in a deficit position can borrow at the central bank's lending facility at i_ℓ and those in a surplus position can deposit their reserves at the deposit facility to earn i_d. A positive spread between i_ℓ and i_d gives banks incentives to borrow and lend directly with each other. These trades can be conducted in two OTC markets, a core market and a periphery market where relationship-lending takes place. Participation in the core market is subject to two frictions. First, banks face matching frictions in finding trading partners. Second, some banks (that we call S for "small") have to pay a cost to access the core market, while others (that we call L for "large") can access it freely.[4] In addition, banks can trade reserves with their long-term partners in the periphery market.[5] Within a relationship, a large bank can provide intermediation service by accessing the core market on behalf of a small bank; the small bank saves on the access cost, and the large bank extracts some rents from providing this service. Furthermore, a relationship allows long-term partners to trade repeatedly over time without the need to search for a

[3]Setting the minimum reserve requirement at zero is just a normalization. Our finding does not rely on this assumption.

[4]For example, small banks usually do not have a liquidity manager and so their opportunity cost of accessing the OTC market is high. Hence, small banks will only enter the OTC market if their gains from trade is sufficiently large. Also, "small" and "large" are merely labels for banks with high and low participation costs. We will show that, in equilibrium, low-cost banks will trade larger volume than high-cost banks in the interbank market, hence justifying the labels "large" and "small".

[5]Below the terms "relationship" and "partnership" are used interchangeably.

new counterparty everyday. However relationships can end for exogenous reasons. In that case, banks have to search for a new partner.

Within this framework, we can explain why we observe arbitrage opportunities in the data. Small banks value long-term relationships which provide liquidity insurance and save their costs of accessing the OTC market. As a result, they are willing to temporarily lower their surplus from trading a loan as long as the long-term gains from keeping a relationship outweigh the short-term loss. Specifically, if the conditions are right, we show that small banks with surplus reserves agree to lend at a rate below i_d. Symmetrically, small banks that need reserves may end up paying a rate above i_ℓ. Therefore, in equilibrium some banks trade below the floor or above the ceiling of the corridor. On the surface there seems to be unexploited arbitrage opportunities. There is none really: small banks are willing to trade at a rate outside the corridor only for small loans with their long-term partners, but not for large loans or with other counterparties. Our stylized model shows that the corridor is "soft", i.e. equilibrium interbank rates can be below i_d or above i_ℓ, when trading frictions are present, even though small banks can access the deposit/lending facilities at no cost. Furthermore our model implies that the occurrence of this outcome depends on aggregate liquidity conditions: A soft corridor is more likely when there is a large aggregate liquidity surplus or deficit.

To get a sense for the performance of the model, we use data from the Money Market Survey Report of the ECB. This survey started in July 2016 and contains the universe of money market trades for the largest 52 banks in the Euro area. The data shows that the money market has a core-periphery structure, very much like that in other jurisdictions. In addition, we find that a significant fraction of loans (38%) from small banks to large banks are conducted at a rate below the deposit facility rate. We parameterize our model by matching several moments in the data, in particular the frequency of trades below the floor. We then conduct several experiments. For example, we study the effects of changing the width and position of the interest rate corridor and the supply and distribution of reserve balances on rates and the trading activities in the core and periphery markets.

A lesson for policy makers is that trades outside the corridor are consistent with a well-functioning core-periphery interbank market. Thus, central banks have no need to worry about eliminating deviations due to long-term relationships. However, one should be careful in interpreting the interbank market rate as a reference for overnight cost of liquidity, because it may also incorporate a relationship premium, which at times can significantly distort the observed overnight rate.

Our work is also one of the first attempts at explaining the endogenous response of the network structure of the interbank market to a change in monetary policy. The network structure that emerges endogenously resembles the core-periphery structure we observe in the data, where most of the trading activities is due to a number of banks that appear to intermediate the trades of others. We show how the interest rate corridor and the distribution of liquidity can affect banks' incentives to build relationships and accordingly the terms and patterns of trades.

Literature

Figure 1 shows the network of the Federal funds market as shown by Bech and Atalay (2008) for September 29, 2006. The market has a core-periphery network structure (or tiered structure) where some banks in the periphery only trade with one bank, the latter possibly trading with many others.[6] Bech and Atalay (2008) point out that, in the US interbank market, "[t]here are two methods for buying and selling federal funds. Depository institutions can either trade directly with each other or use the services of a broker. ... In the direct trading segment, transactions commonly consist of sales by small-to-medium sized banks to larger banks and often take place on a recurring basis. The rate is set in reference to the prevailing rate in the brokered market. In the brokered segment, participation is mostly confined to larger banks acting on their own or a customers behalf."

[6] Sept. 29 2006 was the last day of the third quarter. Bech and Atalay report that a total of 479 banks were active in the market that day. The largest bank in terms of fed funds value traded is located at the center of the graph. The 165 banks that did business with the center bank lie on the first outer circle. The second outer circle consists of the 271 banks that did business with the banks in the first circle, but that did no business with the center bank. The remaining dots are the 42 banks that were more than two links away from the center. Two banks did business only between themselves. Large value links are in yellow and small value links are in red.

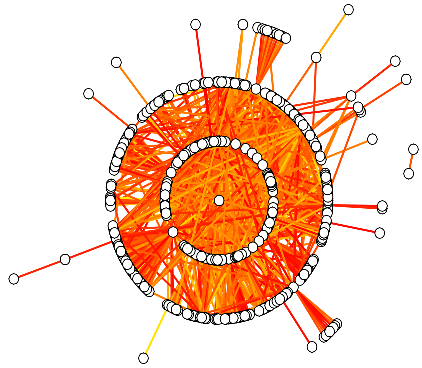

Figure 1: Network structure of the fed funds market (Bech and Atalay, 2008)

Stigum and Crescenzi (2007, Ch. 12) also reports anecdotal evidence of the tiered structure of the fed funds market. In particular, they report that

> "[i]n the fed funds market now, regional banks buy up funds from even tiny banks, use what they need, and resell the remainder in round lots in the New York market. Thus, the fed funds market resembles a river with tributaries: money is collected in many places and then flows through various channels into the New York market. In essence, the nation's smaller banks are the suppliers of fed funds, and the larger bankers are the buyers."

Also

> "[t]o cultivate correspondents that will sell funds to them, large banks stand ready to buy whatever sums these banks offer, whether they need all these funds or not. If they get more funds than they need, they sell off the surplus in the brokers market. Also, they will sell to their correspondents if the correspondents need funds, but that occurs infrequently. As a funding officer of a large bank noted, 'We do feel the need to sell to our correspondents, but we would not have cultivated them unless we felt that they would be selling to us 99% of the time. On the occasional Wednesday when they need \$ 100,000 or \$ 10 million, OK. Then we would fill their need before we would fill our own.'"

Elsewhere, using Bundesbank data on bilateral interbank exposures among 1800 banks, Craig and Von Peter (2010) and Brauning and Fecht (2012) find strong evidence of tiering in the German banking system. Using UK data Wetherlit et. al. (2009) also report the existence of a core of highly connected banks alongside a periphery. Of course, this has important consequences on rates. As Stigum and Crescenzi (2007) note,

Our paper is related to the literature on the interbank market and monetary policy implementation, to the growing literature on financial networks, and to the literature on OTC markets. The first literature on the interbank market includes, Poole (1968), Hamilton (1996), Berentsen and Monnet (2006), Berentsen, Marchesiani, and Waller (2014), Bech and Klee (2011), Afonso and Lagos (2012a,b), and Afonso, Kovner and Schoar (2012), among others. See also Bech and Keister (2012) for an interesting application of the Poole (1968) model to reserve management with a liquidity coverage ratio requirement. While Afonso, Kovner, and Schoar (2012) show some evidence of long term relationship in the interbank market, none of the papers above accounts for it. Rather they all treat banks as anonymous agents conducting random, "spot" trades. So our paper is the first to study the effect of long term relationship on rates. Based on private information, Ennis and Weinberg (2013) explain why some banks borrow at a rate above the central bank's lending rate. Armenter and Lester (2017) explain why banks trade below the deposit rate when some do not qualify for receiving the interest on excess reserves in the US federal funds market. The early literature on financial networks is mostly motivated by understanding financial fragility and has been covered in Allen and Babus (2009), see also Jackson (2010). It includes Allen and Gale (2000) who study whether some banks networks are more prone to contagion than others. Also, Leitner (2005) studies the optimality of linkages, motivated by the desirability of mutual insurance, when banks can fail; while Gofman (2011) and Babus (2013) analyze the emergence and efficiency of intermediaries in OTC markets. In a recent calibration exercise, Gofman (2014) finds that it is suboptimal to limit banks' interdependencies in the interbank market. Elliott, Golub, and Jackson (2014) apply network theory to financial contagion through net worth shocks. Finally, the literature on OTC market includes Duffie, Garleanu, and Pedersen (2005), and Lagos and Rocheteau (2006), among many others.[7] Within this literature, Chang and Zhang (2015) study network formation in asset markets based on heterogeneous liquidity preferences.

> "A few big banks, however, still see a potential arbitrage, 'trading profits,' in selling off funds purchased from smaller banks and attempt to profit from it to reduce their effective cost of funds. Also a few tend to bid low to their

[7] See also Afonso and Lagos (2012), Li, Rocheteau and Weill (2012), Lagos, Rocheteau, and Weill (2013), and Rocheteau and Wright (2013).

> correspondents. Said a trader typical of the latter attitude, 'We have a good name in the market, so I often underbid the market by 1/16."

There is a large empirical literature on the interbank market and we already mentioned a few papers. Furfine (1999) proposes a methodology to extract fed funds transactions from payments data and Armantier and Copeland (2013) test the methodology. Afonso, Kovner and Schoar (2011) study the fed funds market in time of stress. The two papers most related to ours are perhaps the empirical study of Brauning and Fecht (2012) and the theoretical paper of Blasques, Brauning, and van Lelyveld (2015). Brauning and Fecht suggest a theory of relationship lending based on private information, as proposed by Rajan (1992). In good times, banks extract an informational rent, thus explaining why the relationship lending rates are usually higher than the average rate in normal times. In bad times, a lending bank knows whether the borrower is close to failure, and it is willing to offer a discount in order to keep the bank afloat. This argument fails to recognize that in bad times, some borrowers may not be close to failure and the rent that can be extracted from a relationship lender can be even higher then. Moreover, the size of discount involved in these loans is usually not an amount significant enough to matter for the survival of a borrowing bank.[8] Although we do not want to minimize the role of private information, we argue that the simple threat of terminating the relationship can also yield to interbank rate discounts. Blasques, Brauning, and van Lelyveld study a dynamic network model of the unsecured interbank market based on peer monitoring. However, they do not study the impact of the supply of reserves on the structure of the network, or explain why interest rates can fall outside of the corridor.

