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Taylor-rule consistent estimates of the natural rate of interest

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Abstract

We estimate the natural rate of interest for the US and the euro area in a semi-structural model comprising a Taylor rule. Our estimates feature key elements of Laubach and Williams (2003), but are more consistent with using conventional policy rules: we model inflation to be stationary, with the output gap pinning down deviations of inflation from its objective (rather than relative to a random walk). We relax some constraints on the correlation of latent factor shocks to make the original unobserved-components framework more amenable to structural interpretation and to reduce filtering uncertainty. We show that resulting natural rate metrics are more consistent with estimates from structural models.

Keywords: Natural Rate of Interest, Equilibrium Real Rate, Taylor Rule, Bayesian Estimation, Unobserved Components, Beveridge-Nelson Decomposition.
JEL Classifications: C11,E32, E43, E52

Non-technical summary

The natural rate of interest (r^*) plays a key role in assessing the monetary policy stance and explaining secular trends in real yields. It is commonly defined as the rate of interest consistent with the economy operating at its potential level (in the absence of transitory shocks) or its natural level (in the absence of nominal frictions).

Econometric approaches typically estimate r^* from low-frequency stochastic drift components. Laubach and Williams (2003) obtain this trend by drawing on an unobserved components model for real GDP, inflation, and the real rate of interest, but with the latter variable modelled to be exogenous. It is not readily evident how the resulting latent factor estimates can be incorporated into policy rules featuring r^* and an output gap. Specifically, the inclusion of a unit root in inflation (resulting in an accelerationist Phillips curve) is only consistent with stabilising inflation at indeterminate levels (asymptotically, once all transitory shocks have washed out) – while Taylor rules are rather concerned with determining a rate of interest that stabilises inflation in line with an inflation objective.

The contributions of this paper can be summarised as follows: We show how to estimate r^* in a semi-structural model comprising a Phillips curve, and an aggregate demand curve, featuring key elements of the approach originating from Laubach and Williams (2003). Rather than using real interest rates as an exogenously determined process, we close the original framework with a Taylor rule. This requires constructing an output gap that pins down inflation in line with the inflation objective, as incorporated in the Taylor rule – as opposed to the original version, where reaching potential output is consistent with stabilising inflation only at *indeterminate levels*, a feature originating from imposing a unit root in the Phillips curve. To this end, we switch to a non-accelerationist Phillips curve, with the output gap pinning down deviations of inflation from the inflation objective, rather than from a unit-root trend. We further deviate from the original approach by using model-consistent inflation expectations. Finally, we relax some restrictions on (non-trivial) correlations of latent factor shocks. This change makes the model more amenable to structural interpretation; it contributes to aligning econometric and DSGE-based approaches more closely; it can also reduce filtering uncertainty. We adopt a Bayesian estimation strategy which helps to eschew the "pile-up problem" typically quoted as commanding a commonly used – but involved – multi-step Maximum-Likelihood approach.

We document significant differences with r^* estimates for the US and the euro area, as published by Holston, Laubach, and Williams (2017), especially during episodes when inflation

was persistently high and note that inclusion of a Taylor rule yields better consistency with estimates obtained from structural macroeconomic models.

Overall, we validate the common finding that the natural rate of interest has fallen throughout the past two decades, that this decline cannot entirely be accounted for by a lower growth rate in potential output, and that the natural rate appears to have slumped to levels around zero in the wake of the financial crisis.

Beyond these low-frequency trends in real short-term interest rates, it is difficult to provide statistically reliable estimates of r^* at business cycle frequency: uncertainty in natural rate estimates obtained from this modelling class has been shown to be of staggering size. This high degree of statistical uncertainty is model-inherent. It complicates the use of natural rate estimates in real time for policy assessment, as well as a statistical assessment of competing model specification choices.

1 Introduction

The natural rate of interest, henceforth r^*, is central to monetary policy making. It is commonly defined as the rate of interest consistent with the economy operating at its potential or natural level – and this coincidentally being consistent with stable prices. Originating from Wicksell (1898) the concept has received renewed attention after Woodford (2003) included it in the prototypical New-Keynesian framework. Econometric approaches estimate r^* from low-frequency stochastic drift components, yielding a natural rate metric consistent with transitory shocks having washed out.[1] By contrast, structural estimates construct r^* as the real interest rate path resulting from real business cycle shocks in the absence of nominal frictions.[2]

Just as there is no commonly agreed modelling approach to identify potential output or the natural rate of interest in a unique way, there is also no common estimate of the natural rate to gauge the policy stance. It is unclear how to embed prominent estimates originating from Laubach and Williams (2003, henceforth "LW") consistently into conventional policy rules: these estimates produce very different output gaps than conventionally used in Taylor rules; the output gap pins down deviations of inflation from a stochastic trend, so closing the output gap – by aligning real rates to r^* – will stabilise inflation at indeterminate levels rather than around a policy objective. Accordingly, estimates of the natural rate stabilise inflation at indeterminate level (asymptotically, once all transitory shocks have washed out) – while Taylor rules are concerned with determining a rate of interest that stabilises inflation in line with an inflation objective. This inconsistency can be addressed by replacing the exogenously determined real-rate process in LW by an endogenous process for real rates. One way of accomplishing this is to close the model by incorporating a Taylor rule.

DSGE-based estimates (being a function of real business cycle shocks) are rather volatile and thereby go against preferences to smooth nominal interest rate adjustments; even within modelling classes r^* estimates can differ widely, exhibit rather different stabilising properties, and can be subject to a staggering degree of statistical uncertainty (Fiorentini et al. (2018)).

We estimate the natural rate of interest in a semi-structural model comprising a Phillips curve and an aggregate demand curve, thereby featuring key elements of the approach originating from LW, but closed with a Taylor rule. This approach is motivated by two considerations. First, there is a need to adjust the output gap measure in a way that is model-consistent

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[1] see the literature following Laubach and Williams (2003).

[2] see, e.g., Edge, Kiley, and Laforte (2008); Barsky, Justiniano, and Melosi (2014); Curdia, Ferrero, Ng, and Tambalotti (2015); Gerali and Neri (2017).

with the natural rate estimate, as argued by Taylor and Wieland (2016): For example, if observed output is above potential, a rise in the trend growth component should increase potential output and thereby (partly) close the output gap. Including only a time-varying natural rate proxy in the Taylor rule would erroneously prescribe a higher policy rate due to the increase in r^*, while the inclusion of a correspondingly lower output gap would counter the upward pressure. Second, LW construct the natural rate as the low-frequency component in real rates that tends to neither raise nor lower the rate of inflation — irrespective of its level relative to price stability, a feature originating from imposing a unit root in the Phillips curve.

We extend and modify the LW framework in four aspects: First, we estimate all latent factors within one coherent modelling and estimation framework so that they can be included in ad-hoc Taylor rules to gauge the stance of monetary policy.

Second, incorporation of a Taylor rule requires switching to a non-accelerationist Phillips curve, which is different from LW. The resulting measures of slack and of the natural rate of interest will be consistent with achieving price stability, rather than affecting the inflation momentum at *indeterminate* levels of inflation.

A third important difference is the introduction of a model-consistent real rate: LW construct this observable as an exogenous process via the one-period nominal interest rate and use trailing four- or eight-quarter average inflation as a proxy for inflation expectations. Instead, we employ the nominal short-term rate as an observable (in its own right which is possible due to the inclusion of the Taylor rule) and construct the ex-ante real rate using the one-period ahead expected inflation rate from the Phillips curve.

Fourth, we relax specific zero cross-correlation restrictions in the error covariance matrix of latent-factor shocks to allow for a better structural interpretation in the interaction of shocks to potential output, observed output, and the output gap: a shock in potential output no longer requires instantaneous adjustment in output, but can give rise to a sluggish response in output and thereby affect the output gap.

In contrast to LW and the update in Holston, Laubach, and Williams (2017, henceforth "HLW") we employ a Bayesian approach in the estimation procedure. Kiley (2015) and Lewis and Vazquez-Grande (2017) previously used Bayesian econometrics to illustrate challenges with parameter identification in LW.

We choose a Bayesian approach, as it is not subject to 'pile-up'. In this context, 'pile-up' means that when estimating the model simultanously, using Maximum Likelihood, for at least one of the variances of the latent factor shocks the likelihood function peaks at zero. Therefore, LW use a multi-step Maximum-Likelihood approach as in Stock and Watson (1998) – a method that can become increasingly unwieldy as more equations are added and as constraints on cross-correlations of error terms are relaxed. By contrast, a Bayesian approach allows for simultaneous estimation of all model parameters. This method is convenient when extending the model by a policy rule and the labour market and when relaxing cross-equation constraints.

Our results validate the evidence of a secular decline in the natural rate of interest and its slump in the wake of the global financial crisis, as estimated by HLW. Yet we note important differences in latent factor estimates: as for output gaps, we find that our estimates exhibit business cycle dynamics that are better comparable to institutional output gap estimates, while those by LW appear much more persistent. At the same time, relative to the unobserved-components version (restricting latent-factor shocks to be uncorrelated), our output gap metric is more volatile, as expected on the basis of the discussion in Morley et al. (2003). As for estimates of the natural rate of interest we note that, when measured inflation was high and persistent, our estimates are higher than those reported by LW; in the aftermath of the global financial crisis, they are lower; and, overall, they exhibit a more cyclical behaviour.

These differences have important consequences in terms of real rate gap metrics – an indicator of the monetary policy stance. For example, while LW report natural rate gaps to have been positive (i.e. monetary policy to have been restrictive) for the US over the entire period 1979-1991, our natural rate gap estimates exhibit a more cyclical pattern, indicating that policy was restrictive during the Volcker disinflation period, subsequently accommodative, and again rather tight over a brief spell at the end of the 1980s.

In the following Section 2, we describe how to adjust the LW framework to include semi-structural r^* estimates into Taylor rules, how to extend it to capture labour-market dynamics, and how allowing latent factor shocks to correlate makes the model more amenable to structural interpretation. We also provide more detail on the Bayesian estimation strategy. We discuss the main finding of our estimation, including parameter values, strength of identification, and latent factor estimates, in Section 3. Section 4 explores differences in latent factor estimates with those originally obtained in HLW for the US and the euro area and with those obtained by structural models. Overall, in section 5 we conclude that in both currency areas, notwithstanding model-specific differences, estimates of the natural rate of interest exhibit a protracted downward trend since the end of the 1980s and a slump in the wake of the financial crisis.

2 Empirical Framework

2.1 A closed-form unobserved components model for estimating the natural rate of interest and output

We estimate and extend a small-scale semi-structural unobserved components model in the spirit of Clark (1987) and as applied by LW to estimate the natural rate of interest r^*. The major modification introduced by LW is the addition of an (accelerationst) Phillips curve to the output gap / investment-savings relation which transforms the *real* system of equations into a *nominal* model, comparable to the non-policy block of the basic New-Keynesian framework as, e.g. in Galí (2008). This addition becomes meaningful, if we assume prices to be subject to nominal frictions that prohibit instantaneous adjustments. As a result, the nominal interest rate is non-neutral and its choice affects the path of real variables via the Fisher relation

$$i_t = E_t\{\pi_{t+1}\} + r_t \tag{1}$$

with the nominal interest rate i_t, expected one quarter ahead inflation $E_t\{\pi_{t+1}\}$, and the real interest rate r_t generally differing from the natural real rate. In such a framework, the absence of an interest rate specification leaves the real variables indetermined. Since the data for output and inflation are provided for the estimation, however, the nominal interest rate is implicitly given, albeit without any rule-based regularities that can be exploited during the estimation process.

Although the original 2-equation approach offers a larger degree of flexibility, our aim is to determine the dynamics of the latent variables more consistently. We therefore deviate from LW in two important ways. First, broadly consistent with the canonical 3-equation New-Keynesian model as in Galí (2008), we include a Taylor rule with nominal interest rate smoothing as in Woodford (2003)

$$i_t = \rho_i i_{t-1} + (1-\rho_i)(r_t^* + \pi_t^* + \rho_\pi(\pi_t - \pi_t^*) + \rho_y \tilde{y}_t) + \epsilon_t^i, \tag{2}$$

with parameters for interest rate smoothing ρ_i, Taylor rule coefficients ρ_π and ρ_y, a stochastic term ϵ_t^i that captures innovations, and variables for the natural rate r_t^*, observed inflation π_t, the inflation objective π_t^*, and the natural output gap $\tilde{y}_t$ defined as the difference in observed output y_t from potential output y_t^*:

$$\tilde{y}_t \equiv y_t - y_t^*. \tag{3}$$

Second, we assume that the Phillips curve that governs the inflation process is stationary. This is a necessary adjustment relative to LW who use an accelerationist Phillips curve. An inflation process with a unit root would be inconsistent with credible inflation-stabilising monetary policy. The Phillips curve accordingly includes an inflation target π_t^*:

$$\pi_t = (1 - b_\pi)\pi_t^* + \frac{b_\pi}{2}(\pi_{t-1} + \pi_{t-2}) + b_y \tilde{y}_{t-1} + \epsilon_t^\pi \text{ with } 0 < b_\pi < 1, \tag{4}$$

with innovations ϵ_t^π and parameters b_π and b_y.

