B ASSESSING BANKING SYSTEM RISK WITH EXTREME VALUE ANALYSIS

The literature has proposed a number of approaches how to assess the stability of banking systems. In this Feature a novel approach is described that is based on extreme value theory (EVT). EVT is particularly suitable for the analysis of financial instabilities, as it is designed to deal with the occurrence of extremely rare events ("tail risk"). For example, it has been used to examine the severity of stock market crashes, the pricing of catastrophic loss risk in reinsurance or the extent of operational risk in banks. The present application to systemic risk in banking derives a parameter from market returns that can capture the exposure of an arbitrary large number of banks to each other and to aggregate risk. The 25 systemically most important banks are analysed for the euro area and the United States, respectively, between 1992 and 2004. The results suggest that multivariate spill-over risk among banks may be more pronounced in the United States than in the euro area. One explanation for this finding seems to be that cross-border linkages are still weaker in Europe. Exposure to extreme systematic risk, however, is rather similar in the two banking systems. On both sides of the Atlantic the two forms of banking system risk increase during the second half of the 1990s. Increases in spill-over risk in Europe are, however, very gradual. The findings raise interesting policy questions about the relationship between financial integration as well as financial consolidation and the stability of banking systems.

INTRODUCTION

Assessing banking system risk is an essential element in the monitoring of financial stability. This applies to both more bank and more market oriented financial systems. Widespread instability in the banking sector has been associated with depressions and hyper-inflations in economic history.

The present Special Feature briefly reviews in the first section the main literature on sources of banking system instability. The second section describes a novel approach how to assess banking system risk, which is based on extreme value theory. The third section presents an application of this technique to a group of main euro area banks and juxtaposes the results with the ones for a comparable group of banks from the United States. The fourth section discusses some strengths and caveats in the methodology used. The last section concludes.

LITERATURE ON BANKING SYSTEM RISK

Banks are widely regarded as more fragile than other firms. In the economic literature this has been explained with their vulnerability to bank runs, which emerges from their balance-sheet structure that features short-term demandable deposits and long-term illiquid loans.[1] In modern financial systems the fragility of individual banks has been dealt with through regulation and supervision as well as the insurance of retail deposits.

The risk of wider banking system problems is associated with an observed vulnerability of banks to macroeconomic fluctuations and with a number of channels that raise the possibility of contagion among banks. The vulnerability to macroeconomic shocks has been explained by the fact that the value of loan books can fluctuate sharply with the business cycle, while the value of many deposits is not marked to loan book revaluations or states of the business cycle.[2] It has been confirmed in many empirical studies.[3]

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1   See e.g. J. Bryant (1980), "A Model of Reserves, Bank Runs, and Deposit Insurance", *Journal of Banking and Finance,* 4, pp. 335-344; D. Diamond and P. Dybvig (1983), "Bank Runs, Deposit Insurance, and Liquidity", *Journal of Political Economy,* 91 (3), pp. 401-419; or V. Chari and R. Jagannathan (1988), "Banking Panics, Information, and Rational Expectations", *Journal of Finance,* 43, pp. 749-761.
2   See M. Hellwig (1994), "Liquidity Provision, Banking, and the Allocation of Interest Rate Risk", *European Economic Review,* 38 (7), pp. 1363-1389.
3   See e.g. G. Gorton (1988), "Banking Panics and Business Cycles", *Oxford Economic Papers,* 40, pp. 751-781; A. Demirgüç-Kunt and E. Detragiache (1998), "The Determinants of Banking Crises in Developing and Developed Countries", *IMF Staff Paper,* No. 45, pp. 81-109; B. Gonzalez-Hermosillo (1999), "Determinants of Ex-ante Banking System Distress: A Macro-Micro Empirical Exploration of Some Recent Episodes", *IMF Working Paper,* WP/99/33, March.