Section 2 describes the environment. Sections 3 and 4 characterize the equilibrium. We analyze the quantitative implications of the model in Section 5. We discuss several extensions of the model in Section 6 and conclude in Section 7. Proofs can be found in the Online Appendix.

2 Model

We model the daily reserve management problem of banks in a corridor system using an infinite horizon model with discrete time. There is a central bank and two types of banks,

[8] For example, in Canada, the average loan size (suggested by the Furfine algorithm) is CAD 60 million, so that a discount of 10 basis point for a year is only CAD 60,000, a rather trivial amount for those banks in the Canadian payment system.

a measure one of "small" (S) banks and a measure n of "large" (L) banks. While it will be natural to think that $n < 1$, so that there are relatively few L banks, the model can also be used to study the case where $n \geq 1$. All banks are infinitely-lived and discount the future at rate $\beta \in (0, 1)$. Banks are subject to reserve requirements and they have to hold at least $\bar{R}$ units of reserves at the end of each day. For simplicity we normalize $\bar{R}$ to zero. Each unit of reserve is a claim to a numeraire good. Banks are risk neutral and they enjoy utility q from consuming q units of the numeraire (where $q < 0$ if they produce). Each period t consists of settlement as well as 4 sub-periods $s = 1, 2, 3, 4$.

Subperiod	Events
	Settlement
1	Liquidity shocks
2	Building relationship, peripheral trades
3	Core market trades
4	Access to central bank facilities

At the beginning of each period, banks automatically settle their past interbank trades as well as their obligations toward the central bank by producing or consuming the numeraire good. Banks do not default, so after settlement all banks hold zero reserve balances.[9] Following settlement, banks' current business relationships, if any, end with probability $\sigma \in (0, 1)$.

In sub-period 1, banks receive an idiosyncratic liquidity shock. S banks draw a shock ξ from the distribution $F(\xi)$, while L banks draw a shock ε from the distribution $G(\varepsilon)$.

In sub-period 2, banks in a relationship can borrow from and lend to one another. These recurring bilateral links form the *periphery* market, as opposed to the core market that we describe below. We assume that banks in the periphery market use proportional bargaining and S banks obtain a share θ of the trading surplus. We denote the relationship rate between an S bank holding R_S reserves and an L bank holding R_L reserves by $\hat{\imath}(R_{L,} R_S)$.

Single banks do not trade in subperiod 2, but can attempt to find a new business partner.[10] An S (resp. L) bank can pay a cost κ_S (resp. κ_L) to call one L (resp. S) bank to form a relationship starting from the next day. The call is random so any bank that is already in a relationship can receive a call, thus giving this bank a better negotiating position with its current business partner. For simplicity, we assume that a bank who

[9] Our assumption on linear utility ensures that banks are indifferent between consuming the numeraire or carrying reserves forward.

[10] In the Appendix, we consider a setup where a bank with a partner can also search for an additional one. This does not affect the key implications of the model.

initiates a call will not simultaneously receive a call from another bank. Also, random matching implies that the probability that an L bank receives a call increases with the measure of single S banks placing a call (α_S) and decreases with the measure of L banks waiting for a call ($n(1-\alpha_L)$). Similarly, the probability that an S bank receives a call increases with the measure of single L banks placing a call ($n\alpha_L$) and decreases with the measure of S banks waiting for a call ($1-\alpha_S$). Finally, we assume that each bank can maintain at most one relationship. So a bank, who already has a business partner and is contacted by another bank this period, can choose to maintain only one of the two banks as business partner in the next period.

In sub-period 3, all banks, with or without a business partner, choose whether to access the *core market* for reserves. An S bank, single or not, pays a cost γ to access this market. To the contrary, L banks can access this market freely and they always will. The matching function in the core market is such that a bank with a negative reserve position always meets a bank with a positive reserve position.[11] In particular, if there is a measure N^- of banks with negative reserves and a measure N^+ with positive reserves, then the matching function is $\mathcal{M}(N^+, N^-)$. The probability that a bank with negative reserves meets a bank with positive reserves is $\min\{1, \mathcal{M}(N^+, N^-)/N^-\}$. Again, we assume that banks use proportional bargaining and lenders obtain a share Θ of the trading surplus. We denote the lending rate in the core market between a borrower holding R^- and a lender holding R^+ by $r(R^-, R^+)$.

In sub-period 4, banks access the lending and deposit facilities of the central bank. Banks that still carry a reserve deficit will have to borrow reserves at the lending facility and pay the lending rate i_ℓ to cover their deficit. Banks that enjoy a reserve surplus will see it remunerated at the deposit rate $i_d < i_\ell$ by the central bank.

3 Equilibrium

In this section, we set up the decision problem of each type of banks in each market and we define our equilibrium. We first define the payoff of each bank at the settlement stage. Then we define the payoff in each preceding market.

[11] Bech and Monnet (2017) provide a rationale for this type of matching structure.

3.1 End-of-day central bank facilities and next-day settlement

Consider a bank holding excess reserve balance R, with aggregate dues D from previous trade – $D > 0$ denotes an aggregate credit position while $D < 0$ is an aggregate debit position – and a number of business partners $m \in \{0, 1\}$. At the end of the day, a bank holding $R < 0$ has to borrow $-R$ units of reserves from the central bank and pays an interest $-Ri_\ell$ on this loan. Otherwise, this bank deposits excess reserves $R > 0$ with the central bank to earn Ri_d on this amount. In the following settlement stage, the central bank collects or pays the amount of the numeraire good corresponding to these loans and deposits. Also, banks settle their aggregate position towards other institutions by producing and transferring the numeraire good. If their aggregate position is positive, they end up consuming some of the numeraire. Therefore, the value of an L bank just before accessing the central bank facilities is V_4, given by

$$V_4(R, D, m) = \beta \{D + R[1 + i(R)]\} + \beta \bar{V}_1(m) \tag{1}$$

where

$$i(R) = \begin{cases} i_d & \text{if } R \geq 0 \\ i_\ell & \text{if } R < 0 \end{cases},$$

and $\bar{V}_1(m)$ denotes the expected value of an L bank with business partner $m \in \{0, 1\}$ before the separation shock. Similarly, the value of an S bank before accessing the central bank facilities is v_4, given by

$$v_4(R, D, m) = \beta \{D + R[1 + i(R)]\} + \beta \bar{v}_1(m), \tag{2}$$

where $\bar{v}_1(m)$ denotes the value of an S bank with business partner $m \in \{0, 1\}$ before the separation shock. At the beginning of sub-period 1, a fraction σ of banks have their existing relationships broken up exogenously.[12] Then the value functions of the different types of banks are,

$$\bar{v}_1(m) = [1 - \sigma(m)]\, v_1(1) + \sigma(m) v_1(0), \tag{3}$$

$$\bar{V}_1(m) = [1 - \sigma(m)]\, V_1(1) + \sigma(m) V_1(0), \tag{4}$$

where $\sigma(0) = 1$ and $\sigma(1) = \sigma$.

[12] When a bank is hit by this shock, we assume that all of the existing relationships (whether built in the last or earlier periods) are broken at the same time.

3.2 Sub-period 3. The core market

In the core market, banks trade reserves bilaterally. We assume that the matching function does not allow two banks with the same relative position to meet one another. So two banks with reserves surplus will not meet. Therefore, a match will necessarily have a bank holding a negative amount of reserves – naturally we refer to this bank as the borrower – and a bank holding a positive amount of reserves – the lender. We use superscript "+" to denote variables associated with the lender, and "−" to denote variables associated with the borrower.

We use N^+ and N^- to denote respectively the numbers of banks with positive and negative reserve balances in the core market. Let $\Omega^+(.)$ denote the cumulative distribution of banks holding *positive* reserves in the core market. Then $\Omega^+(r)$ is the measure of banks with reserves $R \in [0, r]$. . Similarly, we use $\Omega^-(.)$ to denote the cumulative distribution of banks holding negative reserve in the core market, so that $\Omega^-(r)$ is the measure of banks with reserves $R \in (-\infty, r]$ when $r \leq 0$. In the core market, a bank with a negative position can meet a bank with a positive position with probability $\mu^- = \min\{1, \mathcal{M}(N^+, N^-)/N^-\}$. Also, a bank with a positive position meets one with a negative position with probability $\mu^+ = \min\{1, \mathcal{M}(N^+, N^-)/N^+\}$.[13]

A bank borrowing x units of reserves agrees to repay X units of reserves back to the lender at the beginning of the next day, where (x, X) are determined by proportional bargaining where the lender receives a fraction Θ of the total trade surplus (irrespective of the type of the lender bank). The bargaining solution between two L banks satisfies

$$\begin{aligned}(1-\Theta)[V_4(R^+ - x, D^+ + X, m^+) - V_4(R^+, D^+, m^+)] &= \\ \Theta[V_4(R^- + x, D^- - X, m^-) - V_4(R^-, D^-, m^-)],&\end{aligned}$$

and replace V_4 by v_4 whenever the lender or the borrower is an S bank. The top line term in bracket is the surplus that an L lender holding $R^+ > 0$ and with m^+ business partner obtains when trading x units of reserves today for X units at settlement. The bottom line term in bracket is the surplus from the borrowing bank holding $R^- < 0$ and with m^- business partner. Thanks to linearity, we will show below that the terms of trade (x, X) depend only on the current reserve holdings of the borrower and the lender, R^+ and R^-, and importantly, neither on their type nor on their number of business partners. As a

[13] The derivation of N^+, N^- and Ω^+, Ω^- is provided in the Appendix

consequence the terms of trade will be the same for S and L banks and we denote them by $(x, X)(R^+, R^-)$. Using (1) and (2), the borrower's and lender's trade surplus from a loan (x, X) are

$$\begin{aligned} S^-(R^-, R^+) &= -X + (R^- + x)[1 + i(R^- + x)] - R^-[1 + i_\ell] \\ S^+(R^-, R^+) &= X + (R^+ - x)[1 + i(R^+ - x)] - R^+[1 + i_d] \end{aligned}$$

Then banks choose a loan (x, X) to maximize the trade surplus of the borrower

$$\max_{x,X} S^-(R^-, R^+)$$

subject to the lender getting a fraction Θ of the total surplus, or

$$S^+(R^-, R^+) = (1 - \Theta)[S^-(R^-, R^+) + S^+(R^-, R^+)].$$

We focus on the bargaining outcome described by the following Lemma.[14]

Lemma 1. *The following contract (x, X) solves the bargaining problem in the core market*

$$\begin{aligned} x(R^-, R^+) &= \frac{R^+ - R^-}{2}, \\ X(R^-, R^+) &= (2\Theta - 1)\left(\frac{R^- + R^+}{2}\right)\left[1 + i\left(\frac{R^- + R^+}{2}\right)\right] \\ &\quad + (1 - \Theta)R^+ (1 + i_d) - \Theta R^- (1 + i_\ell). \end{aligned}$$