The remaining equations, as also used by LW, originate from the unobserved components model by Clark (1987): an IS curve is approximated by the process

$$\tilde{y}_t = a_{y,1}\tilde{y}_{t-1} + a_{y,2}\tilde{y}_{t-2} + \frac{a_r}{2}(\tilde{r}_{t-1} + \tilde{r}_{t-2}) + \epsilon_t^{\tilde{y}}, \text{ with } a_r < 0, (a_{y,1} + a_{y,2}) < 1. \tag{5}$$

with the real rate gap defined as $\tilde{r}_t \equiv r_t - r_t^*$. The lag structure in Equations 4 and 5 is adopted from HLW.

The level of potential output follows a random walk:

$$y_t^* = y_{t-1}^* + g_{t-1} + \epsilon_t^{y^*}, \tag{6}$$

where $\epsilon_t^{y^*}$ captures permanent shocks to the level of potential output, while the stochastic drift

$$g_t = g_{t-1} + \epsilon_t^g \tag{7}$$

features a permanent shock to the period-by-period growth rate of potential output ϵ_t^g.

The natural rate of interest r_t^* is then given by

$$r_t^* = 4g_t + z_t, \tag{8}$$

if we use quarterly data and interest rates are annualised. Thereby, in addition to the trend growth rate of the natural rate g_t, any other non-growth determinants are captured by z_t which follows a random walk:

$$z_t = z_{t-1} + \epsilon_t^z. \tag{9}$$

The z_t process will capture deviations between the trend in potential output growth g_t and the lower-frequency component in r_t, thereby allowing risk aversion, safe asset scarcity, or global saving-investment imbalances to drive a permanent wedge between interest rates and growth.

We extend this basic model along a second dimension of economic activity, employing a generalised version of Okun's law as in Clark (1989):

$$\tilde{u}_t = u_{y,0}\tilde{y}_t + u_{y,1}\tilde{y}_{t-1} + u_{y,2}\tilde{y}_{t-2} + \epsilon_t^{\tilde{u}}, \tag{10}$$

choosing a lag length of two as in Kim and Nelson (1999). Observed unemployment u_t is then constructed as the sum of the stationary, cyclical component $\tilde{u}_t$ and trend unemployment u_t^* (included in the Phillips curve via parameter $b_{\tilde{u}}$) characterised by a random walk process:

$$u_t^* = u_{t-1}^* + \epsilon_t^{u^*}. \tag{11}$$

Finally, while the original unobserved components model by Clark (1987) rules out correlation of shocks to latent factors, we will also allow for correlations in shocks to the output gap and the potential level of output, so as to allow for more sluggish responses of output to these shocks.

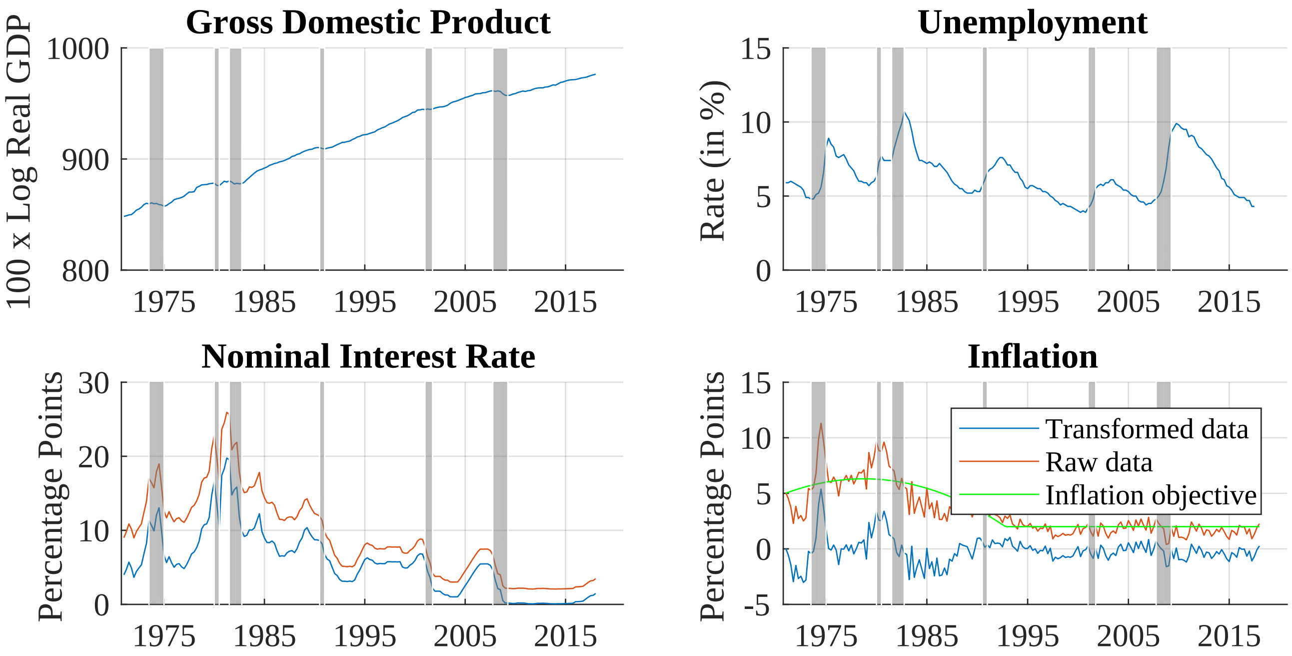
2.2 Empirical Implementation

2.2.1 Data and inflation objectives

We estimate the model both for the US and the euro area (EA) using data from 1971Q1 to 2018Q1 for both economies. The data for the US are identical to LW plus the unemployment rate. For the EA, we use corresponding data from the ECB's Area-Wide Model (updated using the ECB's Statistical Data Warehouse), including real GDP, the 3-month short-term nominal interest rate, the core consumer price inflation (in terms of changes of the CPI or HICP index on a year earlier), and the unemployment rate.

The introduction of a Taylor rule allows us to specify the nominal interest rate as an observable. This eliminates another inconsistency: LW construct the time series for the ex-ante real interest rate via the nominal rate and, as a proxy for expected inflation, the four quarter trailing average of inflation instead of a forward looking variable. We improve upon this proxy by employing the model-consistent one-period ahead expected inflation rate obtained from the stationary Phillips curve, Equation 4, to construct our real interest rate, Equation 1.

Figure 1: US Data

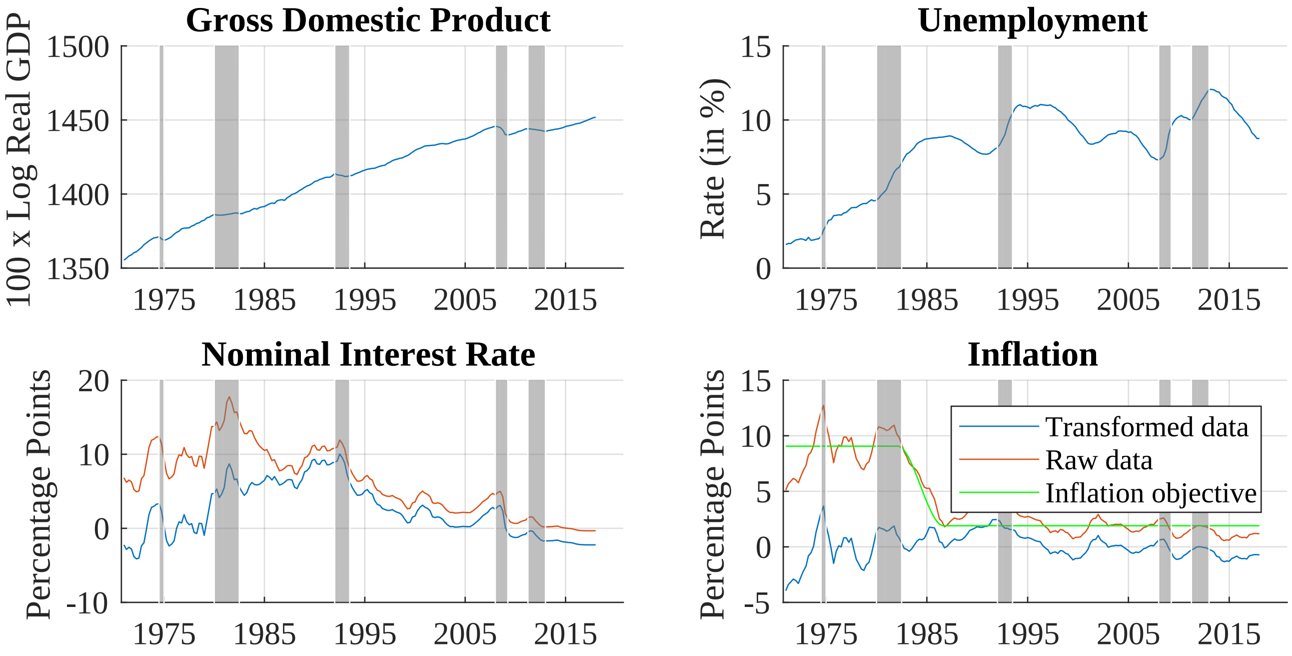


Sources: *Bureau of Economic Analysis, U.S. Bureau of Labor Statistics and Federal Reserve Board of Governors.*
Note: *Shaded areas correspond to recessions according to the National Bureau of Economic Research.*

The Taylor rule and non-accelerationist Phillips Curve depend on corresponding time series for the inflation objective π_t^*. In the absence of an official objective, we construct our own proxies. For the US, we assume that inflation levels desired by the policy makers until 1965 was about 2%. Inflation markedly increased for the following 25 years. We apply a cubic spline to realised inflation as a proxy for the inflation objective. After inflation normalised in the early 1990s, the objective is assumed to be 2%. The resulting data are shown in Figure 1. The precise choice of π_t^* during the 1970s and 1980s is certainly debatable, but it leaves the resulting real interest-rate metric largely unaffected.

For the EA in the 1970s, we assume an inflation objective that can be characterised by the average inflation rate. Towards the late 1970s, inflation decelerated, coinciding with the time of the negotiations to establish the European Monetary System (EMS) which went into effect in 1979. Inflation stabilised at a lower level in the course of the 1980s. At a meeting in Nyborg in 1987, EU finance ministers agreed that price stability would be an important cornerstone of the EMS. The European Central Bank announced a quantitative definition of price stability to be consistent with consumer price inflation below 2% in 1998 and clarified in 2003 that, within this definition, it aims at inflation rates close to 2%. We therefore construct an implicit inflation

Figure 2: EA Data



Sources: *New Area Wide Model database and ECB Statistical Data Warehouse.*
Note: *Shaded areas correspond to recessions according to the CEPR Euro Area Business Cycle Dating Committee.*

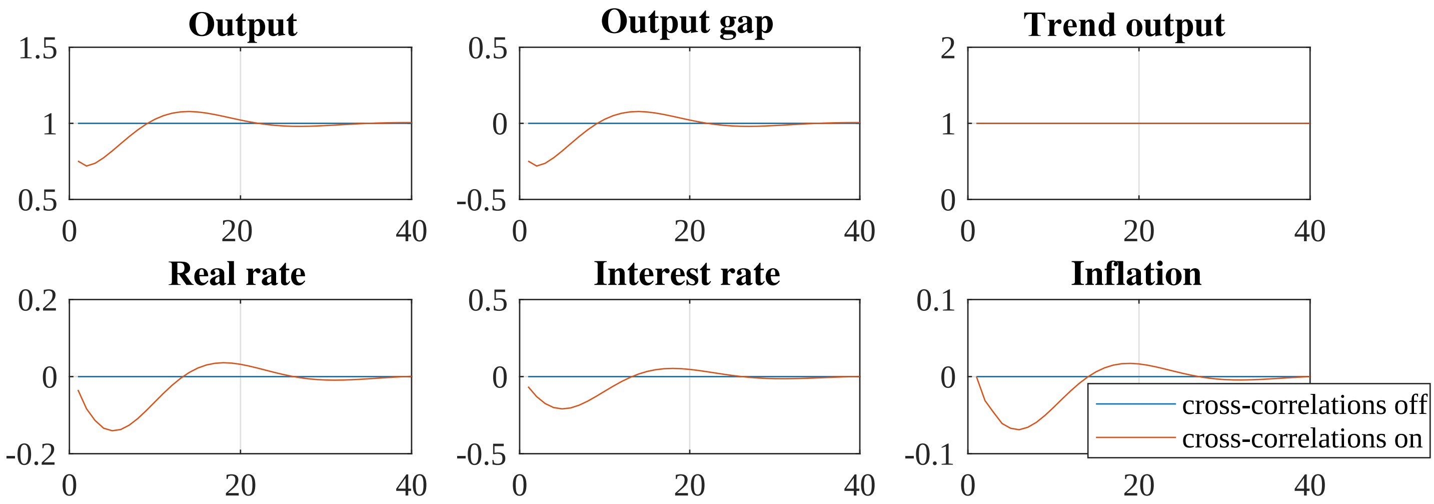
objective corresponding to these three episodes, using a cubic spline to link the average of high inflation rates in the 1970s with the period of price stability beginning in the 1990s. The data are shown in Figure 2.