The risk of bank contagion results from physical exposures among banks, either direct ones through money or other interbank markets or indirect ones through payment and settlement systems, and from asymmetric information among creditors or managers about the health of banks.[4] The prevalence of bank contagion risks is more controversial in the empirical literature. Based on diverse approaches and data samples in terms of time and geographical coverage, some studies find limited or no evidence of bank contagion during crises,[5] whereas other studies point to statistically significant contagion episodes or risks.[6]

There is no space to review these different perspectives and the underlying approaches in greater depth in the present Special Feature, whose main purpose is to present a new avenue of empirical research in this field.[7]

A NEW APPROACH TO ASSESS BANKING SYSTEM RISK

This novel approach is based on statistical methods of extreme value theory (EVT). EVT has been applied to a number of financial stability issues already, because they relate inherently to "tail risk" (very small probability events). Univariate EVT e.g. has been applied to estimate the likelihood of financial market crashes[8] and multivariate EVT to measure the risk of financial market contagion.[9] In the management and pricing of insurance and reinsurance risks EVT has an even longer history, as the distributions of the adverse events covered by this industry tend to be particularly heavy tailed.[10] More recently, it has also been applied to the evaluation of operational risk in financial institutions, such as required e.g. in the new Basel II capital adequacy standards.[11]

ABOUT EXTREME VALUE THEORY

Why is EVT particularly suitable for the analysis of financial stability problems? Widespread instabilities are extremely rare events. For example, stock market crashes that have the severity of Black Monday in 1987 have been

estimated to happen only once or twice a human lifetime. This means that usual data sample sizes are way too small for assessing the likelihood, severity or determinants of widespread crises with regular econometric techniques, as there

4   See Y. Chen (1999), "Banking Panics: The Role of the First-come, First-serve Rule and Information Externalities", *Journal of Political Economy,* 107 (5), pp. 946-968; F. Allen and D. Gale (2000), "Financial Contagion", *Journal of Political Economy,* 108 (1), pp. 1-33; and X. Freixas, B. Parigi and J.-C. Rochet (2000), "Systemic Risk, Interbank Relations and Liquidity Provision by the Central Bank", *Journal of Money, Credit, and Banking,* 32 (3/2), pp. 611-640.

5   See e.g. M. Smirlock and H. Kaufold (1987), "Bank Foreign Lending, Mandatory Disclosure Rules, and the Reaction of Bank Stock Prices to the Mexican Debt Crisis", *Journal of Business*, 60 (3), pp. 349-364; I. Hasan and G. Dwyer (1994), "Bank Runs in the Free Banking Period", *Journal of Money, Credit, and Banking*, 26, pp. 271-288; C. Calomiris and J. Mason (1997), "Contagion and Bank Failures during the Great Depression: The June 1932 Chicago Banking Panic", *American Economic Review*, 87 (5), pp. 863-883; C. Calomiris and J. Mason (2003), "Consequences of U.S. Bank Distress During the Depression", 93, *American Economic Review,* pp. 937-947; or H. Elsinger, A. Lehar and M. Summer (forthcoming), "Risk Assessment for Banking Systems", forthcoming in *Financial Management.*

6   See e.g. A. Saunders and B. Wilson (1996), "Contagious Bank Runs: Evidence from the 1929-33 Period", *Journal of Financial Intermediation,* 5 (4), pp. 409-423; or C. Upper and A. Worms (2004), "Estimating Bilateral Exposures in the German Interbank Market: Is there a Danger of Contagion?", *European Economic Review,* 48 (4), pp. 827-849. ECB (2004), "Cross-border bank contagion risk in Europe", *Financial Stability Review,* December, presented evidence of relevant bank contagion risks among contemporaneous European banks.

7   For a comprehensive survey, see O. De Bandt and P. Hartmann (2002), "Systemic Risk in Banking: A Survey", in C. Goodhart and G. Illing (eds.), *Financial Crises, Contagion, and the Lender of Last Resort*, Oxford, Oxford University Press, pp. 249-297.