The contract specifies that a borrower and a lender will hold the same amount of reserves after trading x, and the payment X compensates the lender for such a trade. From these terms of trade, we can find the implicit pairwise core market rate, $r(R^-, R^+)$, defined by

$$r(R^-, R^+) \equiv \begin{cases} i_\ell - 2(1 - \Theta)(i_\ell - i_d)\frac{R^+}{R^+ - R^-} & \text{, if } R^+ + R^- \leq 0 \\ i_d + 2\Theta(i_\ell - i_d)\frac{-R^-}{R^+ - R^-} & \text{, if } R^+ + R^- \geq 0 \end{cases}.$$

In general, $r(R^-, R^+)$ is within the interest rate corridor defined by the central bank's lending and deposit rates, i.e. $i_d \leq r \leq i_\ell$. When the lender has just enough reserves to compensate the reserve deficit of the borrower, the rate is simply the mid-point of the

[14] Replacing the constraint in the objective function, it is straightforward to see that there can be multiple solutions with equivalent payoffs since $i(R)$ is piecewise-linear. We select the same solution as in Bech and Monnet (2015) who study an equivalent model where banks incur a settlement shock after banks exit the OTC market. They show that the bargaining solution is unique. So we can see our economy as the limit to Bech and Monnet, as the settlement risk becomes negligible. As should become clear below, choosing other solution would not affect the main results of the paper, such as terms of trade in the relationship-lending market or the network structure.

corridor whenever both banks have the same bargaining power, $r = \Theta i_\ell + (1-\Theta)i_d$. As the lender cannot compensate the reserve deficit of the borrower ($R^+ + R^- \leq 0$), the rate will be closer to i_ℓ whenever the lender has all the bargaining power, or little reserves to lend, or the reserves deficit of the borrower is large. In the opposite scenario, the rate will tend to i_d. Naturally, the rate decreases whenever either banks' reserve holdings increase.

It is useful to define the expected gains from trade for a borrower and a lender accessing the core market. The one for a borrower is

$$\Pi^-(R) = \beta\mu^-(1-\Theta)(i_\ell - i_d)\left\{-R[1-\Omega^+(-R)] + \int_0^{-R} R^+ d\Omega^+(R^+)\right\},$$

Whenever $R \leq 0$, the expected gains from trade for the borrower is $\Pi^-(R)$: the borrower meets a lender with probability μ^- and he gets a share $(1-\Theta)$ of the trading surplus. The lender has enough to cover the reserve deficit of the borrower with probability $1-\Omega^+(-R)$. In this case the gains from trading the first R units of reserves are $i_\ell - i_d$. Indeed, up to his reserve deficit R, the borrower values each unit of reserves he borrows at rate i_ℓ, as otherwise he would have to borrow them from the central bank. Symmetrically the lender values at rate i_d each unit of reserves he lends, as this is the rate he would obtain at the deposit facility of the central bank. Beyond R, there is no gains from trade, as the borrower and the lender both value reserves at rate i_d. When the borrower meets a lender that cannot meet his reserves deficit – this happens with probability $\int_0^{-R} d\Omega^+(R^+)$ – the gains from trade $i_\ell - i_d$ extend up to the entire reserves holdings R^+ of the lender, after which both the borrower and the lender value reserves at i_ℓ. The expected gains of being a lender in the core market is

$$\Pi^+(R) = \beta\mu^+\Theta(i_\ell - i_d)\left\{\int_{-R}^{0} -R^- d\Omega^-(R^-) + R\Omega^-(-R)\right\}.$$

A lender meets a borrower with probability μ^+ and he gets a share Θ of the surplus from trade. Using the same reasoning as above, the gain from trade is $i_\ell - i_d$, either on the first R units of reserves whenever the lender cannot cover the reserve deficit of the borrower – an event which occurs with probability $\Omega^-(-R)$ – or on the first $-R^-$ units of reserves whenever the lender has enough reserves to cover the borrower's reserve deficit.

We can now define the expected gain from accessing the core market with R units of reserves as

$$\Pi(R) \quad = \quad \begin{cases} \Pi^-(R), & \text{if } R \leq 0 \\ \Pi^+(R), & \text{if } R > 0 \end{cases}.$$

The characteristics of the expected gains from trade are intuitive. As a lender, holding more reserves increases the gains from trade, but at a diminishing rate, i.e. $\Pi'(R) > 0, \Pi''(R) < 0$ for $R > 0$. For a borrower, holding additional reserves (i.e. having less negative reserves) decreases the gains from trade, but at a decreasing rate, i.e. $\Pi'(R) < 0, \Pi''(R) < 0$ for $R < 0$.[15] Then the value of an L bank at the start of the core market, when it has R units of reserves, aggregate dues $\hat{D}$ from past trades with its business partner, and a number m of business partner, is simply its expected gains from trade in addition to the payoff this bank would obtain without trading in the core market:

$$V_3(R, \hat{D}, m) = \Pi(R) + \beta \left\{ \hat{D} + R(1 + i(R)) \right\} + \beta \bar{V}_1(m). \tag{5}$$

From there, we obtain the marginal values of reserves at the beginning of the core market.

Lemma 2. *$V_3(R, \hat{D}, m)$ is strictly increasing and concave in R. The marginal value of reserves at the start of the core market is*

$$\frac{\partial V_3(R, \hat{D}, m)}{\partial R} = \begin{cases} \beta(1+i_\ell) - \beta\mu^-(1-\Theta)(i_\ell - i_d)[1 - \Omega^+(-R)] > 0 & , \textit{if } R \leq 0 \\ \beta(1+i_d) + \beta\mu^+\Theta(i_\ell - i_d)\Omega^-(-R) > 0 & , \textit{if } R > 0 \end{cases}.$$

Proof. The proof follows from taking the derivative of V_4 with respect to R. However, since the value function has a kink at zero, to ensure that it is strictly increasing and concave with respect to R, we have to make sure that the marginal value remains diminishing when $R = 0$. That is, we need to make sure that

$$\begin{aligned} \beta(1+i_\ell) - \beta\mu^-(1-\Theta)(i_\ell - i_d)[1 - \Omega^+(0)] &> \beta(1+i_d) + \beta\mu^+\Theta(i_\ell - i_d)\Omega^-(0) \\ 1 &> \mu^+\Theta + \mu^-(1-\Theta), \end{aligned}$$

which is satisfied. □

We now turn to the value of S banks at the start of the core market. As it is costly for S banks to participate in the core market, some S banks may prefer to skip the core market altogether. We refer to these banks as being inactive. The value function of inactive S banks is simply

$$v_3^0(R, \hat{D}, m) = \beta \left\{ \hat{D} + R[1 + i(R)] \right\} + \beta \bar{v}_1(m).$$

which is increasing in R, with derivative equal to $\beta(1+i_\ell)$ when $R < 0$ and $\beta(1+i_d)$ when $R > 0$. The value function of active S banks is

$$v_3^1(R, \hat{D}, m) = \Pi(R) + \beta \left\{ \hat{D} + R(1 + i(R)) \right\} + \beta \bar{v}_1(m).$$

[15] This requires $\Omega^{+\prime}(R) > 0, \Omega^{-\prime}(R) > 0$ which we verify later.

Therefore, v_3^1 is also strictly increasing and concave in R, with the same first and second derivatives with respect to R as those of V_3. In particular, notice that trade surpluses of S and L banks in the core market do not depend on the bank's type but only on their reserve holdings.

Naturally an S bank enters the core market if its expected gain from trading there $\Pi(R)$ is bigger than the cost γ of entering the core market. Therefore, the value function of an S bank at the time of choosing whether to participate or not given its reserves R is

$$v_3(R, \hat{D}, m) \quad = \quad \max\{\Pi(R) - \gamma, 0\} + \beta \left\{\hat{D} + R(1 + i(R))\right\} + \beta \bar{v}_1(m). \tag{6}$$

Intuitively, an S bank enters the OTC market only when it is worthwhile, which is the case iff the bank's reserve balance differs significantly from the required level 0. Otherwise, an S bank does not enter, as the expected gain is too small to compensate for γ. So we obtain

Lemma 3. *A bank S enters the core market to borrow iff $R \leq \hat{R}^-$ and enters to lend iff $R \geq \hat{R}^+$, with $\Pi^-(\hat{R}^-) = \Pi^+(\hat{R}^+) = \gamma$.*

Notice that the entry decision does not depend on the S bank's number of business partners, but only on its reserve holdings.

This concludes the analysis of the core market. To summarize: All L banks enter the core market, but S banks only enter the core market whenever their reserve holdings differ sufficiently from zero. A matching function pairs borrowers and lenders who bargain over the terms of trade. Surplus and terms of trade do not depend on their types. And, importantly, all core rates lie within the corridor defined by the central bank rates. We now move on to the periphery market where partner banks trade.

3.3 Sub-period 2. Peripheral (relationship) trades

In sub-period 2, banks with a business partner trade their reserves, taking into account that the core market will open next. Other banks do not do anything and their payoff is given by (5) and (6) with $m = 0$. Now consider two partner banks where bank S holds R_S while bank L holds R_L. We assume that both banks agree on a loan size z from bank S to banks L and a corresponding loan rate $\hat{\imath}$. In addition we assume proportional bargaining with weight θ assigned to bank S. We need to take into account that either of the two banks could have been contacted by another bank before negotiating the loan. We use $c_L = 1$ to denote that bank L has been contacted by another bank, and $c_L = 0$ otherwise, and similarly for bank S. Therefore, we summarize the contact status of the two partners by a vector $\mathbf{c} = (c_L, c_S) \in \{0,1\}^2$. Notice that a bank can credibly threaten to end a relationship only if it has been contacted by another bank. We leave the details for the Appendix, but we still want to state that from (5) and (6) we can find the trading surplus for an S and an L bank as a function of the terms of trade, respectively $TS_S(z, \hat{i}; \mathbf{c})$ and $TS_L(z, \hat{i}; \mathbf{c})$. Then the bargaining problem is

$$\max_{z,\hat{i}} TS_S(z, \hat{i}; \mathbf{c})$$

subject to

$$TS_L(z, \hat{i}; \mathbf{c}) = (1-\theta)(TS_S(z, \hat{i}; \mathbf{c}) + TS_L(z, \hat{i}; \mathbf{c})).$$

Solving the bargaining problem, we obtain the following result.