2.2.2 Allowing cross-correlations in innovations

Clark (1987) assumes innovations across equations to be uncorrelated. This assumption is also routinely made in the context of structural models that start from microfoundations where innovations relate to primitive exogenous forces (without being common causes and, hence, modelled as approximately uncorrelated – see Bernanke (1986)).

This assumption appears to be rather strict in the context of a semi-structural model: Consider a positive realization of the innovation $\epsilon_t^{y^*}$ in Equation 6. This leads to an increase in trend output and – absent any additional unexpected realizations – a contemporaneous one-for-one increase in observed output y_t via the output gap, Equation 3. The interpretation of $\epsilon_t^{y^*}$ as a primitive (permanent) technology shock would be ill-conceived, as we know from quantitative New-Keynesian models positing that frictions cause observed output to respond sluggishly to

Figure 3: Instantaneous 1% increase in potential output



Note: *All responses are denoted in percentage points. Periods are in quarters.*

pure supply shocks, as , e.g. in Warne, Coenen, and Christoffel (2008).

In semi-structural models of this type, innovations are therefore likely to capture endogenous reactions rather than primitive shocks, if the innovations are assumed to be uncorrelated. Weakening this assumption may allow a more structural analysis: the innovation $\epsilon_t^{\tilde{y}}$ can dampen the effect of a positive realization of $\epsilon_t^{y^*}$ on observed output y_t, if the innovations are negatively correlated. This behaviour is more in line with inference from New-Keynesian models and originates from innovations to $\epsilon_t^{\tilde{y}}$ reflecting frictions otherwise not taken into account by the model.

Morley, Nelson, and Zivot (2003) previously discussed econometric implications from relaxing constraints on correlations in latent factor shocks, showing that without these constraints the resulting Beveridge-Nelson decomposition will describe variations in GDP to be accounted for more by the stochastic trend, rather than the cyclical component.

Bearing these aspects in mind, we allow for cross-correlations by centering corresponding priors around zero and allowing correlations to be perfectly positive or perfectly negative – see Table 2. We have compared different variants of relaxing these constraints, in particular those affecting production and unemployment. All resulted in qualitatively similar latent factor estimates and comparable input-output behaviour of the system. Appendix A displays a comparison of latent factor estimates. Figure 3 illustrates that, allowing for cross correlations in shocks, a boost in potential output temporarily opens up a negative output gap and prompts a dip in inflation and real interest rates, while excluding such correlations commands instantaneous adjustment in output and no reaction in inflation and interest rates.

Table 1: Priors and posteriors governing structural parameters

	Prior			Posterior							
				US				EA			
Parameter	Distribution	min	max	5% HPD	Median	95% HPD	HLW	5% HPD	Median	95% HPD	HLW
$a_{y,1}$	Uniform	1	2	1.00	1.15	1.34	1.53	1.01	1.20	1.35	1.67
$a_{y,2}$	Uniform	-1	0	-0.40	-0.18	0.00	-0.59	-0.55	-0.35	-0.18	-0.72
a_r	Uniform	-0.5	0	-0.40	-0.25	-0.07	-0.07	-0.41	-0.25	-0.09	-0.04
b_π	Uniform	0	1	0.63	0.72	0.81	0.67	0.68	0.78	0.87	0.72
b_y	Uniform	0	1	0.05	0.13	0.20	0.08	0.34	0.46	0.60	0.06
ρ_i	Uniform	0	1	0.57	0.69	0.81	–	0.49	0.63	0.76	–
ρ_π	Uniform	1	4	0.00	0.10	0.29	–	0.00	0.03	0.10	–
ρ_y	Uniform	0	2	0.40	0.85	1.40	–	0.40	0.86	1.39	–

Note: *The Beta distribution is scaled to (-1;1). HPD are the highest posterior density intervals. HLW estimates are taken from the original maximum likelihood results (available on the FRBSF website).*

Table 2: Priors and posteriors governing standard deviations of shock processes

	Prior			Posterior							
				US				EA			
Parameter	Distribution	Mean	Variance	5% HPD	Median	95% HPD	HLW	5% HPD	Median	95% HPD	HLW
$\sigma_{\tilde{y}}$	Inverse Gamma	0.67	0.22	0.28	0.53	0.82	0.35	0.26	0.39	0.53	0.29
σ_π	Inverse Gamma	0.67	0.22	0.72	0.79	0.87	0.79	0.20	0.23	0.27	0.97
σ_i	Inverse Gamma	0.67	0.22	0.73	0.82	0.92	–	0.34	0.42	0.49	–
σ_{y^*}	Inverse Gamma	0.67	0.22	0.30	0.56	0.82	0.57	0.51	0.62	0.73	0.39
σ_g	Inverse Gamma	σ_g^{LW}	0.22	0.04	0.07	0.11	0.12	0.02	0.04	0.07	0.05
σ_z	Inverse Gamma	σ_z^{LW}	0.22	0.07	0.52	0.81	0.16	0.48	0.68	0.88	0.31
$\sigma_{\tilde{y}y^*}$	Beta	-1	1	-0.53	-0.25	0.10	–	-0.54	-0.39	-0.24	–

Note: *The Beta distribution is scaled to (-1;1). HPD are the highest posterior density intervals. HLW estimates are taken from the original maximum likelihood results (available on the FRBSF website).*

Importantly the Beveridge-Nelson type of trend-cycle decomposition has proved to lead to higher loading coefficients a_r and b_y in Equations 5 and 4, respectively, thereby mitigating the risk of generating indeterminate states during model estimation, as explained further below. We henceforth focus on the version allowing for cross-correlation in the output gap and the level of potential output only.

2.2.3 Bayesian estimation: Priors on structural parameters and variances

We chose a Bayesian approach to estimating the semi-structural model, because it is not subject to 'pile-up' – in this context the problem that, if the model was estimated simultaneously using maximum likelihood, the likelihood of the variance of one of the shocks to the latent factors would peak at zero. In finite samples, the literature documents non-invertibility problems (with the likelihood function 'piling up' at one) when estimating a moving-average process when the true process is invertible, i.e. the coefficient is smaller than one, see Sargan and Bhargava (1983) and Kim and Kim (2013). A Bayesian approach allows simultaneous estimation of the model and avoids a cumbersome multi-step maximum-likelihood-based approach when adding additional equations and state variables to the original LW approach.

The priors on our parameters are summarised in Table 1 and are chosen based on their economic interpretation: An increase in the real interest rate leads to a decrease of observed output compared to potential output, resulting in a low negative elasticity a_r, as suggested by a standard IS Equation 5. The output gap is a stationary process, leading to autoregressive parameters $a_{y,1}$ and $a_{y,2}$ that are inside the unit circle. The Phillips curve is a stationary autoregressive process with positive feedback captured by b_π. The output gap exerts positive pressure on inflation through coefficient b_y.

Our priors are largely uninformative. To control for the signs of b_y and a_r and to ensure stability in the IS and Phillips-curve equation as well as to ensure that the Taylor principle holds, we limit the prior distributions. The priors on the cross-correlations allow the whole range from perfect correlation to perfectly inverse correlation, but are centered on zero.

For estimation and filtering we used Dynare (Adjemian et al., 2011). Posteriors were generated using a Metropolis-Hastings algorithm that generated two chains with 50.000 draws each. The initial 25.000 draws were discarded to account for a 50% burn-in period. Initial values are taken from the maximum likelihood estimates in HLW where available, while the average of the prior distribution is used otherwise.

3 Results

3.1 Parameter estimates

We present posterior estimates for the US and the EA in Tables 1 and 2 and compare them with those obtained in HLW. In Figures 32–39 (Appendix B) we illustrate differences in posterior against prior distributions across model specifications. Apart from the difference in the estimation strategy and the inclusion of a Taylor rule and a non-accelerationist Phillips curve, the differences to HLW can be understood as a consequence of allowing shocks in the cyclical and permanent component of output to correlate, causing gap measures to be smaller and less persistent, and from using model-consistent inflation expectations.

We find that the sum of autoregressive coefficients in the IS curves, $a_{y,1}$ and $a_{y,2}$, are quite close to unity, indicating persistent business cycle dynamics (but less so than in HLW).

The coefficient on the interest rate gap a_r is stronger than estimated by HLW, possibly owing to differences in real rate metrics as a consequence of our use of model-consistent inflation expectations and because of smaller business cycle amplitudes than in HLW.

Likewise, the coefficient on the output gap in the Phillips Curve, b_y, is also larger than reported in HLW, and particularly so for the euro area.

Notwithstanding the detrending of inflation rates by our proxy of implicit inflation objectives, inflation still exhibits significant persistence, as reflected in estimates of b_π.

The parameter estimates of the Taylor rule coefficients fulfill the Taylor principle. The smoothing parameter, ρ_i, is estimated to be lower than commonly reported in the DSGE literature (e.g. Smets and Wouters (2007) estimate ρ_i to be 0.81 in the euro area).

As for estimated variances of latent factor shocks, Tables 1 and 2 validate that r_t^* is mostly driven by shocks in the non-growth component z_t. Shocks to the growth component g_t are estimated to be small, giving rise to a rather smooth evolution in potential output growth. By comparison variances of shocks to the non-growth component of z_t are larger, reflecting a complete lack of convergence in the low-frequency components of real growth and the real rate of interest, as also previously conjectured in Hamilton et al. (2016). The high ratio of σ_z/σ_g, in particular for the euro area, is qualitatively similar to HLW, but even more pronounced. This result originates from including the Taylor rule and thereby modelling r_t^* to follow the path of real rates (rather than potential output growth) more closely: deviations of r_t from r_t^* (real rate gaps) are stationary by construction.

Correlations in shocks to the cyclical and the level component of output $\sigma_{\tilde{y}y*}$ are strongly negative, giving rise to shocks in potential output to affect slack – and the other way around. This evidence vindicates our conjecture that these cross correlations in latent-factor shocks are empirically relevant.

Tables 1 and 2 and Appendix B (Figures 32–39) indicate that, notwithstanding our use of largely flat priors, the data are sufficiently informative of posterior parameter densities and parameters appear to be sufficiently well identified.

Overall, our estimates suggest significantly higher loading coefficients of the output gap into the Phillips curve and the real rate gap into the aggregate demand equation than reported by Holston et al. (2017). Fiorentini et al. (2018) show that loading coefficients smaller than 0.1 in absolute value give rise to very large state variances, and thereby staggering natural rate uncertainty. They illustrate how weak loading parameters tend to violate the observability condition – a regularity condition for state space models necessary to filter states from observables – and prompt the variance of shocks affecting the non-growth component z_t to be extremely large. By contrast, as our loading coefficients are higher, filtering uncertainty should be attenuated.

3.2 The natural rates of interest and output growth

Filtered estimates of the natural rate of interest and the growth rate of potential output for the United States and the euro area are shown in Figures 4 and 5.[3]

Confidence bands are constructed from posterior densities and, given the use of Metropolis-Hastings, reflect *only parameter uncertainty*. Without taking into account shock and filtering uncertainty as well, these confidence bands cannot be seen as reflecting natural rate uncertainty. Constructing such confidence bands capturing all sources of uncertainty (using, instead of Metropolis-Hastings, the Gibbs Sampler in combination with the Durbin-Koopman simulation smoother) can lead to error bands spanning several percentage points (for such error bands, see e.g. Brand et al., 2018, p. 37, based on a specification with cross-correlation of shocks restricted to zero).

According to our estimates, in both the US and the euro area, r^* and g display a downward trend and reach all-time lows in the aftermath of the financial crisis. In the US, since the crisis r^* has rebounded somewhat and recently remained at levels around zero. In the euro area the recovery pattern in r^* is more tepid and features a double-dip into negative territory in the wake of the two crises (the banking and, subsequently, the sovereign debt crisis).

Figures 6 and 7 show corresponding output and real rate gaps for the US and the euro area. For both regions, the initial peak-to-trough plunge in capacity utilisation is estimated to have been around four percentage points in the US and three percentage points in the euro area.[4] The second recession experienced in the euro area in the wake of the sovereign debt crisis is marked by a second dip of the output gap into negative territory. Natural rate gap

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[3] Recession dating follows the National Bureau of Economic Research for the US and the CEPR Euro Area Business Cycle Dating Committee for the EA.

[4] The wide fluctuations in the output gap in the initial phase of the sample is to be discounted due to the initialisation of the Kalman filter.