8   See e.g. D. Jansen and C. de Vries (1991), "On the Frequency of Large Stock Returns: Putting Booms and Busts into Perspective", *Review of Economics and Statistics,* 73, pp. 18-24; or F. Longin (1996), "The Asymmetric Distribution of Extreme Stock Market Returns", *Journal of Business,* 69 (3), pp. 383-408.

9   See ECB (2005), "Financial market contagion", *Financial Stability Review",* December.

10 See e.g. P. Embrechts and N. Veraverbeke (1982), "Estimates for the Probability of Ruin with Special Emphasis on the Possibility of Large Claims", *Insurance: Mathematics and Economics*, 1, pp. 55-72; J. Beirland and J. Teugels (1992), "Modeling Large Claims in Non-life Insurance", *Insurance: Mathematics and Economics,* 11 (1), pp. 17-29; or T. Mikosch (2006), *Non-Life Insurance Mathematics: An Introduction with Stochastic Processes*, Berlin, Springer Verlag.
    Heavy or fat tailed distributions refer to statistical distributions in which extreme events, such as large losses or financial crises, are much more frequent than under the widely used normal distribution.

11 See P. de Fontnouvelle, J. Jordan and E. Rosengren (2005), "Implications of Alternative Operational Risk Modeling Techniques", *NBER Working Paper*, No. 11103, February.

are no or only a few relevant observations of the main phenomenon of interest.

EVT is precisely geared towards overcoming this problem. It is a discipline in statistics that analyses the behaviour of tails of statistical distributions, i.e. the probabilities and sizes of the most extreme and rare outcomes. A central result of this discipline is the fundamental theorem of extreme value theory.[12] It describes the families of distributions to which the behaviour of minimum and maximum outcomes (here financial returns) of other distributions converge asymptotically. These results can be used e.g. to assess the probability and size of the most extreme negative outcomes of financial variables. The left tails or minima are the relevant area where to look for crises. As the tails display certain regularities under quite weak assumptions, this assessment can be done even if crises are not in the sample or only a small number of times. The reason is that these regularities allow determining the shape of the tail in its entirety. Once one has estimated the tail, it is easy to calculate the probability or severity of specific crisis situations.

In economics and finance the use of EVT has emerged with the fundamental contributions of Mandelbrot and Fama.[13] These authors detected that the frequency of stock returns does not follow a Gaussian normal distribution, as it exhibits "fat tails". The fact that the tails of those distributions are thicker than for the normal means that very large and very small returns are more frequent. For example, there is an over-proportionate occurrence of crashes (crises). This observation is not limited to stock returns, but characterises a wide range of financial data. The potentially drastic consequences of severe financial crises for consumption, investment and growth, underlines the importance of techniques such as EVT that allow to analyse these extreme outcomes.

MEASURING BANKING SYSTEM RISK WITH EVT

In the present Special Feature the application of EVT to banking system stability is presented, following the novel approach by Hartmann,

Straetmans and de Vries (2005).[14] In line with previous literature, it uses changes in market valuations of banks to assess system risk. In contrast to the previous banking literature, however, it does so focusing entirely on extreme downturns in banks' market values, large crashes in their stock prices, so that there cannot be any doubt about the critical nature of the situations considered. Moreover, since system risk is to be assessed a multivariate approach has to be chosen.

Suppose a system is composed of N banks. The conditional probability that any subset of these N banks faces a critical situation given that other banks face a critical situation (*extreme spill-over* or contagion *risk*) is based on the ratio of the two joint crash probabilities for the two subsets of banks considered.[15] These joint probabilities can be described with tail dependence parameters η that can be calculated for any number of banks. The η parameter captures any dimension of dependency between the respective N banks for which it is estimated (bivariate, trivariate and up to N-dimensional). The η can also be used to describe *extreme systematic risk*, bivariate conditional probabilities of bank crashes given that the market as a whole crashes; these are the so called tail-βs.[16] In the present Special Feature

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12 It was proven by B. Gnedenko (1943), "Sur la distribution limite du terme maximum d'une serie aleatoire", *Annals of Mathematics,* 44, pp. 423-453.