Proposition 4. (Tiering structure) *There are reserves thresholds $\bar{R}^- < 0$ and $\bar{R}^+ > 0$ such that if $R_S + R_L \in [\bar{R}^-, \bar{R}^+]$ then bank S does not enter the core market. Otherwise bank S enters the core market. The size of the loan between two banks in the periphery market is*

$$z(R_S, R_L) = \begin{cases} R_S & \text{if } R_S + R_L \in [\bar{R}^-, \bar{R}^+] \\ (R_S - R_L)/2 & \text{otherwise} \end{cases}. \tag{7}$$

This result is intuitive. First, the loan size z is chosen to maximize the total surplus given the banks' reserves, while the rate $\hat{i}$ is chosen to split the surplus according to the surplus sharing protocol. Suppose the aggregate reserve balance of the two partners $R_S + R_L$, is large in absolute terms. If only bank L accesses the core market, it runs the risk of not meeting a counterparty, if at all, able to deal with such a large balance. In this case, the penalty is (relatively) large. The S and L banks can share the risk by splitting $R_S + R_L$ even if this means bank S has to pay γ to access the core market. Therefore splitting balances is an insurance against the frictions in the core market. Similarly, suppose the reserve balance of the pair is close to zero, so that the two partners together do not have much of a reserve surplus or deficit. Then splitting these balances still would not justify that bank S pays the entry cost in the core market. In this case, bank L trades $-R_S$ from bank S in the periphery market, with the result that bank S satisfies its reserve requirement exactly, and only bank L enters the core market. This is the "tiering" outcome for which it is crucial that bank S has to pay a cost of accessing the core market. The thresholds aggregate reserve holdings $\bar{R}^-$ and $\bar{R}^+$ are defined such both banks are indifferent between splitting reserves and both entering or only the L bank entering with the entire aggregate reserve holdings of the two partners.[16]

Finally, we obtain the following expression for the pairwise periphery interest rates,

Lemma 5. *The pairwise periphery market rate for a loan between two partner banks is*

$$1+\hat{i}(R_S,R_L,\mathbf{c}) = \frac{\Phi(R_S,R_L)}{z(R_S,R_L)} + \frac{\mathcal{V}(\mathbf{c})}{z(R_S,R_L)}, \tag{8}$$

where

$$\begin{aligned}\beta\Phi(R_S,R_L) \quad &\equiv \quad (1-\theta)\left[v_3(R_S,0,0) - v_3(R_S - z(R_S,R_L),0,0)\right] \\ &\qquad +\theta\left[V_3(R_L + z(R_S,R_L),0,0) - V_3(R_L,0,0)\right]\end{aligned}$$

and

$$\mathcal{V}(\mathbf{c}) \quad \equiv \quad (1-\sigma)\{\theta(1-c_L)c_S\left[V_1(1)-V_1(0)\right] - (1-\theta)(1-c_S)c_L[v_1(1)-v_1(0)]\}.$$

The pairwise periphery market rate is the sum of two terms. The first term on the right-hand side of (8) Φ/z, is a weighted sum of the bank S' opportunity cost of giving up z and bank L's benefit of obtaining z in the current period. This first component is always between the two policy rates i_d and i_ℓ. The second term $\mathcal{V}/z$ is the weighted sum of the S bank's cost of giving up the relationship and the L bank's benefit of staying in the relationship. This term depends on $\mathbf{c} = (c_L, c_S)$. When the L bank has been contacted by another S bank but the S bank has not ($c_L = 1, c_S = 0$), then the S bank values the partnership more than the L bank does ($\mathcal{V} < 0$). In this case, the L bank has a backup partner and thus can threaten credibly to end the current relationship if the deal breaks down. On the contrary, if $c_L = 0, c_S = 1$, then $\mathcal{V} > 0$ and the S bank can threaten credibly to end the current relationship.

Notice that whenever bank S is a lender ($z > 0$), $\mathcal{V}/z$ is negative if bank S values the partnership with bank L more than bank L does. In this case, the agreed rate is driven down, i.e. bank L is able to extract more rents from bank S when the latter benefits from the relationship or when the weight θ assigned to bank S, is small. Similarly, whenever

[16] This model in a sense endogenizes bank sizes: even when banks face the same shock distribution, $G = F$, the L banks will appear more active in the interbank market, hence justifying the labels "L" and "S." To see that, compare the average trading activities of an S bank and an L bank in a relationship: (i) in the periphery, expected trade size is the same for S and L banks in absolute terms; (ii) when both banks enter the core market, the expected trade size is the same for S and L banks because they split their total balance and bring the same amount to the OTC; (iii) when only the L bank enters the OTC market, the L bank has a higher expected trade size.

bank S is a borrower ($z < 0$) the agreed rate increases as bank S values the partnership more.[17] Naturally, and although this is not directly apparent in (8), the partnership is more valuable whenever the cost of accessing the core market is large and creating partnership is rather difficult.[18] Clearly the magnitude of $\mathcal{V}$ is the reason why the relationship rate can fall below i_d or go above i_ℓ. We formalize this result in the following corollary,

Corollary 6. *Pairwise periphery market loans rate may fall outside of the corridor,*

$$\text{if } i_\ell - i_d < -\frac{\mathcal{V}(\mathbf{c})}{z(R_S, R_L)} \text{ then } \hat{i} < i_d,$$

$$\text{if } i_\ell - i_d < \frac{\mathcal{V}(\mathbf{c})}{z(R_S, R_L)} \text{ then } \hat{i} > i_\ell.$$

This concludes the analysis of trades between two partner banks. To summarize, bank S may end up using bank L as an intermediary to access the core market. In addition, bank S may even agree to pay a premium to continue partnering with bank L when $c_S = 0$ and $c_L = 1$. In this case the pairwise periphery rate could well hover above i_ℓ whenever the S bank is borrowing from the L bank, or below i_d whenever bank S is a lender. We have the opposite outcome when $c_S = 1$ and $c_L = 0$.

3.4 Sub-period 2. Calls for New Partners

We assume that each bank can only maintain at most one relationship across periods. Hence, at the beginning of each period, a bank has either 0 or 1 relationships. In sub-period 2, before the periphery market opens, single banks can search for a new partner to build a relationship starting from the next period. After this process, a bank has either 0, 1 or 2 relationships for the next period. Those banks with two relationships can have a backup partner for next period which may improve the bank's position in the bargaining with the existing partner in the current periphery market. At the end of each period, a bank with two partners has to choose to continue with only one of the partners. The other partnership will be terminated.

[17] Interestingly, this is consistent with the finding by Ashcraft and Duffie (2007) that "[t]he rate negotiated is higher for lenders who are more active in the federal funds market relative to the borrower. Likewise, if the borrower is more active in the market than the lender, the rate negotiated is lower, other things equal."

[18] This is not apparent in the expression for $\mathcal{V}$ but γ and κ as well as the matching function in the OTC and relationship market affect $v_1(1)$ and $v_1(0)$ indirectly, so that $v_1(1) - v_1(0)$ is increasing in γ or κ or the market tightness in the different markets.

S banks can pay a cost κ_S to contact a bank L. At this stage, all single banks S are the same, so either they all pay the cost, or none of them do, or they use a mixed strategy if they are indifferent. Similarly, single banks L can pay a cost κ_L to contact a bank S.

To be precise, at the start of sub-period 1, a measure $n(1-N_L)$ of L banks have a business partner and a measure nN_L of banks L is single, a measure $(1-N_S)$ of S banks have a business partner and a measure N_S of banks S is single. Suppose a measure α_S of single banks S and α_L of single banks L search for a business partner. The following table shows the probabilities that a non-searching bank will be contacted by a single bank to build a new partnership:

Banks not searching	Prob. of contact
L	$\rho_{L0} = \min\{1, \frac{\alpha_S N_S}{n(1-\alpha_L N_L)}\}$
S	$\rho_{S0} = \min\{1, \frac{n\alpha_L N_L}{1-\alpha_S N_S}\}$

For example, there are $\alpha_S N_S$ S banks searching and $n(1-\alpha_L N_L)$ banks L not searching. Hence the probability that an L bank will be contacted by a single bank is given by ρ_{L0}. The following table considers the case for searching banks:

Searching banks	Prob. of contact	Fraction of singles	Prob. of finding a new partner
L	$\min\{1, \frac{1-\alpha_S N_S}{n\alpha_L N_L}\}$	$\frac{(1-\alpha_S)N_S}{1-\alpha_S N_S}$	$\rho_{L1} = \frac{(1-\alpha_S)N_S}{1-\alpha_S N_S}\min\{1, \frac{1-\alpha_S N_S}{n\alpha_L N_L}\}$
S	$\min\{1, \frac{n(1-\alpha_L N_L)}{\alpha_S N_S}\}$	$\frac{nN_L(1-\alpha_L)}{n(1-\alpha_L N_L)}$	$\rho_{S1} = \frac{nN_L(1-\alpha_L)}{n(1-\alpha_L N_L)}\min\{1, \frac{n(1-\alpha_L N_L)}{\alpha_S N_S}\}$

For example, there are $n\alpha_L N_L$ banks L who can potentially match with $1-\alpha_S N_S$ non-searching banks S. Thus a searching bank L can find an bank S with probability $\min\{1,(1-\alpha_S N_S)/(n\alpha_L N_L)\}$. Among these non-searching banks S, a fraction $((1-\alpha_S)N_S)/(1-\alpha_S N_S)$ of them are single. Hence, the probability that a searching bank L can find a new partner to build a relationship is given by ρ_{L1}.[19]

So the measure of L banks with a business partner are all L banks (a) who had a partner last period and did not lose it, (b) who did not have a partner last period, searched and found a new partner that they did not lose, and (c) who did not have a partner last period, got contacted and did not lose the new partner:

$$n\,(1-N_L) = \underbrace{n(1-N_L)(1-\sigma)}_{(a)} + \underbrace{\alpha_L n N_L \rho_{L1}(1-\sigma)}_{(b)} + \underbrace{nN_L(1-\alpha_L)\rho_{L0}(1-\sigma)}_{(c)},$$

[19]Since there is no voluntary breakups in equilibrium, a single bank L cannot build a new relationship for the next period with a non-single bank S .

or

$$N_L = \frac{\sigma}{[\sigma + \rho_{L0}(1-\sigma) + \alpha_L(\rho_{L1} - \rho_{L0})(1-\sigma)]} \tag{9}$$

Similarly, the measure of banks S with a partner is

$$1 - N_S = \underbrace{(1-N_S)(1-\sigma)}_{(a)} + \underbrace{\alpha_S N_S \rho_{S1}(1-\sigma)}_{(b)} + \underbrace{N_S(1-\alpha_S)\rho_{S0}(1-\sigma)}_{(c)},$$

or

$$N_S = \frac{\sigma}{\sigma + \rho_{S0}(1-\sigma) + \alpha_S(\rho_{S1} - \rho_{S0})(1-\sigma)} \tag{10}$$

S banks search whenever the expected benefits is higher than the search cost κ_S. Hence, as the relationship starts the next day, the fraction of single banks S searching is

$$\alpha_S = \begin{cases} 1 & \text{if } [\rho_{S1} - \rho_{S0}](1-\sigma)\beta[v_1(1) - v_1(0)] > \kappa_S \\ 0 & \text{if } [\rho_{S1} - \rho_{S0}](1-\sigma)\beta[v_1(1) - v_1(0)] < \kappa_S \\ [0,1] & \text{otherwise} \end{cases}$$

Similarly, the fraction of single L banks searching is

$$\alpha_L = \begin{cases} 1 & \text{if } [\rho_{L1} - \rho_{L0}](1-\sigma)\beta[V_1(1) - V_1(0)] > \kappa_L \\ 0 & \text{if } [\rho_{L1} - \rho_{L0}](1-\sigma)\beta[V_1(1) - V_1(0)] < \kappa_L \\ [0,1] & \text{otherwise} \end{cases}$$

Notice that there cannot be a steady state equilibrium with relationships where all single banks are searching, $\alpha_S = \alpha_L = 1$. In this case a single bank searching would only contact banks who already have a partner, and she would be better off not searching trying its luck at being contacted by one of the searching single banks. Similarly, since banks lose their partners with probability σ, a steady state equilibrium with relationship exists only if single banks search, so we cannot have $\alpha_S = \alpha_L = 0$.