Figure 4: Estimates of the natural rate and trend growth in the US

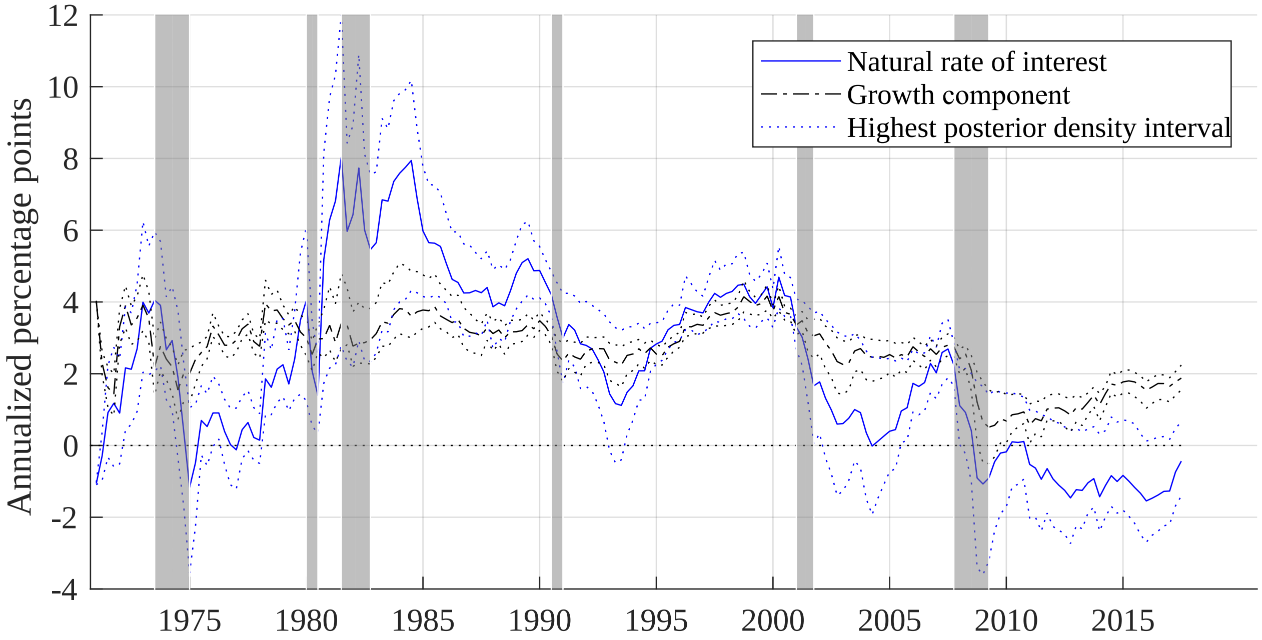


Figure 5: Estimates of the natural rate and trend growth in the EA

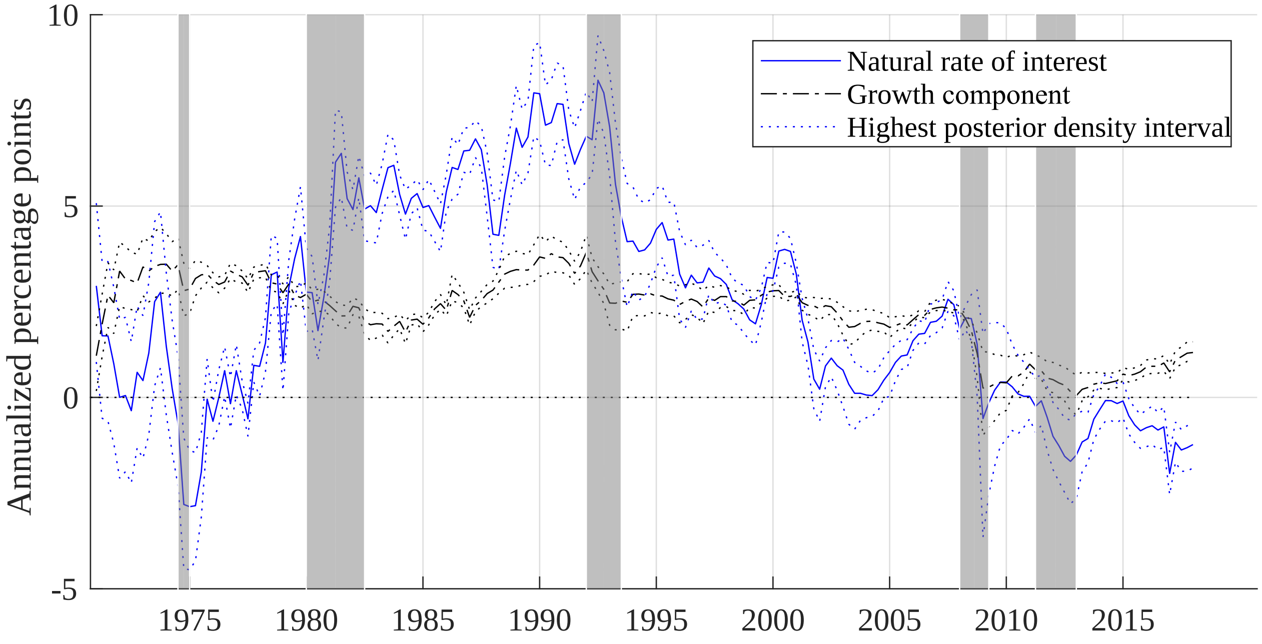
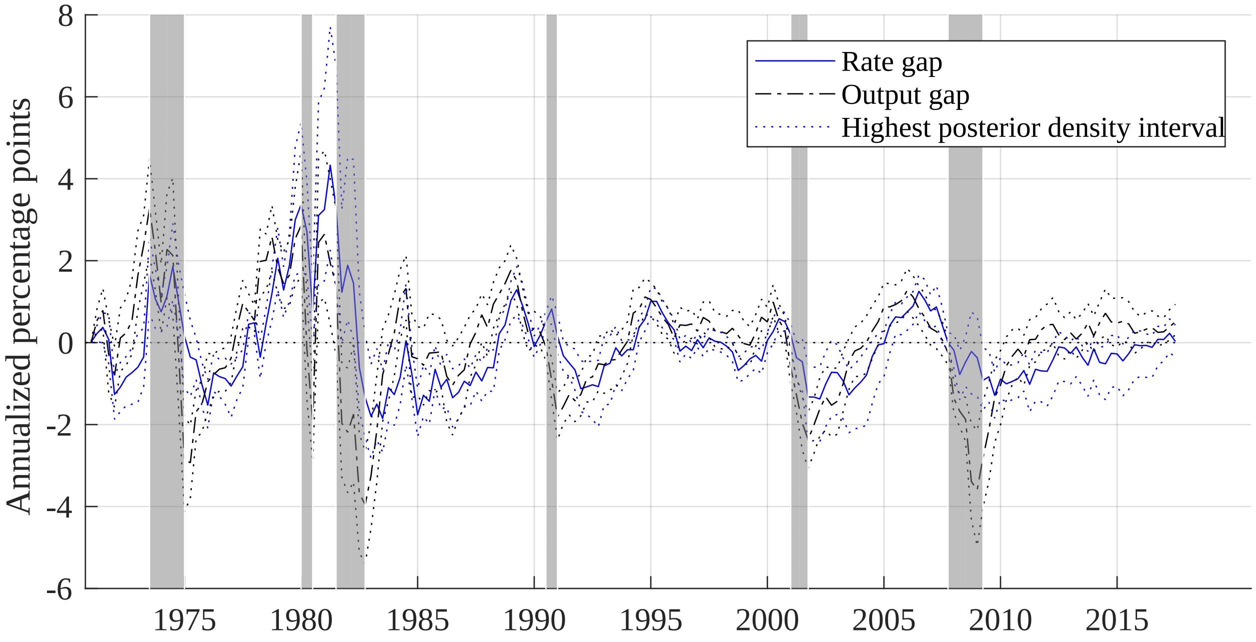


Figure 6: Estimates of the output gap and the real rate gap in the US



estimates suggest that monetary policy has been estimated to be largely accommodative since 2009, with the exception of the period of ECB rate hikes in 2011 in the context of strong growth and elevated inflation rates.

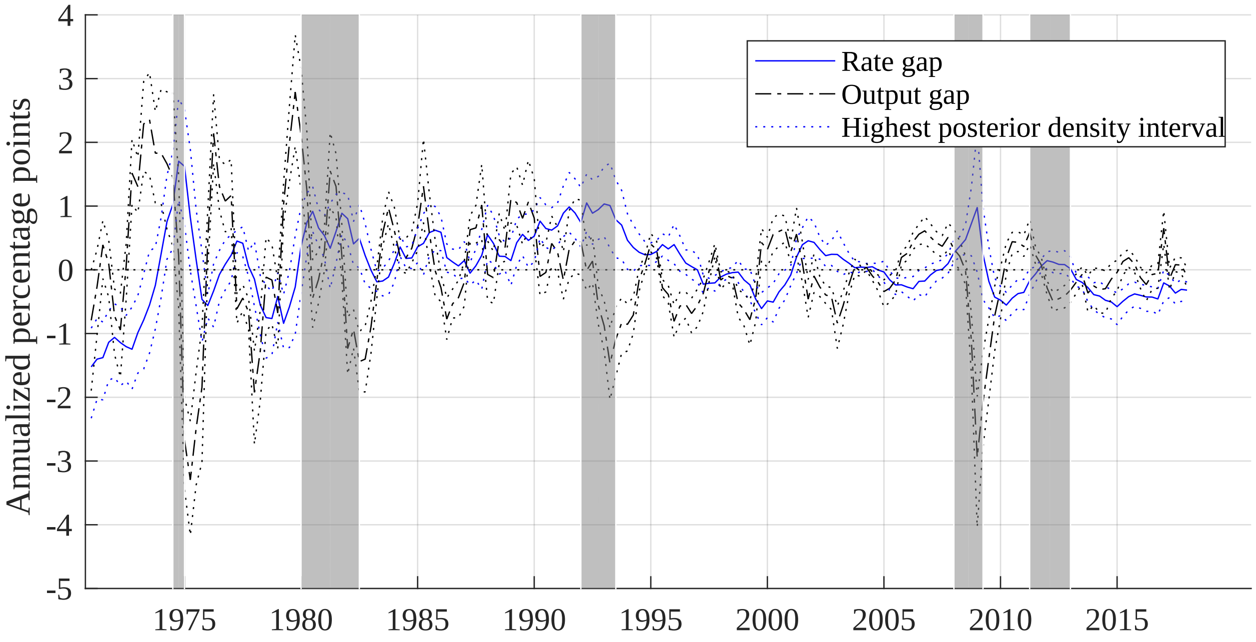
4 Comparison to other natural rate estimates

4.1 Comparison with estimates from Holston et al. (2017)

Given the differences in the Phillips curve specification and the use of a Taylor rule, it is worthwhile comparing our latent factors and natural rate gaps with those obtained by HLW.

Figures 8 and 9 display output gap estimates for the US and the EA in comparison with HLW. There is a striking discrepancy during the 1980s, especially for the EA, when the LW methodology delivers a large and highly negative output gap, while our methodology results in above-potential output. This difference is due to our choice of a non-accelerationist Phillips curve. The accelerationist Phillips curve used by LW and HLW interprets a 20 year stint below potential output as a prolonged episode of *dis*inflation (rather than *de*flation, as one might

Figure 7: Estimates of the output gap and the real rate gap in the EA



wrongly conjecture judging from a conventional interpretation of economic slack measures). Hence the difference of a non-zero mean output gap in LW and HLW and an output gap that is centred around zero. The Taylor-rule-consistent metric appears more amenable to conventional interpretation of business cycle dynamics, and more consistent with official output gap estimates.

Accordingly, our natural rate estimates displayed in Figures 12 and 13 show episodes featuring very significant discrepancies compared to those by LW and HLW. During the 1980s, our approach yields significantly higher r^* estimates and, after 2008, in the wake of the financial crisis, somewhat lower r^* estimates than LW. These differences need to be seen in the light of their different stabilising properties: By construction tracking r^* in LW would have stabilised inflation – but only around its stochastic trend, while tracking our metric would have stabilised inflation in line with the inflation objective. By implication r^* estimates are sensitive to different choices for π^*. Differences in such choices play a more important role during the Great Inflation than in recent decades as monetary policy became explicitly geared to maintain price stability.