13 See B. Mandelbrot (1963), "The Variation of Certain Speculative Prices", *Journal of Business,* No. 36, pp. 394-419; and E. Fama (1965), "The Behavior of Stock Market Prices", *Journal of Business,* 38, pp. 34-65.

14 P. Hartmann, S. Straetmans and C. de Vries (2005), "Banking System Stability: A Cross-Atlantic Perspective", *NBER Working Paper*, No. 11698, October. This approach builds on the theoretical work by A. Ledford and J. Tawn (1996), "Statistics for Near Independence in Multivariate Extreme Values", *Biometrika,* 83 (1), pp. 169-187; and G. Draisma, H. Drees, A. Ferreira and L. De Haan (2001), "Tail Dependence in Independence", EURANDOM report 2001-014.

15 This simple characterisation holds under the assumption of a common crisis percentile across banks, but a similar one can be found for a common crisis quantile. Percentiles refer to the probabilities of certain outcomes, here extreme negative returns, and quantiles to their sizes. Once one of the two are fixed, the others follow from the observed or estimated distribution.

16 S. Straetmans, W. Verschoor and C. Wolff (2003), "Extreme US Stock Market Fluctuations in the Wake of 9/11" (paper presented at the American Finance Association Meetings 2004, San Diego CA, 3-5 January) introduce the concept of tail-β in relation to asset pricing theory.

these two types of ηs are used as the basic summary statistics for the assessment of banking system risk.

The tail dependence parameter η has some advantageous properties. It varies between 0 and 1. $1/N$ describes the case of asymptotic independence (low system risk) and 1 describes the case of asymptotic dependence (high system risk). Moreover, it can be estimated with well-known univariate techniques.[17] As the resulting estimator for the tail dependence parameter η is asymptotically normally distributed, it is relatively straightforward to define confidence intervals and conduct structural stability and cross-sectional tests. With these tests one can determine whether banking system risk has changed over time or whether it is different across different banking systems.

AN APPLICATION TO THE EURO AREA AND THE US

The above approach is applied to the systemically most important listed banks of the euro area and the US. Systemic importance is assessed on the basis of different measures for size and interbank lending for the period 1992 to 2004. This procedure leads to the selection of overall 25 euro area and 25 US banks, which account for similar shares of the two respective banking systems.

Tail dependence parameters η are estimated from daily bank stock returns. For robustness different percentiles of the return distribution are tried to describe critical situations, but only the results for the percentile $p=0.0005$ are displayed in the Feature. The associated crisis quantiles correspond to 10 to 20% daily crashes in the bank stocks considered.[18] Such levels are close to the worst negative outturns for those banks over the whole sample. The sample covers a number of individual bank crises and a number of more general situations of financial turmoil, but this would not be necessary with the chosen approach.

MULTIVARIATE EXTREME SPILL-OVERS

First consider tail dependence among all euro area banks and all US banks, as indicators of spill-over risk. The 25-dimensional parameter estimates for both cases over the whole sample period are $\eta_{EA}=0.17$ and $\eta_{US}=0.39$. Given that independence would imply $\eta=1/25=0.04$, there seems to be some system risk from extreme spill-overs in both cases, but it is higher in the US than in the euro area. A cross-sectional test rejects the null hypothesis that both tail dependence parameters are equal at the 1% level, so that the difference is statistically significant. It is interesting to note that estimates of tail dependence parameters for some single euro area countries are of a similar order of magnitude as the estimate for the US. This result suggests that most of the difference in extreme banking spill-over risk between the euro area and the US is explained by lower cross-border risks in Europe.