3.5 Sub-period 1. Liquidity Shocks

Recall that in subperiod 1, S banks get a liquidity shock $\xi \sim F(\xi)$, while L banks get a liquidity shock $\varepsilon \sim G(\varepsilon)$. We now define the expected payoffs of different types of banks at the beginning of subperiod 1 *before* they know their liquidity shocks. Let the dues from the periphery market loan be $\hat{D}(R_S, R_L; \mathbf{c})$, with

$$\hat{D}(R_S, R_L; \mathbf{c}) \quad = \quad [1 + \hat{\imath}(R_S, R_L; \mathbf{c})]z(R_S, R_L)$$

given by (8). Then the expected value of a bank L with a partner before the liquidity shock is

$$V_1(1) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_{\mathbf{c}} V_3(\varepsilon + z(\xi,\varepsilon), -\hat{D}(\xi,\varepsilon;\mathbf{c}), 1) dG(\varepsilon) dF(\xi).$$

where $z(\xi, \varepsilon)$ is given by (7). Similarly, the expected value of a bank S with a partner before the liquidity shock is

$$v_1(1) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E_{\mathbf{c}} v_3(\xi - z(\xi, \varepsilon), \hat{D}(\xi, \varepsilon; \mathbf{c}), 1) dG(\varepsilon) dF(\xi).$$

Using (5) the expected value of a single bank L before the liquidity shock is

$$V_1(0) \quad = \quad \int_{-\infty}^{\infty} \Pi(\varepsilon) + \beta[(\varepsilon)(1 + i(\varepsilon))] dG(\varepsilon) + V(0),$$

where $V(0) = \max\{\rho_{L1}\beta\bar{V}_1(1) + \beta(1 - \rho_{L1})V_1(0) - \kappa_L, \rho_{L0}\beta\bar{V}_1(1) + \beta(1 - \rho_{L0})V_1(0)\}$ determines the value of searching when single. Similarly, using (6) the expected value of a single S bank before the shock is simply

$$v_1(0) \quad = \quad \int_{-\infty}^{\infty} \max\{\Pi(\xi) - \gamma, 0\} + \beta\left\{(\xi)(1 + i(\xi))\right\} dF(\xi) + v(0),$$

where $v(0) = \max\{\rho_{S1}\beta\bar{v}_1(1) + \beta(1-\rho_{S1})v_1(0) - \kappa_S, \rho_{S0}\beta\bar{v}_1(1) + \beta(1-\rho_{S0})v_1(0)\}$ includes the option to search for a business partner for tomorrow. Notice that in the equation defining $v_1(0)$ there is no β in front of $v(0)$ because the decision to search is done today.

This completes our description of the four sub-periods.

4 Characterization of the equilibrium

We can now define an equilibrium. For any variables Z, it is convenient to define the vector $\mathbf{Z} = \{Z^-, Z^+\}$. Then,

Definition 1. *A steady-state equilibrium is a list* $\{\alpha_S, \alpha_L, N_L, N_S, \mathcal{V}, \bar{\mathbf{R}}, \hat{\mathbf{R}}, \mathbf{N}, \boldsymbol{\Omega}\}$ *consisting of the fractions* α_S, α_L *of single banks* S *and* L *searching, the measures* nN_L *and* N_S *of single banks, the value of having a partner* $\mathcal{V}$*, the reserves thresholds* $\bar{\mathbf{R}}$ *defining intermediation and core market access for* S *banks with a partner, the reserves thresholds* $\hat{\mathbf{R}}$ *defining core market access for single* S *banks, the measure of banks with positive and negative reserves in the core market* $\mathbf{N}$ *and the distribution of reserves in the core market* $\boldsymbol{\Omega}$*, such that, given the policy rates* i_d *and* i_ℓ*, the choice searching for a partner, reserves holdings, and entry by* S *banks in the core market are optimal and consistent with this list and the bargaining solutions.*

It is easy to see the possibility of multiple equilibria. For instance, suppose $\kappa_S = \kappa_L = 0$. Then either all single banks S searching for a partner, or all single banks L searching for a partner is an equilibrium. There is no point for a bank L to search if all (single) banks

S are already searching (and inversely). Since it is the empirically relevant case, in the sequel, we characterize the equilibrium for the case where banks L do not search, that is we set $\alpha_L = 0$.

To characterize the equilibrium, we use Γ_L to denote the the expected benefit for a bank L of having a business partner for one period only, and similarly for Γ_S. These values only depend on $\{\bar{\mathbf{R}}, \hat{\mathbf{R}}, \mathbf{N}, \mathbf{\Omega}\}$. Therefore, guessing $\{\bar{\mathbf{R}}, \hat{\mathbf{R}}, \mathbf{N}, \mathbf{\Omega}\}$, we obtain Γ_L and Γ_S. Then the following proposition gives us $\{\alpha_S, \alpha_L, N_S, N_L, \mathcal{V}\}$, from which we can check if the initial guess is satisfied. Appendix **??** describes the numerical algorithm for finding the equilibrium of the model. In the proof we show that $\alpha_L = 0$ if

$$\kappa_L > (1-\sigma)\beta\frac{\Gamma_L}{1-\beta(1-\sigma)},$$

which we now assume holds. Also, define $N_L^*(n, \alpha_S)$ – the fraction of banks L that are single when there is a total measure n of banks L and a fraction α_S of single banks S search for a new partner – as solving the equilibrium identity that the number of L banks with a partner must equal the number of S banks with a partner,

$$\sigma(1-N_L) = (1-\sigma)\alpha_S\frac{1-n(1-N_L)}{n}N_L$$

and the fraction of single bank S then is

$$N_S^*(n, \alpha_S) = 1 - n(1 - N_L^*(n, \alpha_S)).$$

We first characterize the equilibrium when there is a large number of L banks.

Proposition 7. *Suppose $n > 1/(2-\sigma)$. Then there are three possible equilibrium outcomes with $\alpha_L = 0$:*

1. $\alpha_S = 0$ *(S banks do not search) whenever*

$$\kappa_S \quad \geq \quad (1-\sigma)\beta\frac{\Gamma_S}{1-\beta(1-\sigma)} \equiv \bar{\kappa}_S$$

in which case

$$E\left[\mathcal{V}(\mathbf{c})\right] = 0$$

2. $\alpha_S = 1$ *(all single S banks search) whenever*

$$\underline{\kappa}_S \equiv \frac{N_L^*(n,1)(1-\sigma)\beta}{1-\beta(1-\sigma)\left[1-\frac{N_S^*(n,1)}{n}(1-\theta)\right]}\Gamma_S \geq \kappa_S$$

in which case

$$E\left[\mathcal{V}(\mathbf{c})\right] = \frac{-(1-\sigma)(1-\theta)(\Gamma_S+\kappa_S)}{1-\beta(1-\sigma)(1-N_L^*(n,1)-(1-\theta)\frac{N_S^*(n,1)}{n})}$$

3. $\alpha_S \in (0,1)$ *(some single S banks search) whenever*

$$\bar{\kappa}_S \geq \kappa_S \geq \underline{\kappa}_S$$

in which case α_S *is implicitly defined by*

$$\frac{N_L^*(n,\alpha_S)(1-\sigma)\beta\Gamma_S}{1-\beta(1-\sigma)\left[1-\alpha_S\frac{N_S^*(n,\alpha_S)}{n}(1-\theta)\right]} = \kappa_S$$

and

$$E\left[\mathcal{V}(\mathbf{c})\right] = \frac{-(1-\sigma)(1-\theta)(\Gamma_S+\kappa_S)}{1-\beta(1-\sigma)(1-N_L^*(n,\alpha_S)-(1-\theta)\alpha_S\frac{N_S^*(n,\alpha_S)}{n})}$$

Intuitively, when κ_S is high, banks have no business partner because it is too costly for S banks to find one. When κ_S is low, all single banks S will try to find a business partner. For intermediate level of κ_S, single banks S are indifferent between searching for a business partner and staying single. Finally, the number of S banks actively searching for a business partner has to be consistent with the steady state number of partnerships.

We now characterize the equilibrium when there are relatively few L banks overall. The main difference with Proposition 7 is that all banks L may always be contacted by a single bank S, even though not all single banks S are searching. Although the intuition is similar to the one above, we state Proposition 8 for completeness and because it may be the relevant case when we analyze the data.