The discrepancies in different approaches notwithstanding, all natural rate estimates share

Figure 8: Output gaps in the US: Comparison to LW

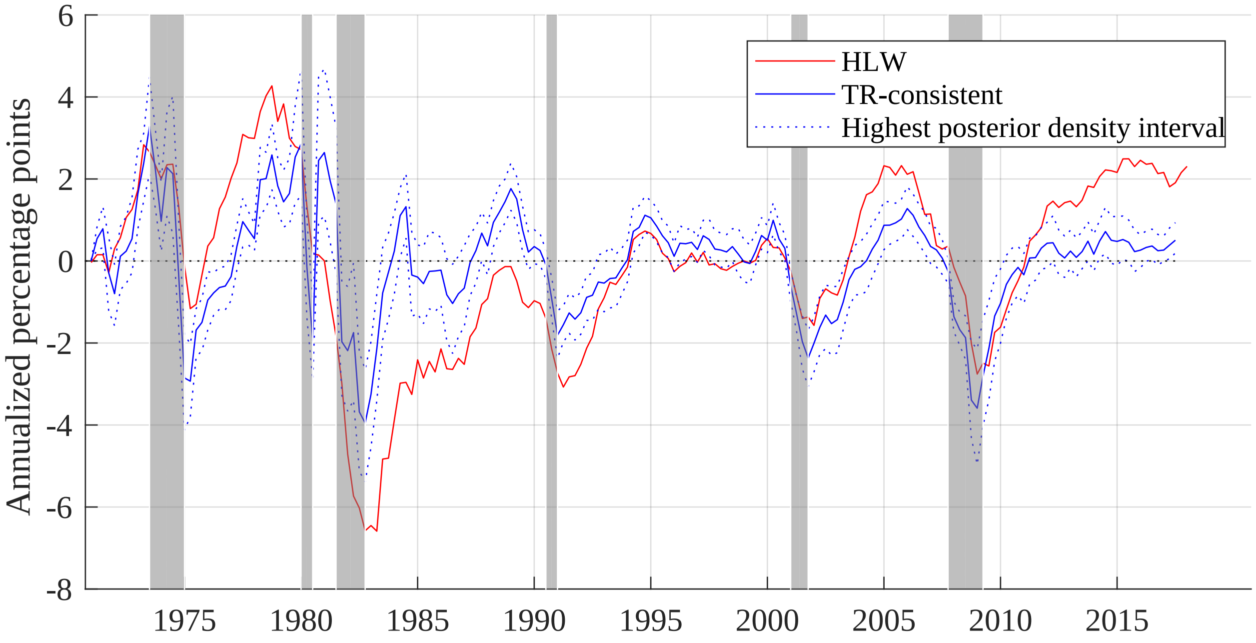


Figure 9: Output gaps in the EA: Comparison to LW

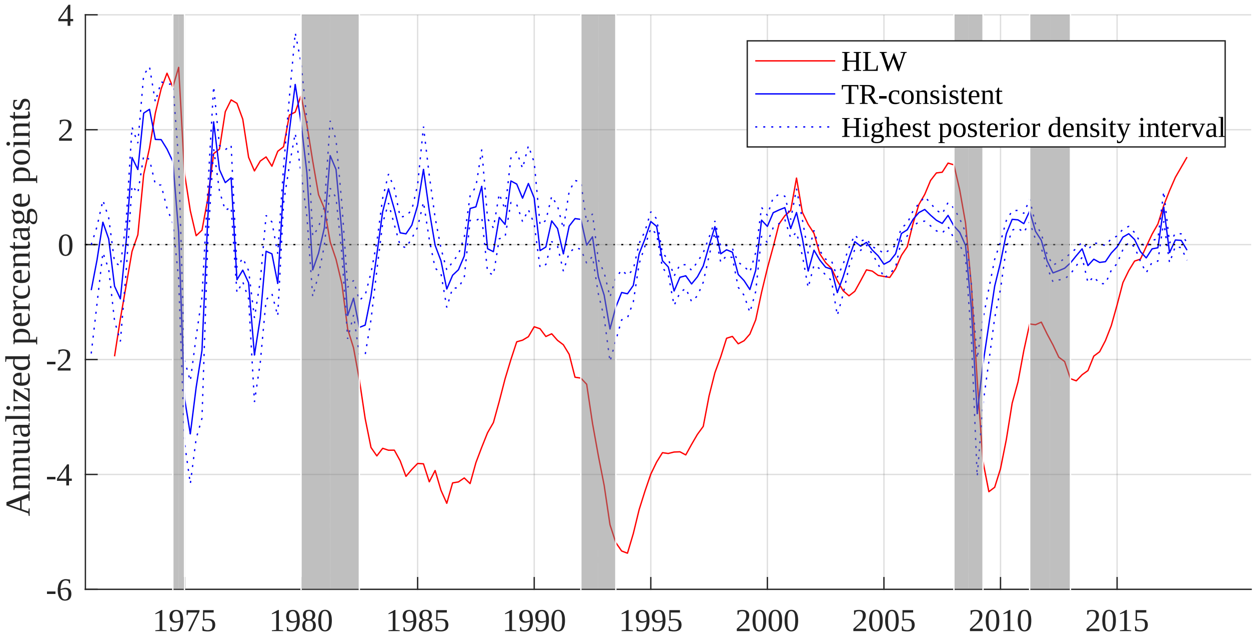


Figure 10: Non-growth components in the US: Comparison to LW

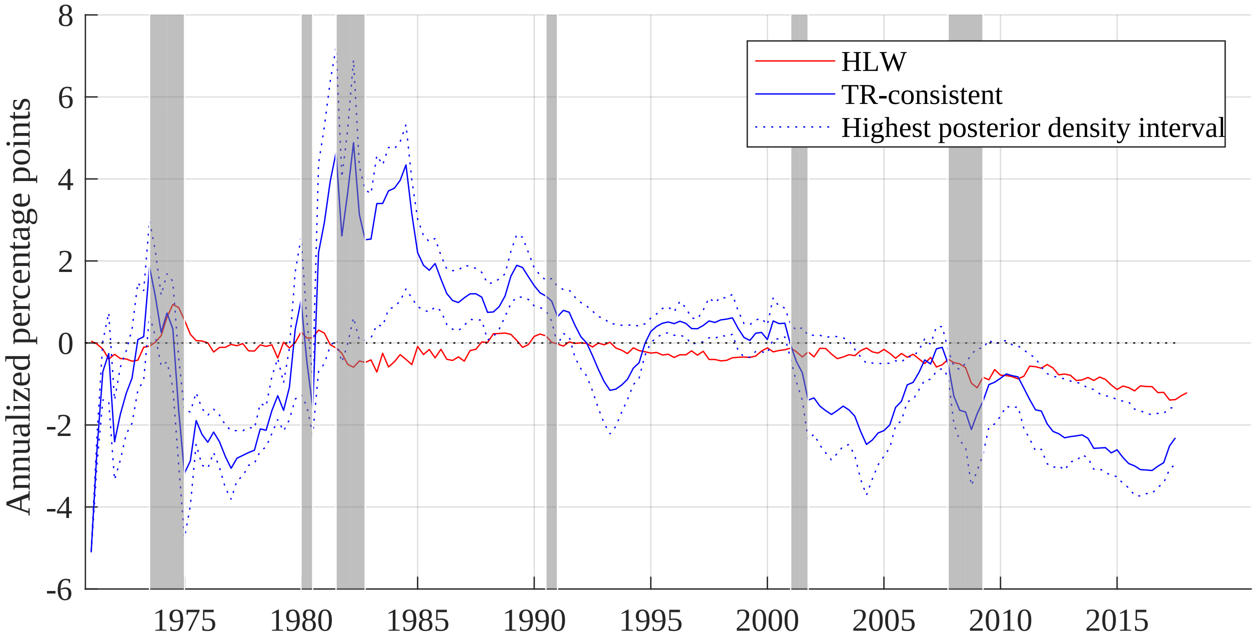
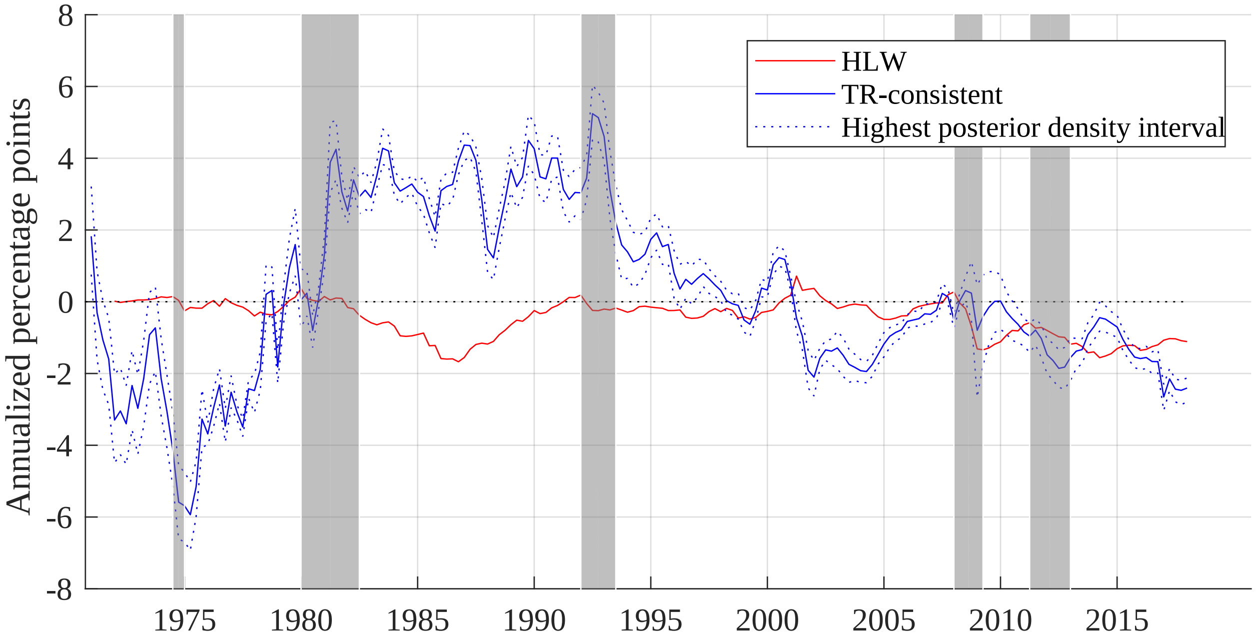


Figure 11: Non-growth components in the EA: Comparison to LW



a long-term decline both in the US and the EA, see Figures 12 and 13, respectively, and a precipitous collapse in the aftermath of the global financial crisis, above and beyond the estimated decline in potential output growth, likely reflecting non-growth related effects, such as flight-to-safety, global saving-investment imbalances, or higher risk aversion, as suggested by Del Negro et al. (2017) and Del Negro et al. (2018).

We show a comparison of real rate gaps from HLW and our approach for the US and the euro area in Figures 14 and 15. By virtue of the stationarity of the Phillips curve and the inclusion of the Taylor rule (in combination with the non-growth component z_t remaining a random walk) our r^* metric tracks the real rate of interest much more closely than the one by HLW. Accordingly, natural rate gaps $r_t - r_t^*$ are much less persistent and exhibit stronger co-movements with the business cycle.

For the US directional differences are most pronounced following the Volcker-disinflation policies in the early 1980s. With the Great Inflation being conquered, our metric assesses monetary policy to have been accommodative during the 1990s, while the HLW metric assesses it to be on the tight side, reflecting ongoing deceleration in inflation rates.

For the euro area similar directional differences are discernible during the first five years of EMU (1999-2004). Through the lens of HLW the initial deceleration and subsequent acceleration in inflation rates implies policy to have been tight and subsequently accommodative. In turn our metric traces the course of policy in the wake of falling commodity prices and other favourable supply side effects and the subsequent reversal in such effects in a more consistent manner: Policy was eased in the context of below-objective inflation, subsequently tightened in the context of robust growth and inflation readings above the upper ceiling on the ECB's definition of price stability (owing among other factors to rising commodity prices and a weak exchange rate), and later on eased only gradually, notwithstanding weakening economic growth.

In the aftermath of the crisis, for the euro area, HLW report monetary policy to have been accommodative ever since 2009, while our real rate gap estimates capture the narrative that policy was eased as of 2009, turned neutral or slightly restrictive during the period of ECB rate hikes in 2011 in the context of strong growth and elevated inflation rates, and was subsequently eased again.