Let us turn to the evolution of banking system spill-over risk over time. Chart B.1 and B.2 show recursive estimates of tail dependence parameters between 1994 and 2004 for the euro area and the US, respectively.[19] The dashed lines refer to estimates from data that are cleaned from the clustering of volatility (GARCH effects), which are typical for financial return data, whereas the solid lines are for original data.[20]

17 See B. Hill (1975), "A Simple General Approach to Inference about the Tail of a Distribution", *The Annals of Statistics,* 3 (5), pp. 1163-1173.
18 For daily (weekly) data a 0.0005 percentile means that a critical situation occurs on average every 2,000 days (weeks) or every four (forty) years. Compare this to the 0.1 or 0.05 percentiles that have to be used in standard econometric approaches, where a crisis is assumed to happen every 10 or 20 days (weeks).
19 Recursive estimates mean that η is first derived for a (small) data window at the start of the sample, and then further points on the curves are derived for ever larger data windows until the end of the sample is reached.
20 In the specific case of Chart B.1 the two lines are so close that they are basically indistinguishable.

Chart B.1 Evolution of multivariate extreme spill-over risk in the euro area banking system

— original
⋯⋯ declustered
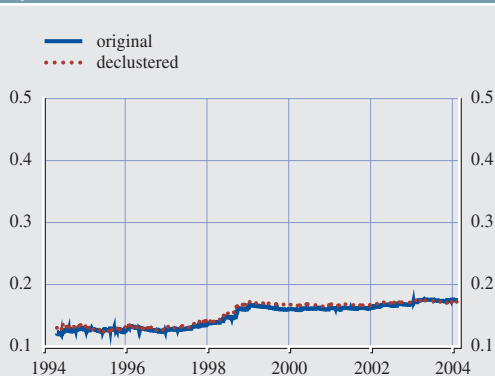
Source: Figure 1 of P. Hartmann, S. Straetmans and C. de Vries (2006), "Banking System Risk: A Cross-Atlantic Perspective", in M. Carey and R. Stulz (eds.), *Risks of Financial Institutions*, Chicago IL, Chicago University Press and National Bureau of Economic Research. Reprinted with kind permission by the Chicago University Press.

Chart B.2 Evolution of multivariate extreme spill-over risk in the US banking system

— original
⋯⋯ declustered
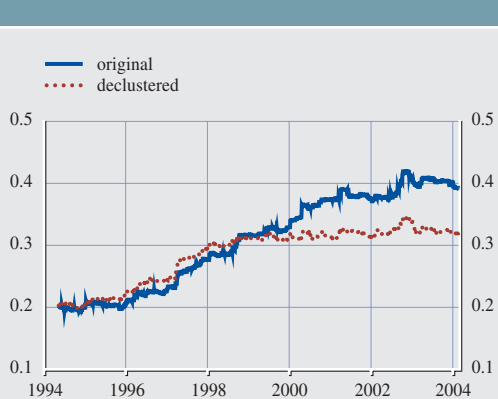
Source: See Chart B.1.

The euro area shows a slight increase of multivariate spill-over risk among the systemically most important banks. This process is very gradual, exhibiting a weak acceleration in the second half of the 1990s.

The charts reflect the higher level of system risk in the United States. But also the dynamics is stronger, as extreme spill-over risk increases somewhat more forcefully for most of the sample period. De Nicolo and Kwast (2002) draw similar conclusions based on a more standard correlation analysis and suggest that strong banking consolidation may in part explain the increase in risk.[21] The same explanation may not be as relevant for Europe as it is for the US, as consolidation has been less extensive and exhibited only a limited cross-border dimension.

The application of structural stability tests confirm that the changes observed in Charts B.1 and B.2 constitute statistically significant upward breaks in systemic risk.[22] They also indicate that no further increases of η_{EA} happens at the time of the introduction of the euro when a common euro area money market was created. If anything, the tests suggest a slight decrease in multivariate spill-over risk at that time.