Proposition 8. *Suppose* $n \leq 1/(2-\sigma)$. *Then there are three possible equilibrium outcomes with* $\alpha_L = 0$*:*

1. $\alpha_S = 0$ *(S banks do not search) whenever*

$$\kappa_S \quad \geq \quad \bar{\kappa}_S$$

in which case

$$E\left[\mathcal{V}(\mathbf{c})\right] = 0$$

2. $\alpha_S = 1$ *(all single S banks search) whenever*

$$\underline{K}_S \equiv \frac{\sigma n(1-\sigma)\beta}{\left[1-n(1-\sigma)\right]\left[1-\beta\theta(1-\sigma)\right]}\Gamma_S \geq \kappa_S$$

in which case

$$E\left[\mathcal{V}(\mathbf{c})\right]=\frac{-(1-\sigma)(1-\theta)\left(\Gamma_S+\kappa_S\right)}{1-\beta(1-\sigma)\left[\theta-\sigma\frac{n}{(1-n+n\sigma)}\right]}$$

3. $\alpha_S\in(\frac{n}{1-n+\sigma n},1)$ *(some single S banks search and all banks L have a trading partner before the separation shock) whenever*

$$\frac{\sigma(1-\sigma)\beta}{\left[1-\beta\theta(1-\sigma)\right]}\Gamma_S\geq\kappa_S\geq\underline{K}_S$$

in which case

$$\alpha_S=\frac{\sigma n(1-\sigma)\beta}{\left[1-n(1-\sigma)\right]\left[1-\beta\theta(1-\sigma)\right]}\frac{\Gamma_S}{\kappa_S}$$

and

$$E\left[\mathcal{V}(\mathbf{c})\right]=\frac{-(1-\sigma)(1-\theta)\left(\Gamma_S+\kappa_S\right)}{1-\beta(1-\sigma)\left[\theta-\sigma\frac{n}{\alpha_S(1-n+n\sigma)}\right]}$$

4. $\alpha_S\in(0,\frac{n}{1-n+\sigma n})$ *(some single S banks search and some banks L have a trading partner before the separation shock) whenever*

$$\bar{\kappa}_S\geq\kappa_S\geq\frac{\sigma(1-\sigma)\beta}{\left[1-\beta\theta(1-\sigma)\right]}\Gamma_S$$

in which case α_S is implicitly defined by

$$\frac{N_L^*(n,\alpha_S)(1-\sigma)\beta\Gamma_S}{1+\beta(1-\theta)(1-\sigma)\alpha_S\frac{N_S^*(n,\alpha_S)}{n}-\beta(1-\sigma)}=\kappa_S$$

and

$$E\left[\mathcal{V}(\mathbf{c})\right]=\frac{-(1-\sigma)(1-\theta)\alpha_S\frac{N_S^*(n,\alpha_S)}{n}\left(\Gamma_S+\kappa_S\right)}{1-\beta(1-\sigma)\left[1-N_L^*(n,\alpha_S)-(1-\theta)\alpha_S\frac{N_S^*(n,\alpha_S)}{n}\right]}$$

5 Quantitative evaluation

In this section, we first report some stylized facts relevant to our model using data from the ECB. We then parameterize the model and use it to perform some quantitative exercises studying the effects of monetary policy on the functioning of the interbank market.

5.1 Data

Most interbank overnight loans are conducted over the counter, so it is usually very difficult or even impossible to obtain reliable transaction-level data covering most of interbank market trades. Previous empirical studies lacked direct measures of interbank trades and relied on indirect inference using an algorithm based on the work of Furfine (1999). However serious concerns were raised about the appropriateness of this approach for some jurisdictions because its application can involve significant errors (Armantier and Copeland, 2012). To avoid this problem, we obtained access to transaction level data about interbank loans in the Eurosystem from the money market statistical reporting (MMSR) data set.

The MMSR dataset shows transaction-level data of the 52 largest Euro Area banks by balance sheet size. Since 2016, these banks (Reporting Agents, or RA) have to report all of their trading activities on the money market, including the trades they conducted with banks that do not have to report their money market activities (non-RA). RA cover about 80 percent of Euro Area money market activities and thus are large or very large banks. We look specifically at overnight transactions taking place with banks located in the Euro Area with a minimum trade volume of 1 million euros. Importantly, because all banks are located in the Euro Area, all RA and non-RA banks in our subsample have access to the ECB deposit and lending facilities. The sample covers the period 1 July 2016 until 1 July 2018. During this period, the deposit facility rate (DFR) was -0.4 % and the marginal lending facility rate was 0.25%.

The main advantage of using this data set is that it contains confirmed transaction data and therefore it is not subject to the type I and type II errors of data constructed using the Furfine algorithm. There are some limitations however. First, the dataset shows transactions from the perspective of RA, transacting either with other RA or with relatively smaller counterparties (non-RA). Therefore while we can have a sense for non-RAs activities, we do not observe transactions conducted among non-RA. Another shortcoming is attributed to the sample period covered by MMSR. The data only starts in July 2016, in an environment characterized by abundant liquidity in the money markets and therefore one with relatively few incentives to trade. In addition, there were no changes in policy rates during this period.

There are altogether 39 RA and 1,115 unique non-RA who trade with at least one RA in the sample. Figure 2 reports the number of trading partners of non-RA and RA. Panel (a) considers non-RA and shows the share of volume by number of trading partners of non-RA. It shows that non-RA have a median number of 2 partners that are RA and more than 80% of non-RA have 3 partners or less. Non-RA with only one RA partner account for about 30% of the total trading volume involving non-RA. Panel (b) focuses on RA and reports the share of volume by number of trading partners. The median number of trading partners for RA is 182. The most active six banks have at least 88 partners and they account for 89.51 % of the reported trading volume. These statistics are consistent with a core-periphery structure where banks in the core (RA banks) trade with many counterparties while banks in the periphery (non-RA banks) have a small number of business partners.

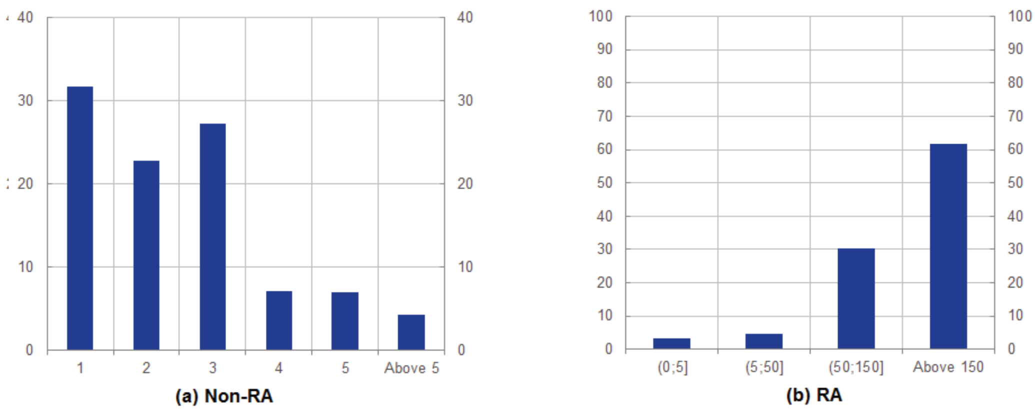

Figure 2: (a) Share of volume of non-RA by number of RA counterparties, (b) Share of volume of RA by number of counterparties

Over the sample period, RA reported altogether 157,098 trades conducted with non-RA. Table 1 below reports some statistics concerning these trades. Most of these trades (94%) are loans from RA to non-RA. Surprisingly, while the average interest rates are above the DFR, a significant (statistically different from zero) fraction of trades are conducted below the DFR. Almost all these trades are loans from non-RA to RA; On the other hand, RA rarely lend to non-RA below the DFR. Among the loans from non-RA to RA, roughly 39% are conducted below the DFR.

Table 1: Summary Statistics

	Non-RA to RA	RA to non-RA
No. of transactions	10099	146999
Percentage of total	6.43%	93.57%
Average rates	-0.38%	-0.34%
Average size (millions)	53	28
Fraction of trades below DFR	38.83%	0.06%

Parameterization and Equilibrium Statistics

In this section, we parameterize the model to match certain targets observed from the

MMSR data set. Given the difference in their trading activities, we interpret RA as banks L in the model and non-RA as banks S. We interpret transactions between one RA and a non-RA as trades in the periphery market in the model, while trades between two RA take place in the core market in the model.

The period length is one day. The annualized discount factor is set to 0.97. The lending and deposit rates are chosen to match the annualized policy rates $i_d = -0.4\%$ and $i_\ell = 0.25\%$ during the sample period. The rest of the parameters are chosen to match moments from the MMSR data. To match the observation that the grand majority (about 94%) of trades between a RA and a non-RA are loans from RA to non-RA, we find that liquidity shocks ε for banks L are drawn from a normal distribution $\mathcal{N}(10, 2.5)$, while liquidity shocks ξ for banks S are drawn from a distribution $\mathcal{N}(-3.9, 2.58)$. Therefore, banks S tend to have liquidity outflows (i.e., potential liquidity demanders) and banks L tend to have liquidity inflows (i.e., potential liquidity providers).

Table 2: Parameter Values

Parameter	Definition	Value
β	discount factor	0.9999
i_ℓ	lending facility rate	0.0000068
μ_F	S bank's mean reserve balances	-3.90
η_F	variance of S bank's reserve balances	2.58
μ_G	L bank's mean reserve balances	10.00
η_G	variance of L bank's reserve balances	2.50
Θ	lender's bargaining power in core market	0.5
θ	S bank's bargaining power in periphery market	0.88
n	measure of L banks	0.1
σ	probability of relationship separation	0.003
γ	core market participation cost	0.0002
κ_S	S bank's costs for building a new relationship	0.00001
κ_L	L bank's costs for building a new relationship	0.00001

The cost for banks to search and build a new relationship is $\kappa_S = \kappa_L = 0.00001$, and the probability of separation is $\sigma = 0.003$. In the core market, the number of matches is given by $\min\{N^+, N^-\}$, where N^+ is the total measure of banks (S or L) with excess reserves, while N^- is the total measure of banks with a reserve deficit. The cost for banks S to search in this market is $\gamma = 0.0002$.[20] Borrowers and lenders have equal bargaining

[20] If we equate the average loan size (from RA to non-RA) in the model to that in the data we find that one unit of reserve in model is equivalent to €26 millions in the data. This implies that the participation cost of $\gamma = 0.0002$ in the model corresponds to approximately €2500 in the data. Similarly, the implied search costs κ_S, κ_L are equivalent to around €126 in the data.

powers $\Theta = 0.5$ in core trades and S banks have bargaining power $\theta = 0.88$ when trading in the periphery market.[21] Since $n < 1/2$, the equilibrium corresponding to these parameter values is the one we characterize in Proposition 8.

Table 3: Implications of Model

	Data	Model
Fraction of trades where banks L are borrowers	6.43%	6.51%
Median rate when banks L borrow	-0.39%	-0.40%
Median rate when banks L lend	-0.34%	-0.32%
Fraction of loans below i_d when banks L borrow	38.83%	35.00%
Fraction of loans below i_d when banks L lend	0.06%	0.00%
Median no. of relationships of banks S	2	2

Overall, the chosen parameter values allow us to match the set of moments of the loan rates distribution reported in Table 3. Importantly, the model is able to generate a significant fraction of loans trading below the floor as in the data.