Figure 12: r^* in the US: Comparison to LW

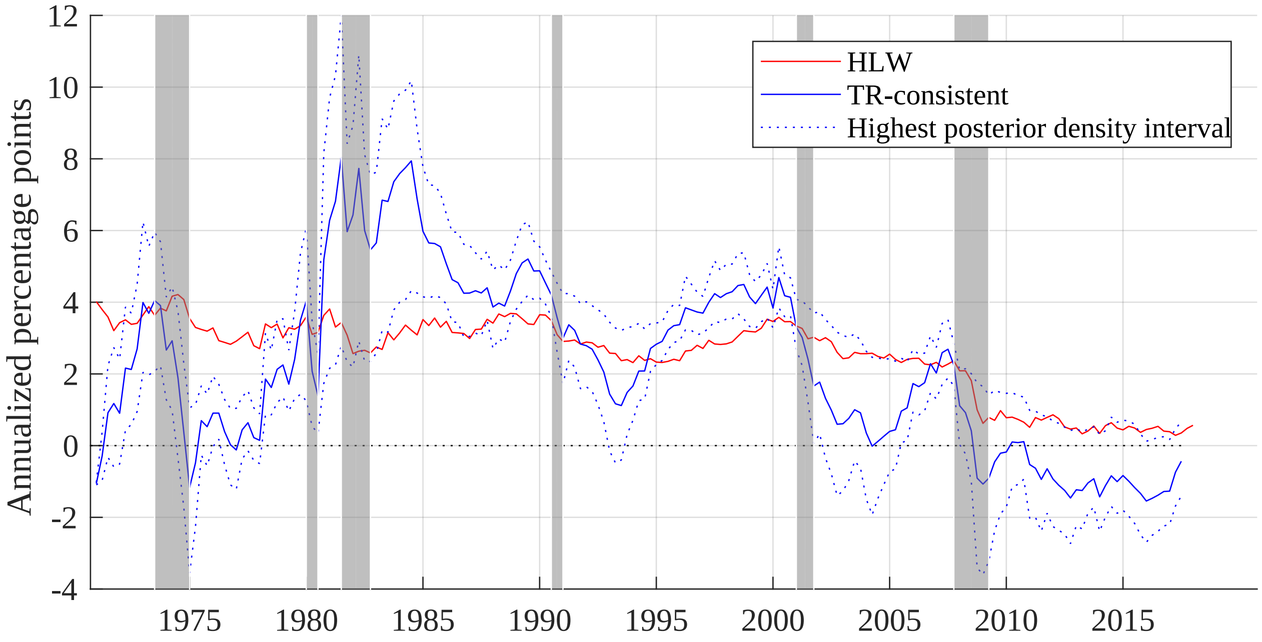


Figure 13: r^* in the EA: Comparison to LW

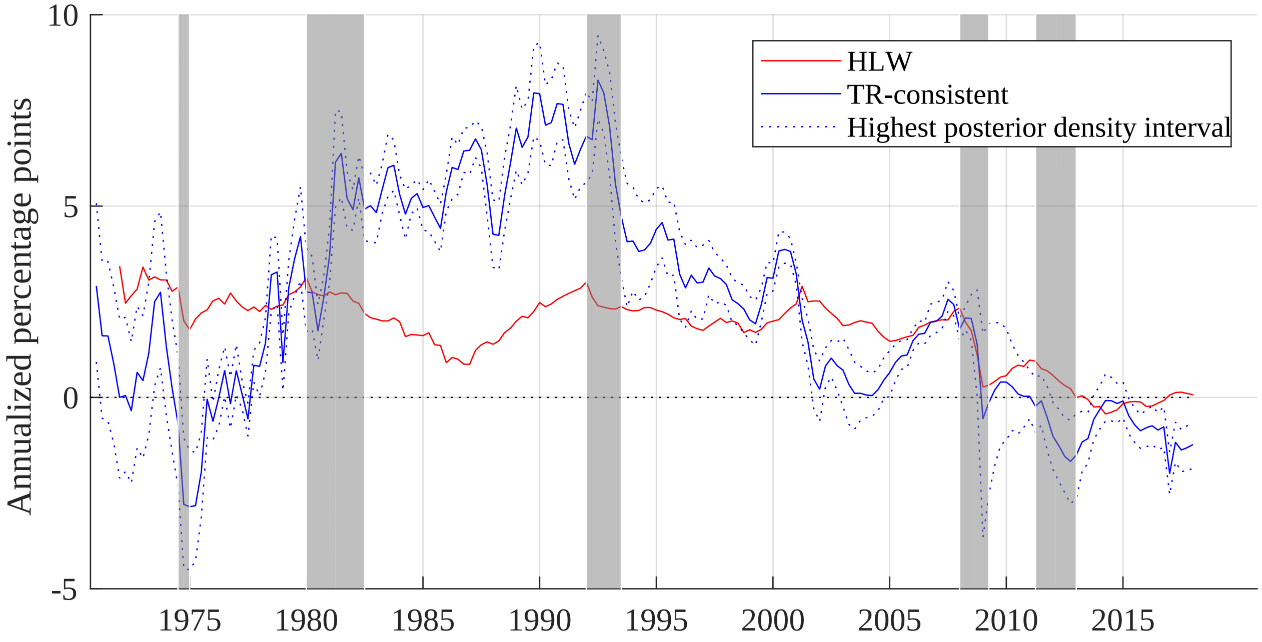
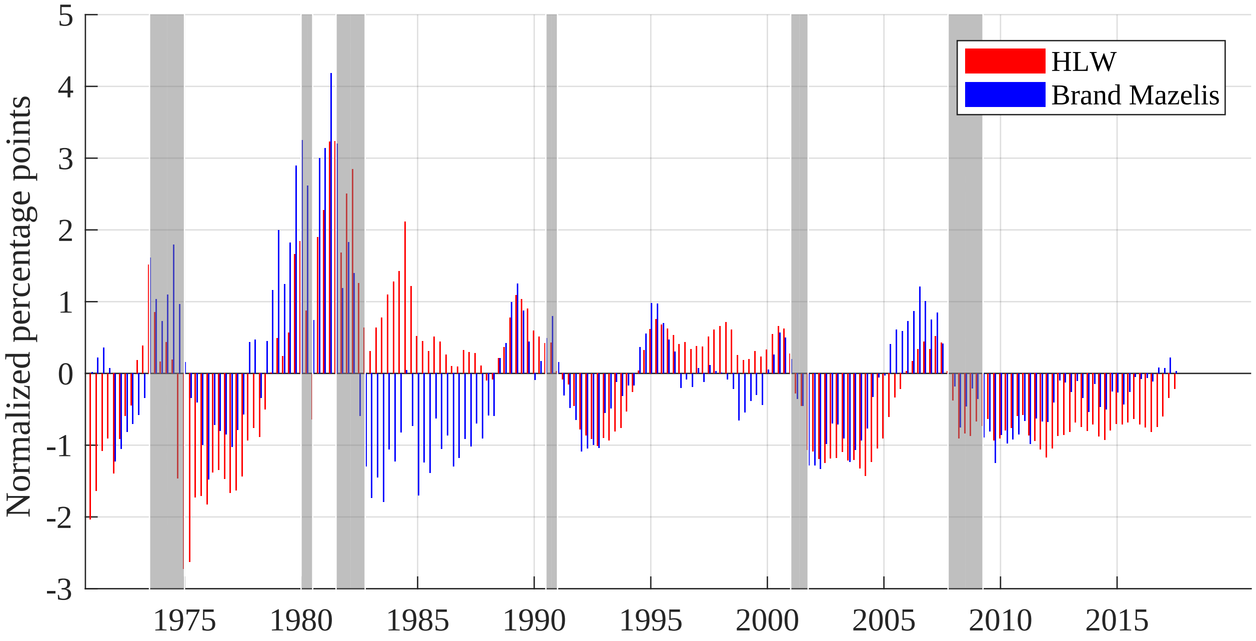
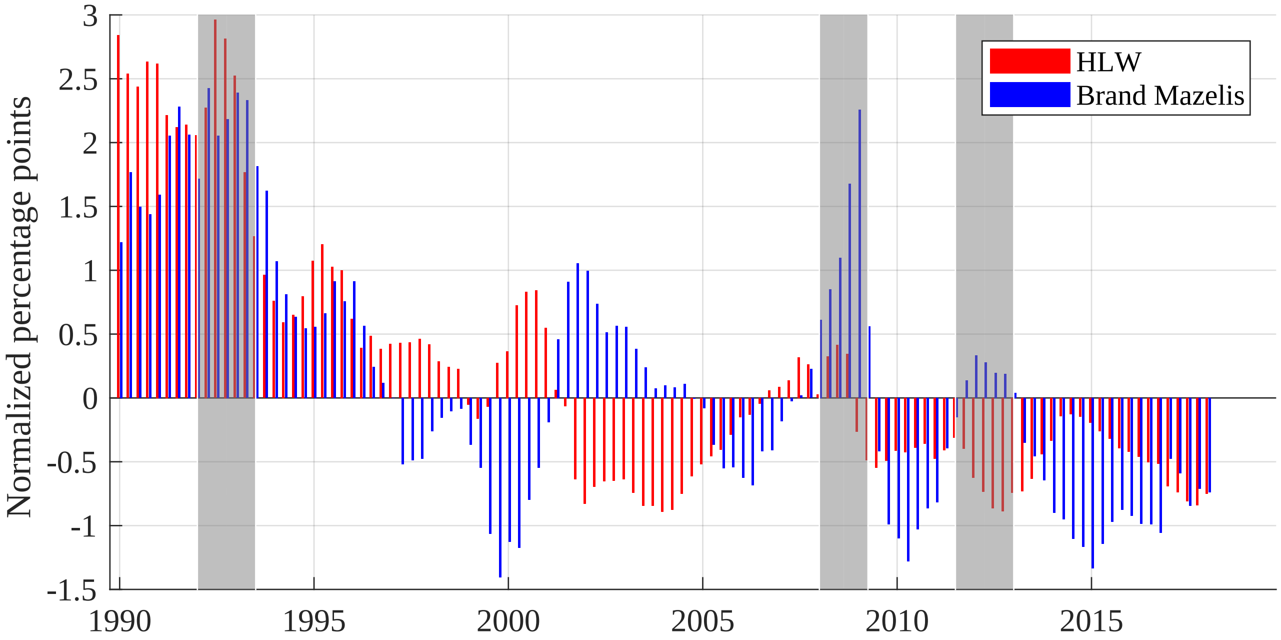


Figure 14: Real rate gaps in the US



***Note:** rate gaps are normalised by the sample standard deviation.*

Figure 15: Real rate gaps in the EA



***Note:** rate gaps are normalised by the sample standard deviation.*

4.2 Comparison with natural rate estimates obtained from using structural models

We compare Taylor-rule consistent r^* estimates with those from structural models for the US, see Figure 16, and the EA, see Figure 17. Gerali and Neri (2017) provide estimates for both economies, while Hristov (2016) focuses on the latter with two different model types. Both figures show that our metric of the natural rate of interest tracks those from structural estimates quite closely, albeit in a somewhat smoother fashion.[5]

Structural models commonly infer the natural rate of interest from a counterfactual simulation that abstracts from 'inefficient' shocks, such as price and wage mark-up shocks, and shocks to the nominal interest rate. For this purpose, observable time series that serve as input are deconstructed into primitive shocks. Depending on the size and feature of the structural model, different structural shocks may serve as explanatory drivers.

A disadvantage of the structural approach is that the dynamics of the model are dictated by the micro-founded approach which tends to deliver a high degree of internal consistency, but may at times lead to an inadequate description of the data. In case of misspecification, primitive shocks may be incorrectly attributed, thereby masking the true realization of the unobservables. Our semi-structural approach bypasses this micro-founded framework in favor of a more agnostic one. Differences in the latent variables should therefore be expected and, based on their manifestations, may allow inference on the relative influence of different shock types.

Particularly interesting is the analysis of the non-growth component z_t, which is an aggregate of different structural factors. One of these is the preference for safe assets, which has a structural counterpart in the risk premium shock in the model by Gerali and Neri (2017). In the euro area σ_z is estimated to be larger than in the US (and conversely σ_g to be smaller). In our analysis, the non-growth component is comparatively important to describe the elevated level of r^* during the 1980s and early 1990s and drives the natural rate higher than estimated by LW (Figure 10 and 11). A large part of the hump-shaped increase in r^* during this episode might be due to the use of synthetic interest rates for the euro area: this variable is strongly affected by exchange rate and inflation risk premia in the context of recurring EMS crises.

According to the analysis by Gerali and Neri (2017) the risk premium shock is also relatively more important for the euro area than for the US. For the US, the explanatory power of the risk

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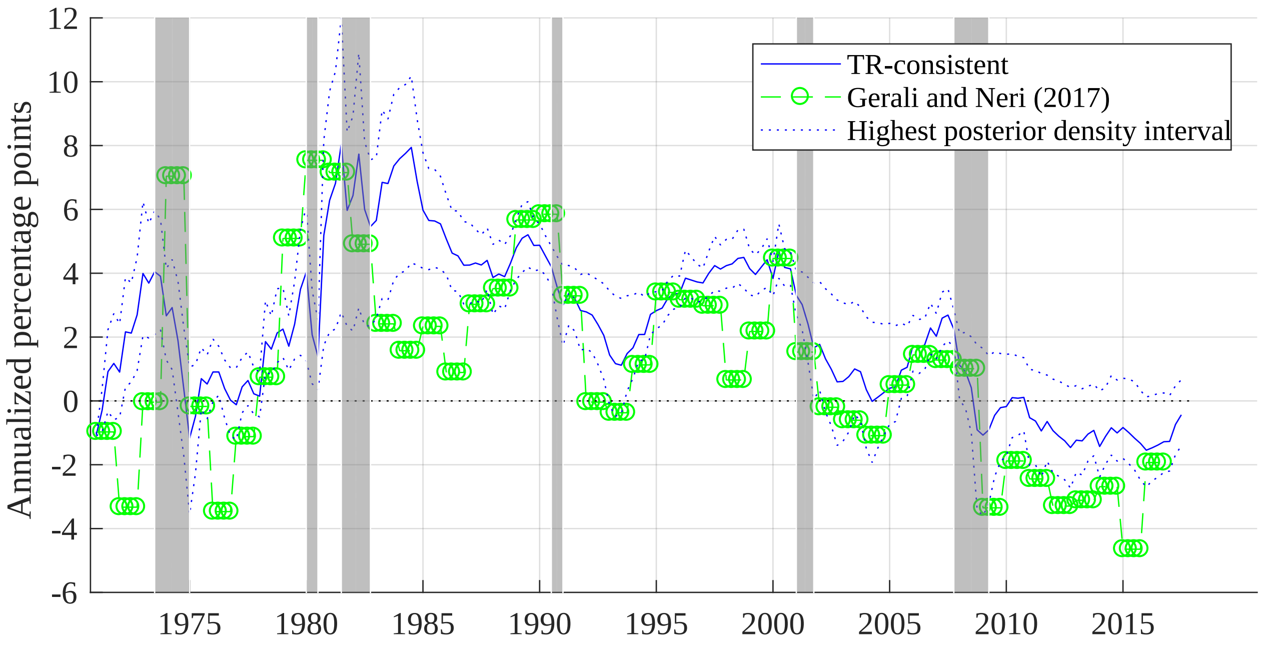
[5] Note that we display filtered estimates throughout. Two-sided estimates would by definition smooth out more volatile periods.