EXTREME SYSTEMATIC RISK

Next consider the exposure of euro area and US banks to extreme systematic risk, as approximated by crashes in measures of aggregate risk (tail-βs). In this Feature the results for the general stock indices of the euro area and the US are reported,[23] but other indicators of aggregate risk give very similar results. We focus on the bivariate parameters η, which describe the extreme dependence between individual bank stocks and the market risk factor and determine the tail-βs.

Charts B.3 and B.4 are derived in two steps. First, for each bank ηs are estimated recursively over time. Second, for each point in time, the average η is derived for the 25 banks of each area/country.[24] The dashed and solid lines represent again GARCH-corrected and non-corrected data, respectively. Hence, each point

21 G. De Nicolo and M. Kwast (2002), "Systemic Risk and Financial Consolidation: Are They Related?", *Journal of Banking and Finance,* 26, pp. 861-880.
22 The test used is the recursive one presented by C. Quintos, Z. Fan and P. Philipps (2001), "Structural Change Tests in Tail Behaviour and the Asian Crisis", *Review of Economic Studies,* 68, pp. 633-663, and applied to η.
23 The indices are total return indices from Thomson Financial Datastream.
24 As each η estimator is asymptotically normally distributed, also the averages are normal.

Chart B.3 Evolution of extreme systematic risk in the euro area banking system

original
declustered
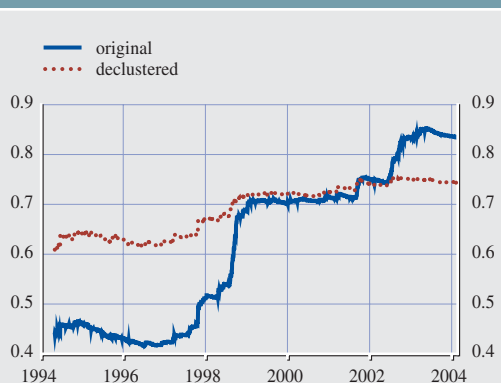
Source: Figure 3 of P. Hartmann, S. Straetmans and C. de Vries (2006), "Banking System Risk: A Cross-Atlantic Perspective", in M. Carey and R. Stulz (eds.), *Risks of Financial Institutions*, Chicago IL, Chicago University Press and National Bureau of Economic Research. Reprinted with kind permission by the Chicago University Press.

Chart B.4 Evolution of extreme systematic risk in the US banking system

original
declustered
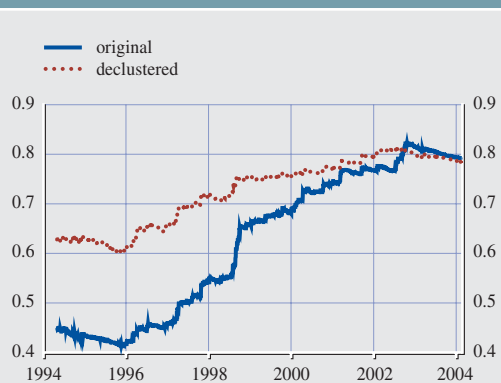
Source: See Chart B.3.

on one of the lines represents an estimate of the average extreme systematic risk in the respective banking system since the start of the sample.

A first observation is that exposure to extreme systematic risk seems to be quite similar in the two banking systems. The average bivariate ηs over the whole sample period are 0.83 for Europe and 0.79 for the US. A test cannot reject the null hypothesis that the two values are statistically indistinguishable. This contrasts with the multivariate spill-over results reported above. The systemically most important banks in the euro area and the US seem to be exposed to a similar extent to severe macroeconomic risk. It should, however, be kept in mind that behind the average tail dependence reported in the two charts, there are also some differences for different banks. In particular, smaller and more regional banks tend to be less exposed to fully area-wide shocks than larger and more diversified banks. Moreover, when looking at the relatively high values of η one needs to remember that for this bivariate measure the case of asymptotic independence is already reached at η=0.5.