Figure 3 plots the distributions of interest rates in the core and the peripheral markets. Note that all core trades are conducted within the corridor while some banks S lend to banks L at a rate that is below the floor represented by the DF rate (-0.4%). In equilibrium, the relationship premium is $|\mathcal{V}| = 8.2408 \times 10^{-6}$, which is 21.56% of the average interest payment $\hat{i}z$. The model implies that, over a two-year period, a bank S has a median number of business partners equal to 2, as in the data. Figure 4 shows the model distribution of the number of partners for S banks. We also simulate trading activities in a system with 550 banks (500 S and 50 L) for two years and plot the network graph in Figure 5. Nodes denote banks and edges denote trading links. The size of a node represents a bank's betweenness centrality.[22] As shown in the graph, the calibrated model exhibits a core-periphery structure, as observed in the data.

5.2 Comparative Statics

We now generate some comparative statics using our parameterization of the model.

5.2.1 Interest rate corridor

Effects of an increase in $i_\ell - i_d$ (mid-point unchanged)

[21] Setting a high θ helps generate loans trading below the floor by making a relationship more valuable to banks S.

[22] Betweenness centrality measures the extent to which a node lies on paths between other nodes. The measure is based on the number of shortest paths passing through a node. Intuitively, the removal of nodes with high betweenness from the network will have larger disruption to the flows between other nodes.

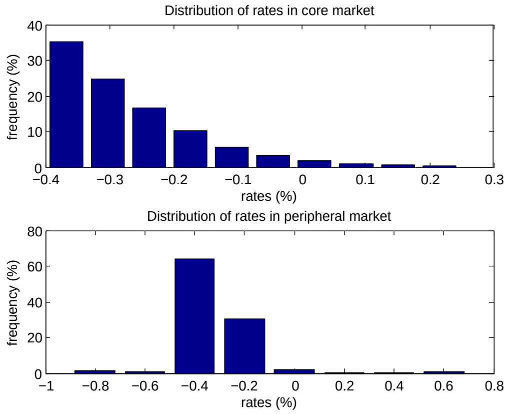

Figure 3: Interest Rate Distribution

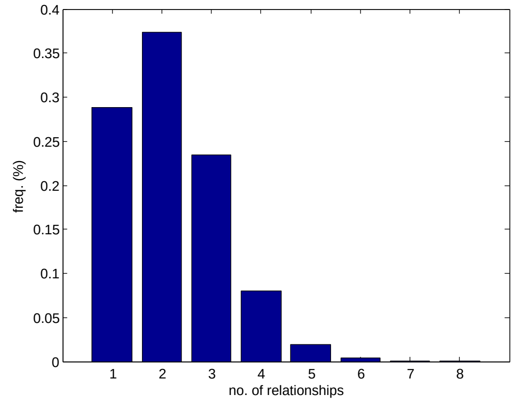

Figure 4: No. of Relationships of S banks (simulated over a two-year period)

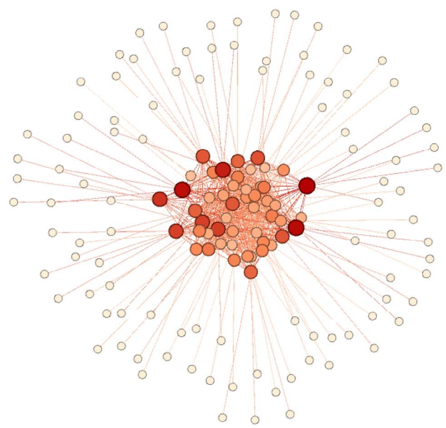

Figure 5: Simulated Network (550 banks for 500 periods)

Berentsen and Monnet (2008) show that widening the corridor or channel, defined by the difference in the policy rates $i_\ell - i_d$ is a contractionary monetary policy. Suppose the central bank keeps the mid-point of the corridor unchanged but widens the width of the corridor. What are the effects on the interbank market? Here when $i_\ell - i_d$ increases, the central bank lending and deposit facilities become less attractive relative to conducting an interbank transaction. As shown in Figure 6, the value of a relationship for bank S, $v_1 - v_0$, increases and banks S' incentives to find a business partner rises (higher α_S), implying a higher number of relationships. As a consequence, the number of single banks L, N_L, drops. Then both the fraction of loans trading below the floor and the relationship premium increase.

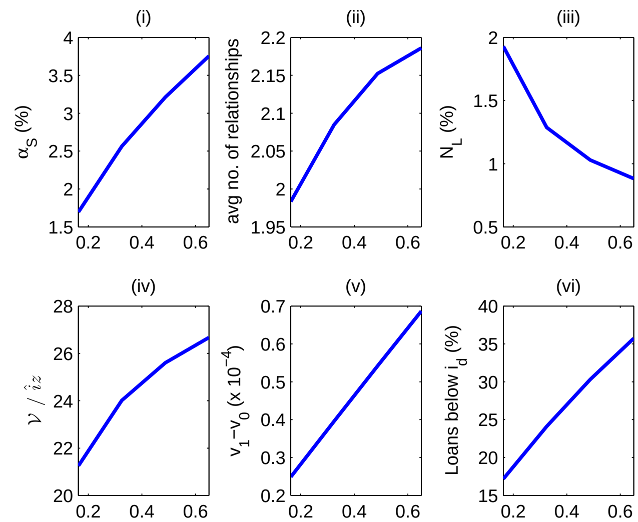

Figure 6: Effects of corridor width $i_\ell - i_d$ on (i) the fraction α_S of single banks S searching, (ii) the average number of relationships, (iii) the fraction N_L of single banks L, (iv) the relationship premium as a fraction of interest payment, $|\mathcal{V}|/\hat{i}z$, (v) the value of a relationship for bank S, (vi) the fraction of loans trading below the floor.

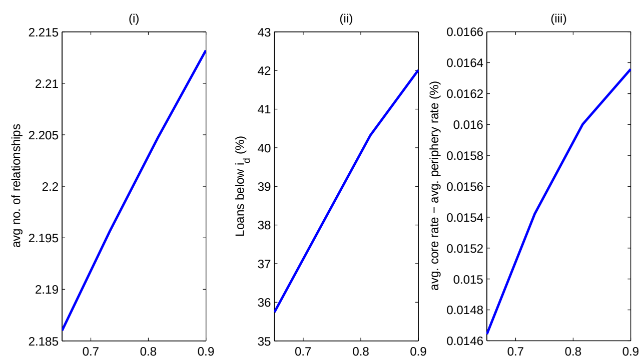

Figure 7: Effects of increasing width (i_d fixed)

Effects of an increase in $i_\ell - i_d$ (floor unchanged)

The above finding suggests that, when designing its "exit strategy", the central bank should consider also the width of the corridor. For example, Figure 7 examines the case where the central bank raises the interest rate target by increasing i_ℓ from 0.25% to 0.5% but leaving i_d unchanged at -0.4%, with the corridor widening from 0.65% to 0.9%. Naturally the average rates increase both in the core market and in the periphery market. But as the corridor widens and the number of relationships rises, more trades are conducted below the floor. As a result, the average rate in the core market rises faster than that in the periphery market. This divergence may be difficult for market participants to interpret and hence may cause confusion.

5.2.2 Reserve balances

Effects of an increase in μ_F, banks S need less reserves on average

Increasing μ_F, leaving everything else constant, is equivalent to shifting to the right the distribution of banks S initial reserve shock.[23] Since the average initial reserve positions of banks S is less negative, banks S need to trade less in the interbank market. As shown in Figure 8, the average rate in the core market drops as the net demand for reserve balances decline. S banks' incentive to find a partner also drops (lower α_S) and as a result the number of banks with a partner decreases (N_L rises). This implies that that the fraction of loans trading below the floor drops. The relationship premium $|\mathcal{V}|$ drops in general but the premium as a fraction of interest payment, $|\mathcal{V}|/\hat{i}z$, goes up when the average trade size z becomes smaller.

Effects of an increase in μ_G, banks L need less reserves on average

Again, when μ_G increases, a bank L's initial reserve balances become more positive, the average rate in the core market drops as the net demand for reserve balances declines. Since banks S tend to be short in reserves, their incentives to build relationships rise (higher α_S) as shown in Figure 9. The number of relationships goes up (N_L drops). This implies that both the fraction of loans trading below the floor and the relationship premium increase.

Effects of changing the supply and distribution of reserves

As shown in the two cases above, the effects of increasing initial reserve balances can be very different depending on the distribution of these balances. Figure 10 reports the effects of changing the aggregate reserve balances $\Delta\mu$ and how these effects depend on the allocation of these additional balances. We capture the liquidity allocation by a parameter λ such that $\Delta\mu_F = \lambda\Delta\mu_G$. In other words, for a given increase in aggregate balances $\Delta\mu$, a higher λ means that a larger fraction of new reserves are allocated directly to banks S. We plot the average interest rates weighted by the size of transactions in the core market, in the periphery market and in both markets. Naturally, a liquidity expansion drives the rates down while a contraction drives the rates up. Since the interbank market starts with abundant reserves, a liquidity contraction generates a bigger marginal effect than an expansion. Also, the marginal effect of $\Delta\mu$ is bigger when λ is small because funds can reach the interbank market more directly through banks L. This consideration is relevant for comparing different liquidity provision options in normal times as well as designing an

[23] Recall that one unit of reserve in model can be interpreted as €26 millions.

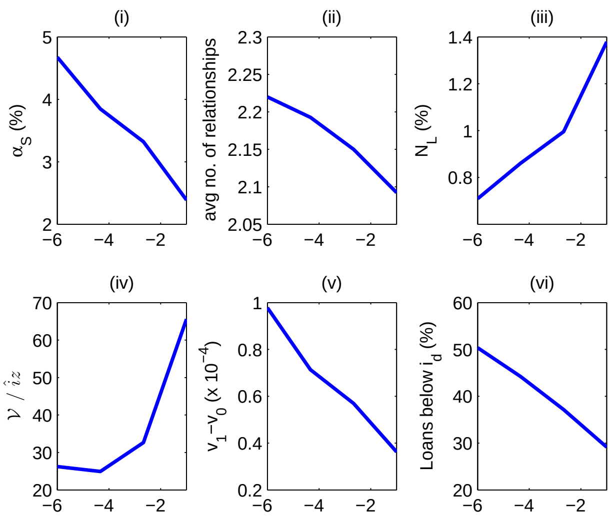

Figure 8: Effects of small banks' mean reserve balances μ_F on (i) the fraction α_S of single banks S searching, (ii) the average number of relationships, (iii) the fraction N_L of single banks L, (iv) the relationship premium as a fraction of interest payment, $|\mathcal{V}|/\hat{i}z$, (v) the value of a relationship for bank S, (vi) the fraction of loans trading below the floor.

exit strategy for absorbing extra liquidity from the system.

5.2.3 Effects of κ_S

Naturally, increasing the search cost for new partner, κ_S, reduces banks S' incentives to find a new partner (Figure 11). Hence there are more single banks in equilibrium (higher N_L and N_S) with two effects on the rates traded in the periphery market. First, as fewer banks S are looking for partners, fewer banks L can credibly threaten to drop their existing partnership. As a result, the fraction of loans trading below the floor drops. Second, as the costs of finding a new partner is higher, the value for banks S of an existing partnership increases. Hence banks L with an outside option can now ask for a higher relationship premium.