Figure 16: r^* in the US: Comparison to structural estimates



Note: *Gerali and Neri (2017) only provide annual data for US estimates of the natural rate of interest.*

premium shock is only of secondary importance for variations in r^*. In their analysis, shocks to the marginal efficiency of investment (which can be seen as a proxy for credit frictions) make up a larger part.

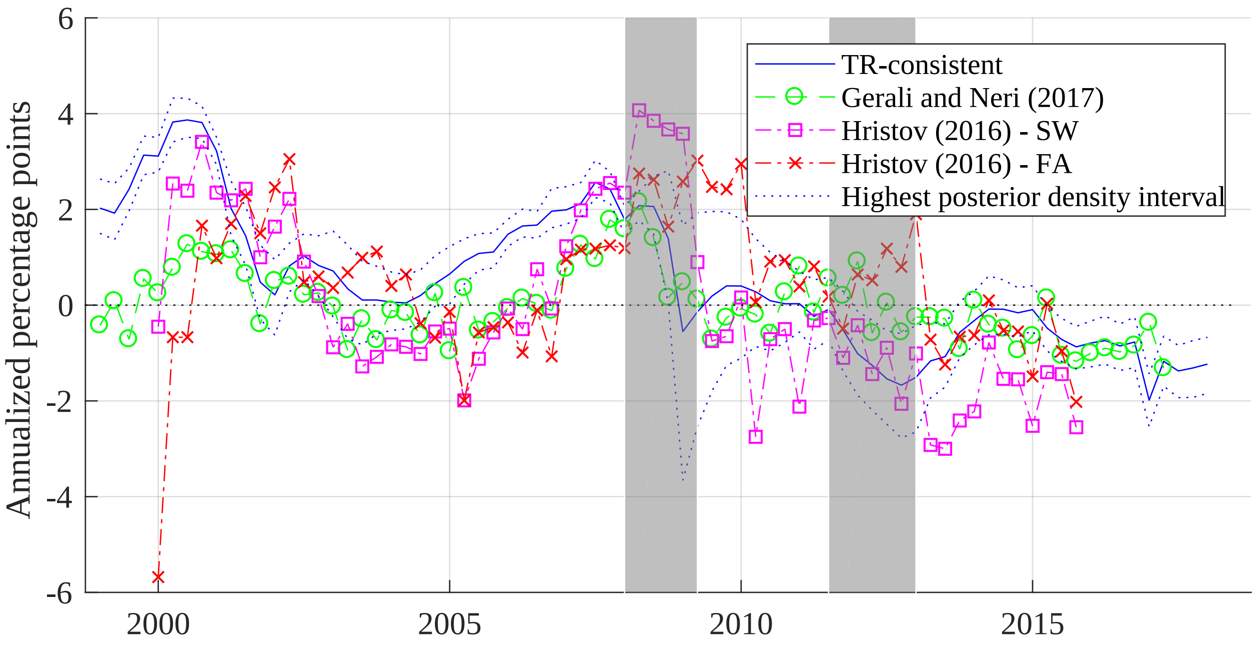
Overall, our estimates of the relative importance of the non-growth component z_t compared to the growth component g_t corroborate the evidence identified by Gerali and Neri (2017) that a risk-premium component appears to have been a more important driving force of the natural rate in synthetic euro area data than in the US data.

With this background of the different methodological approaches in mind, we find that including a Taylor rule in semi-structural approaches like LW seems pivotal to achieve greater consistency with structural estimates of r^* – both in terms of its downward trend and eventual drop into negative territory in the wake of the global financial crisis.

4.3 Comparison with institutional output gap estimates

As expected when allowing for correlations in latent factor shocks, our model-dependent output gap estimates are more volatile: The closer resemblance to Beveridge-Nelson cycles typi-

Figure 17: r^* in the EA: Comparison to structural estimates



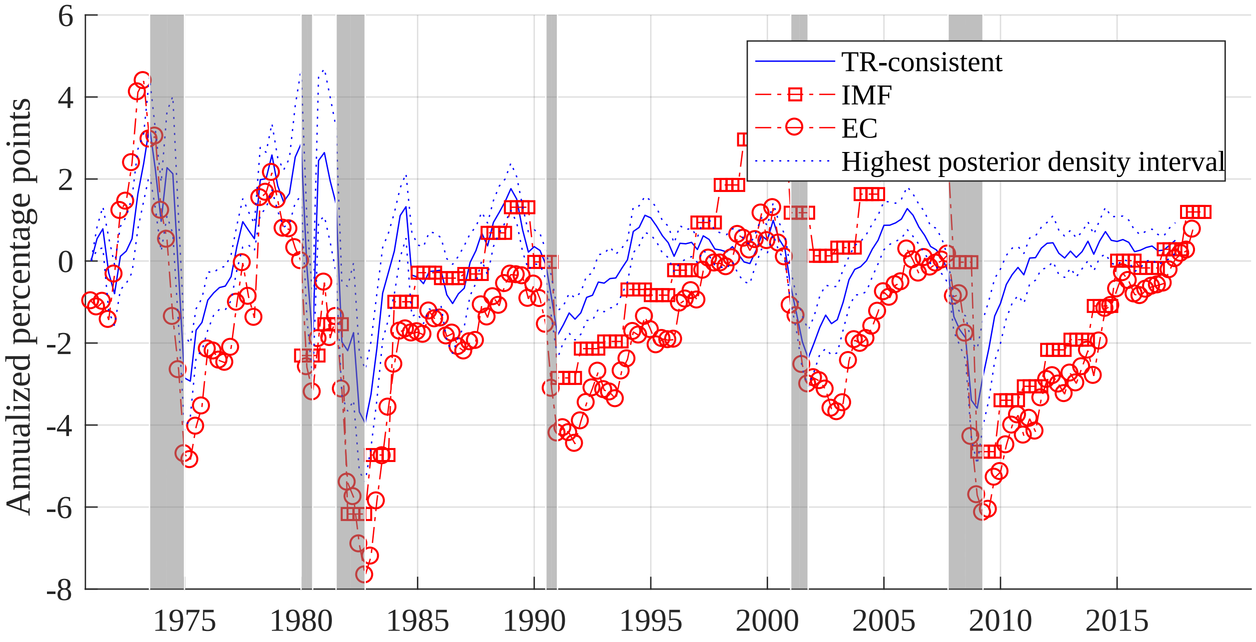
Note: *For Hristov (2016), SW refers to the plain Smets-Wouters type model, while FA refers to the Smets-Wouters model with a financial accelerator.*

cally imparts higher volatility to the cyclical component, as stochastic drifts are more important drivers of output fluctuations as discussed in Morley et al. (2003).

Figures 18 and 19 plot model-specific output gaps against institutional ones. While the model-specific estimates co-move with institutional ones and, by and large, there is consistency in the timing of business cycle turning points, there are visible differences in the size of slack at specific points in time: in the aftermath of the crisis, the model estimates slack to be more swiftly absorbed than official estimates. In particular, in the euro area the second recession following the sovereign debt crisis is estimated to have caused a much smaller negative output gap than estimated by the IMF or the European Commission.

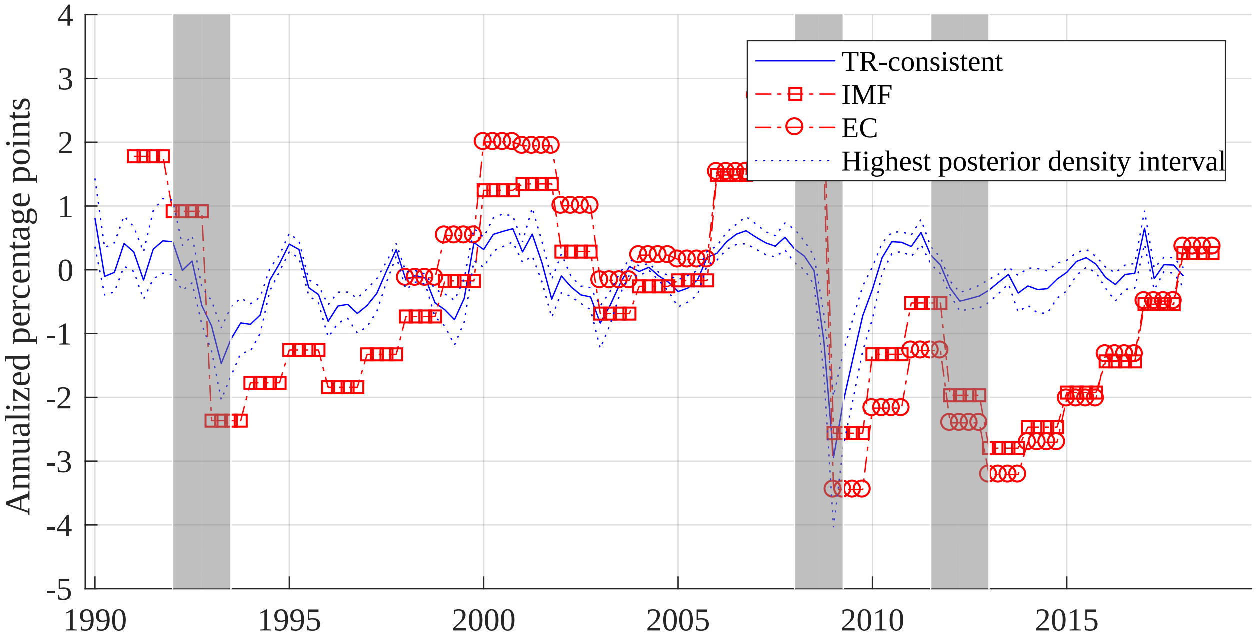
Appendix A provides a comparative overview of latent factor estimates from model variants including or excluding the labour market (as in Equation 10) and from allowing latent factor shocks to correlate or not. All estimates co-move strongly, but show that variations in the size of output or unemployment gaps measures depend on whether latent factor shocks are allowed to correlate or not. Including the labor market in our framework further aligns the output gap series with institutional estimates, especially towards the post-Financial Crisis years in the US.

Figure 18: Output gaps in the US: Comparison to institutional measures



Source: *Congressional Budget Office (CBO); BEA; International Monetary Fund (IMF).* ***Note****: In percent of potential output. The CBO output gap is constructed as the difference between their potential output estimate and the observed GDP series provided by the Bureau of Economic Analysis (BEA).*

Figure 19: Output gaps in the EA: Comparison to official measures



Source: *European Commission (EC); International Monetary Fund (IMF).* ***Note:*** *In percent of potential output.*

5 Conclusion

We have provided estimates of the natural rate of interest for the US and the euro area obtained from a simple, closed, semi-structural model comprising an aggregate demand curve, a non-accelerationist Phillips curve, and a Taylor rule with nominal interest rate smoothing. Compared to LW, the model is more akin to the canonical 3-equation New-Keynesian model, and estimates of the natural rate of interest are more aligned with structural ones.

Relative to the approach by LW who specify the real rate of interest to be an exogenous process, our estimates are more consistent with conventional policy rules. First, we use a non-accelerationist Phillips curve and, hence, impose stationarity of inflation around its objective, as required in a Taylor rule. Second, we obtain all latent factor estimates from one coherent modelling framework, obviating the combination of natural rate estimates with inconsistent output gap measures obtained from extraneous sources.

We find significant differences to natural rate estimates by LW during episodes when inflation was persistently high, likely reflecting their choice of an accelerationist Phillips curve. Particularly, during those episodes, we find similarities with estimates obtained from structural macroeconomic models, as, for example, in Gerali and Neri (2017). Since the mid-1990s, as monetary policy became better geared to achieve low and stable inflation, the differences of our estimates with those by LW appear much less important.

Our model-dependent output-gap estimates correlate with institutional ones, but allowing for cross-correlations in shocks yields a rather volatile metric, as variations in output are constructed to be rooted more firmly in stochastic drifts. Yet allowing for such cross-correlations is supported by the data, makes the semi-structural approach more amenable to structural interpretation and helps to reduce filtering uncertainty arising from flat aggregate demand and Philipps curves – the latter having been identified as a key weakness in estimating r^* in common semi-structural approaches (Fiorentini et al., 2018).