Also the evolution over time is quite similar on the two continents. Both banking systems exhibit

a relatively clear increase of exposure to aggregate risk. This picture is confirmed with break tests on individual banks' ηs. Almost all banks in the sample exhibit a significant upward break.

DISCUSSION AND CAVEATS

The approach presented in the present Special Feature has a number of desirable properties for the analysis of financial stability. For example, contrary to most other approaches it can capture the rare events that are of greatest interest for the analysis of financial instabilities. It is fully multivariate and therefore appropriate for the system dimension. The semi-parametric estimation approach does not rely on strong assumptions. For example, the assumption of particular parametric distributions for financial variables can be generally problematic for crisis situations, in particular in a cross-country context. More specifically the approach does not rely on the assumption of normally distributed returns or linear concepts such as correlation, which typically lead to considerable biases in the assessment of single and joint tail events.

Nevertheless, with the present approach it is still relatively easy to derive confidence bands

and test for statistical significance of the results. Although sometimes not explicitly reported, all the results in the Feature are highly statistically significant. Finally, while it was not done so for the present article, the approach can be extended to the estimation of crisis probabilities and they can be refined to detailed spill-over probabilities for whatever individual or groups of banks are of interest.

The methodology has also some caveats. First, an issue of concern in the EVT literature is the number of observations that enter the parameter estimates that determine the tail shape. While there are methods to determine this number optimally, results can be sensitive to the choice of method. Second, from time to time the optimal number of observations used does not constitute a large sample. So, small sample properties are of interest and have been dealt with in the underlying working paper. Third, the estimators presented here are first-order approximations. In some circumstances, the second order terms could have some importance.

Also with respect to the specific application conducted here, a number of issues need to be kept in mind. First, the analysis is based on market data. So, the estimations of systemic risk are only precise to the extent that bank stocks are accurately priced. Second, market data limit the scope of the analysis to listed banks. In particular in Europe, however, there are still a number of important public banks, co-operative banks or large networks of co-operative banks for which stock returns are unavailable. Third, a significant part of the sample period relates to one long cycle. Particularly due to the unavailability of European stock data, it is not possible to conduct the analysis for a longer period covering a larger number of cycles. And last, the two measures of system risk presented (extreme spill-over risk and extreme systematic risk) are to some extent related. While they clearly measure system risk from different angles, they should not be interpreted as drawing a perfect line between aggregate risk and contagion.

In sum, while applications of extreme value theory prove to be important for a variety of analyses relating to the stability of financial systems, there are also other approaches that can be fruitfully considered.

CONCLUDING REMARKS

This Special Feature illustrated how the theory of extreme values can be used to assess the stability of banking systems. It presented a tail dependence parameter that can be used to determine two forms of systemic risk, interbank spill-overs and exposure to extreme systematic risk. The parameter is relatively easy to estimate, lends itself to confidence intervals and statistical testing and gives rise to intuitive graphical illustrations of the evolution of systemic risk.

The approach was then applied to the 25 systemically most important banks in both the euro area and the United States during the period 1992 to 2004. Keeping the caveats listed above in mind, the analysis leads to a number of tentative conclusions. First, multivariate spill-over risk in the US banking system seems to be more pronounced than in the euro area system. This feature is partly related to the still relatively weak spill-over risk linkages between banks across European borders. Second, extreme systematic risk in both banking systems are rather similar. This seems interesting to note in relation to empirical literature that seems to have found more robust evidence showing the relevance of macro shocks for banking crises than the relevance of interbank contagion. Third, from a policy perspective it seems particularly important that the indicators in this Feature suggest an increase in banking system risk over the period considered. This increase was, however, relatively limited for spill-over risk in Europe.

All in all, the results underline the importance of macro-prudential analysis that pays attention to the area-wide dimension in Europe. An interesting and important question for future research is whether the ongoing process of financial integration in Europe will further

increase spill-over risks among European banks, e.g. to the levels already observed in the United States today.