5.2.4 Effects of θ

As shown in Figure 12, a partnership becomes more valuable for banks S (higher $v_1 - v_0$) when their bargaining power increases. As a result they search more for partners and more relationships are built (higher α_S and lower N_L). Then a larger fraction of loans will be traded below the floor of the corridor since more banks L will be contacted and benefit from the threat of quitting an existing partner. There are two opposite effects on the relationship premium. First, the premium rises with $v_1 - v_0$. Second, the premium

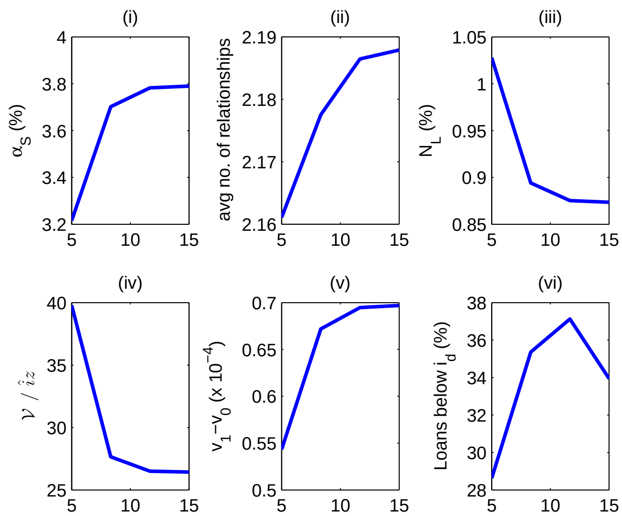

Figure 9: Effects of large banks' mean reserve balances μ_G on (i) the fraction α_S of single banks S searching, (ii) the average number of relationships, (iii) the fraction N_L of single banks L, (iv) the relationship premium as a fraction of interest payment, $|\mathcal{V}|/\hat{i}z$, (v) the value of a relationship for bank S, (vi) the fraction of loans trading below the floor.

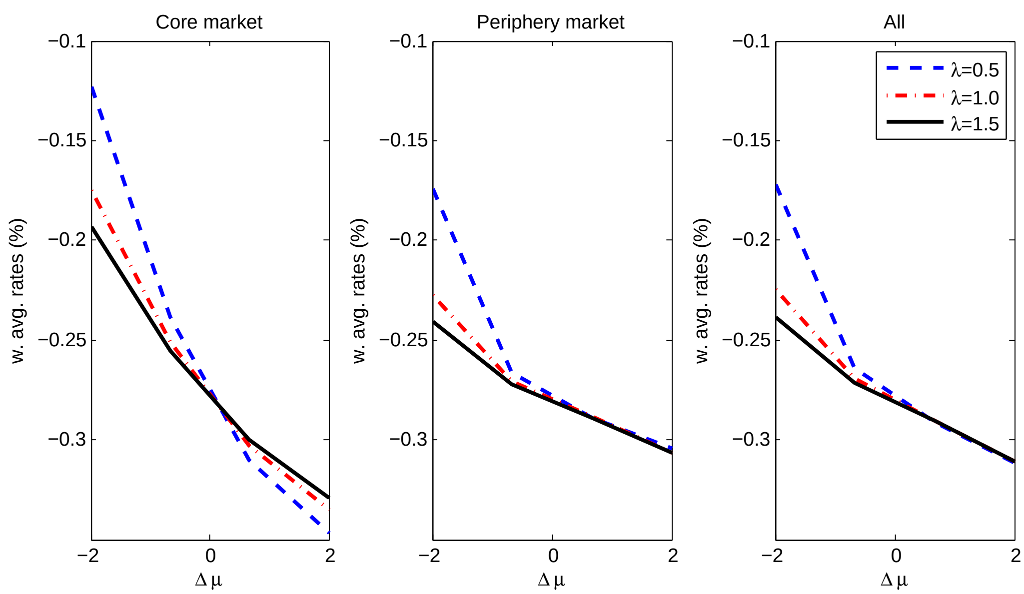

Figure 10: Effects of aggregate reserve balances $\Delta\mu$ for different allocation λ

drops as the bargaining share of the L banks declines. The second effect dominates and the relationship premium always decreases with θ.

Discussion

In this section, we discuss a few potential extensions and the importance of various assumptions for our results.

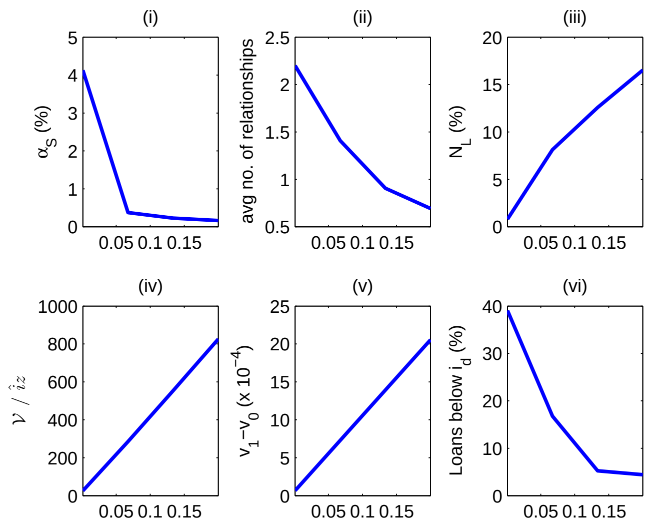

Figure 11: Effects of small banks' search cost for new partner $\kappa_S(\times 10^2)$ on (i) the fraction α_S of single banks S searching, (ii) the average number of relationships, (iii) the fraction N_L of single banks L, (iv) the relationship premium as a fraction of interest payment, $|\mathcal{V}|/\hat{i}z$, (v) the value of a relationship for bank S, (vi) the fraction of loans trading below the floor.

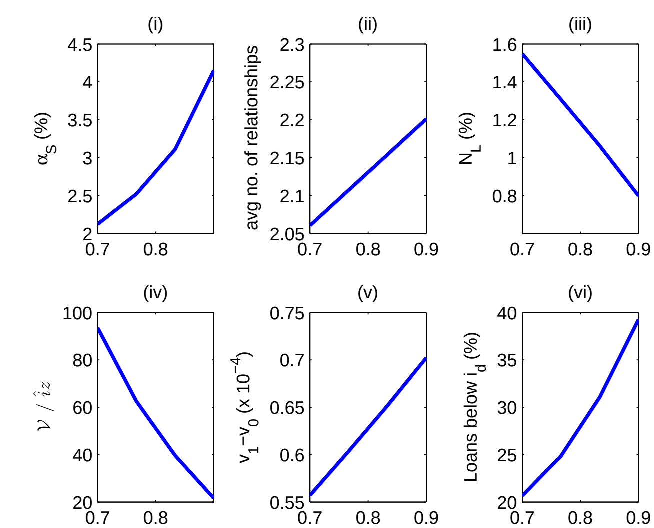

Figure 12: Effects of small banks' bargaining power θ on (i) the fraction α_S of single banks S searching, (ii) the average number of relationships, (iii) the fraction N_L of single banks L, (iv) the relationship premium as a fraction of interest payment, $|\mathcal{V}|/\hat{i}z$, (v) the value of a relationship for bank S, (vi) the fraction of loans trading below the floor.

First of all, the structure of the costs to access the core market is key to explain the core-periphery network structure of the interbank market. Since trading in the core market is beneficial, we needed to add some frictions so that some banks choose not to trade there. Costly participation is a realistic feature of the interbank market and it is motivated by evidence suggesting that cost saving considerations induce small banks to access the interbank market through large correspondent banks (e.g. Stigum and Crescenzi, 2007). This cost structure could also be considered as a short-hand for other frictions such as imperfect information and limited commitment. Our calibrations shows that this cost does not have to be large to generate the core-periphery structure observed in the data.

Second, while we assume a particular relationship building process, a relationship premium will arise as long as one side of a partnership values the continuation of the relationship more than the other side. In our current model, only single banks are allowed to initiate a new relationship. In the Appendix, we consider a more general setup in which banks with a relationship can also search for a backup partner. This extension does not change the key implications of our model (e.g. the existence of a relationship premium). In addition, while we allow banks to have a backup relationship in the periphery market, they can only maintain at most one relationship across periods. This is obviously a strong assumption as banks may have an incentive to maintain multiple relationships over time. Extending the model to include several relationships would be interesting. One would however have to tackle the question of how to model bargaining with several counterparties, or deal with a sequential bargaining problem and decide the order in which banks would bargain, e.g. in a random or a fixed sequence in each period. Given the model is already rich, we chose to bypass these questions by assuming that banks can only maintain one partner over time. In a more general setting where maintaining multiple relationships is allowed, we expect that the key findings will not change as long as some small banks, given the cost structure and shock process, choose to maintain only one relationship in equilibrium. In that case, trades in the periphery market will still involve a relationship premium whenever only one side of the transaction has a backup partner. We would argue that this is a relevant situation for the European interbank market as the MMSR data shows that a significant fraction of non-RA have only one RA partner.

Third, the model has a simple trading structure: banks can only conduct one trade in the periphery market and another trade in the core market each period. We impose this simplifying assumption merely to keep the model analytically tractable. Allowing multiple rounds of trades in the core market will make the model more complicated but should not affect the main implications of the paper. Chiu and Monnet (2016) consider an extension in which there is an OTC market open in sub-period 2 so that single banks can conduct two rounds of OTC trades each period. Another potential extension is to allow partners in a relationship to trade multiple times in a period (e.g. allowing them to retrade after the core market). This extension would increase the relationship premium, thus making it easier to obtain rates outside of the corridor. Hence, the current model adopts the minimal set of assumptions necessary to deliver the network structure.

Finally, we have also examined a few other extensions in Chiu and Monnet (2016) including endogenous initial balances, a competitive core market, counterparty risks, relationships between same types and transitional dynamics across steady states.

6 Conclusion

This paper presents a model of the unsecured segment of the interbank money market in which banks have incentives to build long-term relationships. The network of interbank loans arises endogenously, with monetary policy affecting not only the terms of trades but also the trading network. Using a unique data set from the ECB, we summarize some new, reliable stylized facts about interbank trading activities in the Europe area. We show that our model can help explain some puzzling observations in the data, such as why the floor of the corridor is "soft." We then parameterize the model to perform policy-relevant exercises regarding the design of the corridor system. The model is simple and tractable and, we believe, captures some important features of the interbank market. As such, it can be a useful tool for policy-makers, e.g., to investigate quantitatively the effects of running and "exiting" the floor system.

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