Overall, we validate the common finding that the natural rate of interest has fallen throughout the past two decades and that it has slumped in the wake of the financial crisis. This decline cannot entirely be accounted for by a lower growth rate in potential output. But beyond these broad trends, it is difficult to provide statistically reliable estimates of r^* at business cycle frequency: uncertainty in natural rate estimates obtained from this modelling class has been shown to be of staggering size. This high degree of statistical uncertainty is model-inherent. It complicates the use of natural rate estimates in real time for policy assessment, as well as a statistical assessment of competing model specification choices.

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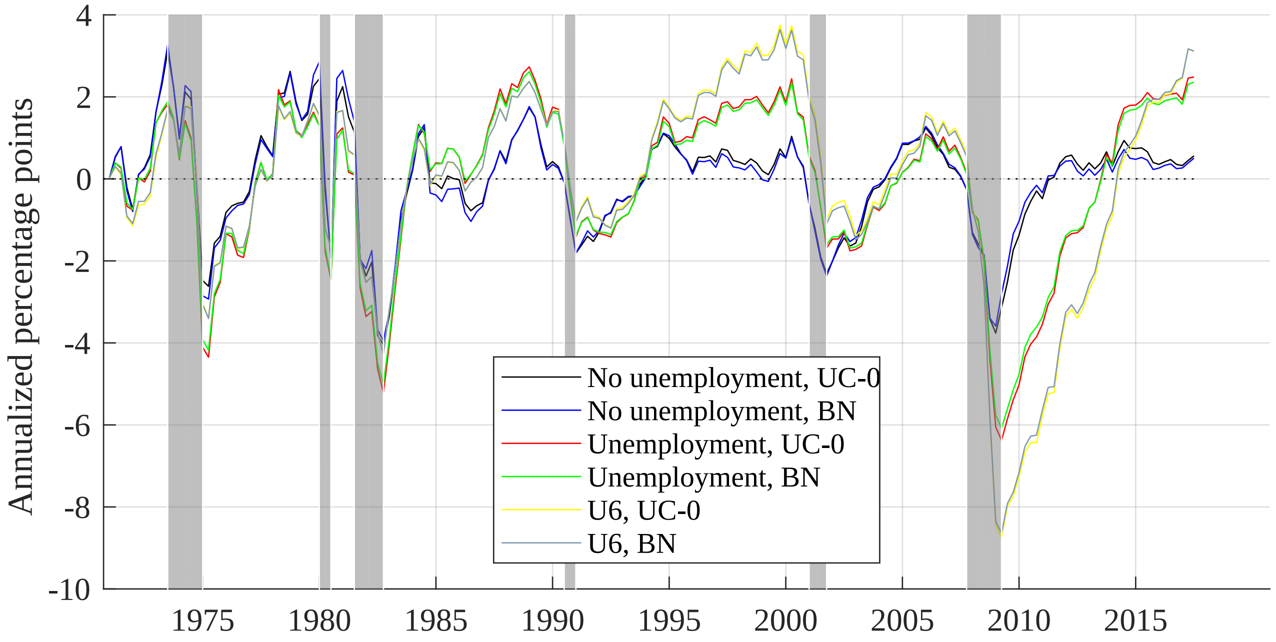
Appendix A Comparison across different model specifications

This appendix provides a comparative overview of latent factor estimates from model variants including or excluding the labour market (as in Equation 10) and from allowing latent factor shocks to correlate or not.

Regarding employment data in these exercises, for US unemployment we use total unemployment, plus all marginally attached workers plus total employed part-time employees from the U.S. Bureau of Labor Statistics. This series only goes back until 1994, but generally behaves like the regularly reported unemployment rate with a level differential. We therefore extend our unemployment rate with the regularly reported one, corrected by the level differential in 1994:Q1. For EA unemployment we employ a similarly broad measure of unemployment and extend it to the beginning of our sample by combining it with the regular unemployment series, corrected by the differential to broad unemployment in 2005:Q1 (referred to as 'U6' in the chart legends).

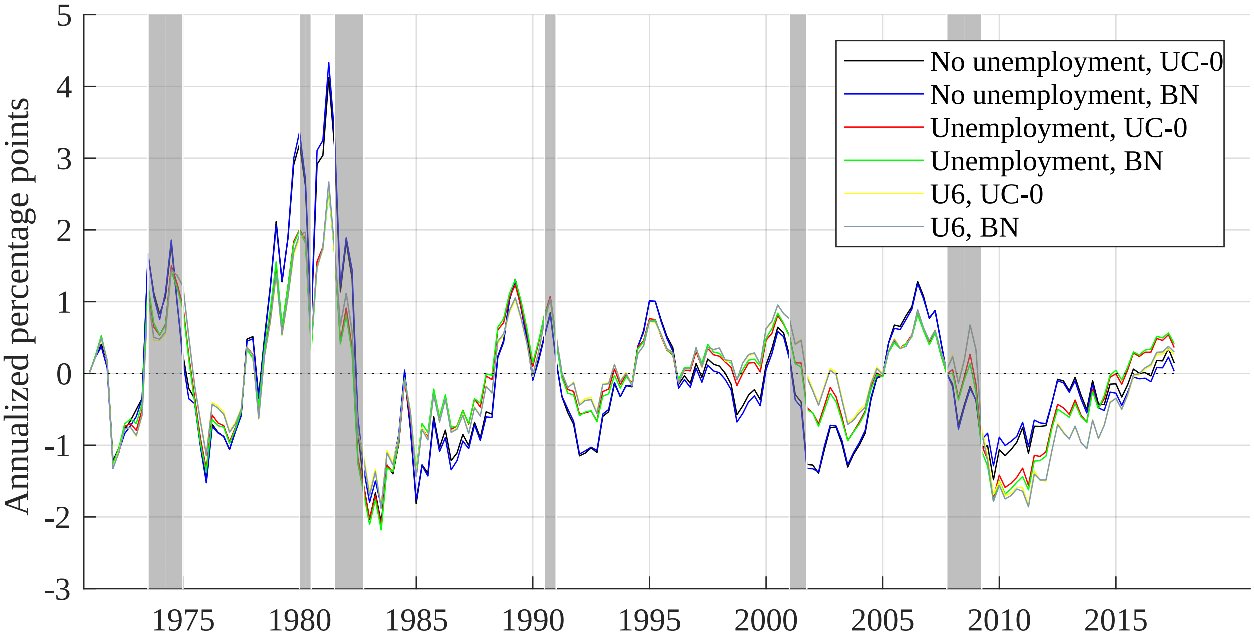
In the following figures 'no unemployment' refers to the 'preferred' model version without the labour market (featured in most of the analysis presented in the main body of this paper), 'UC-0' refers to the model without correlations in latent factor shocks, 'BN' refers to the version allowing for such correlations, as in the Beveridge-Nelson decomposition. Specifically, these correlations capture interactions between cyclical components and their respective counterparts in levels.

Figure 20: Comparison across output gaps for the US



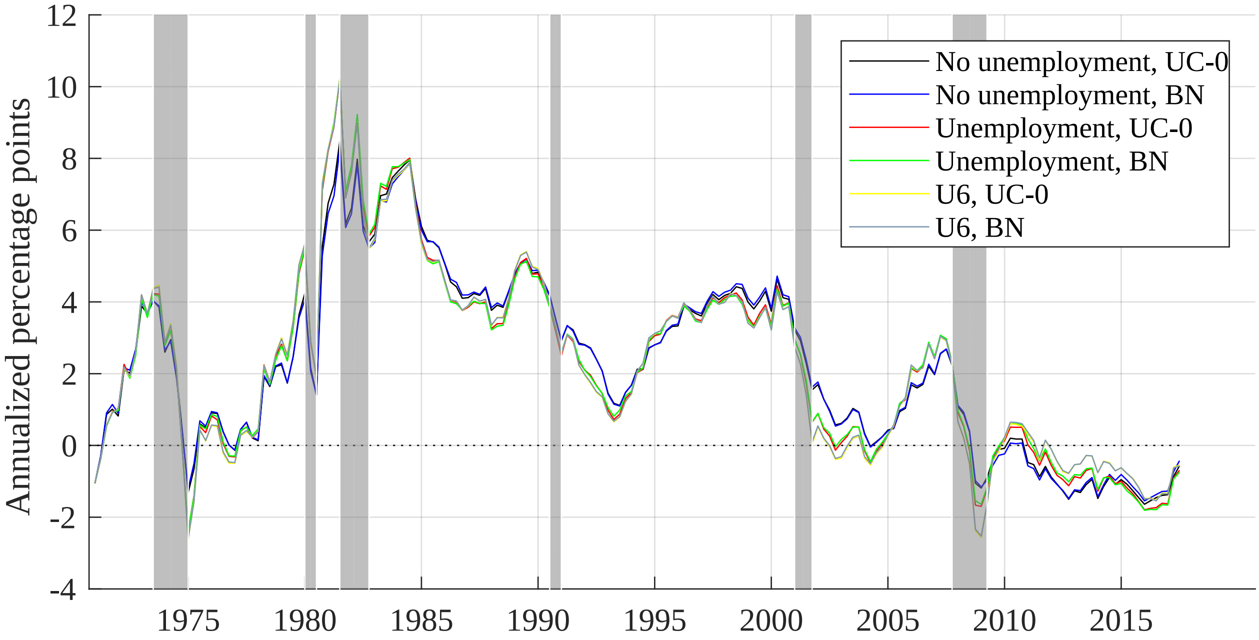
***Note**: UC-0 refers to the plain unobserved components model, while BN refers to the (Beveridge-Nelson) specification without restrictions of uncorrelated trend and cycle innovations. Unemployment refers to the specification including unemployment in the Phillips-Curve. U6 uses an unemployment rate that includes discouraged workers, all other marginally attached workers and those workers who are part-time purely for economic reasons.*

Figure 21: Comparison across rate gaps for the US



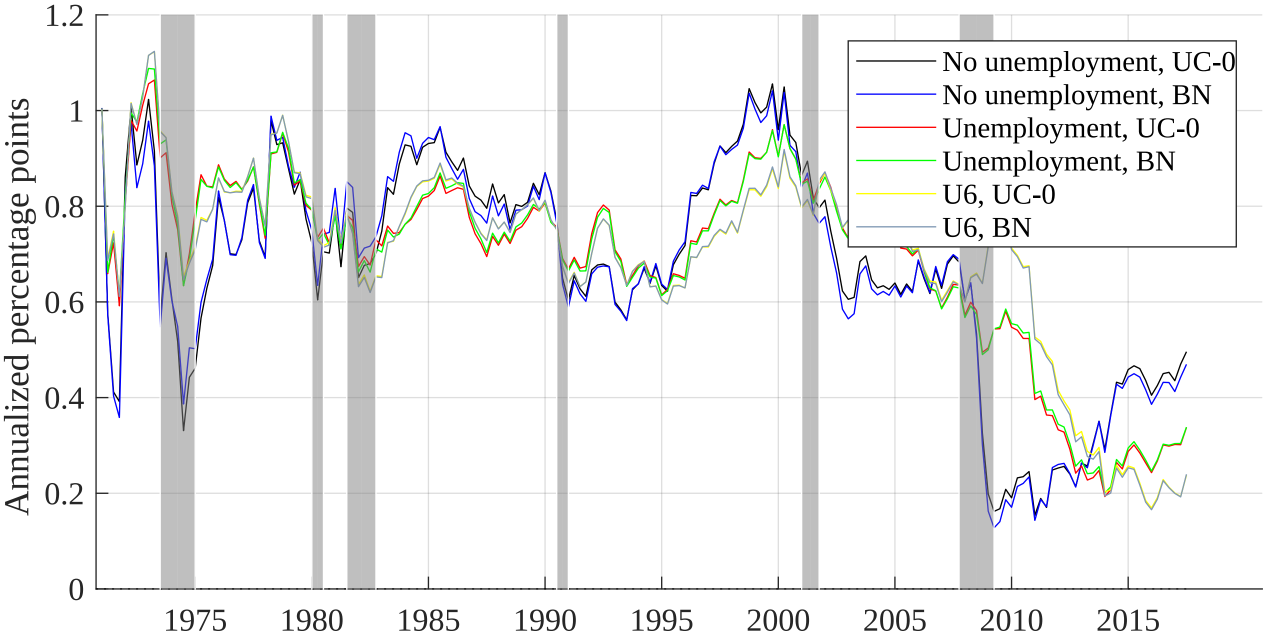
***Note**: UC-0 refers to the plain unobserved components model, while BN refers to the (Beveridge-Nelson) specification without restrictions of uncorrelated trend and cycle innovations. Unemployment refers to the specification including unemployment in the Phillips-Curve. U6 uses an unemployment rate that includes discouraged workers, all other marginally attached workers and those workers who are part-time purely for economic reasons.*

Figure 22: Comparison across natural rates for the US



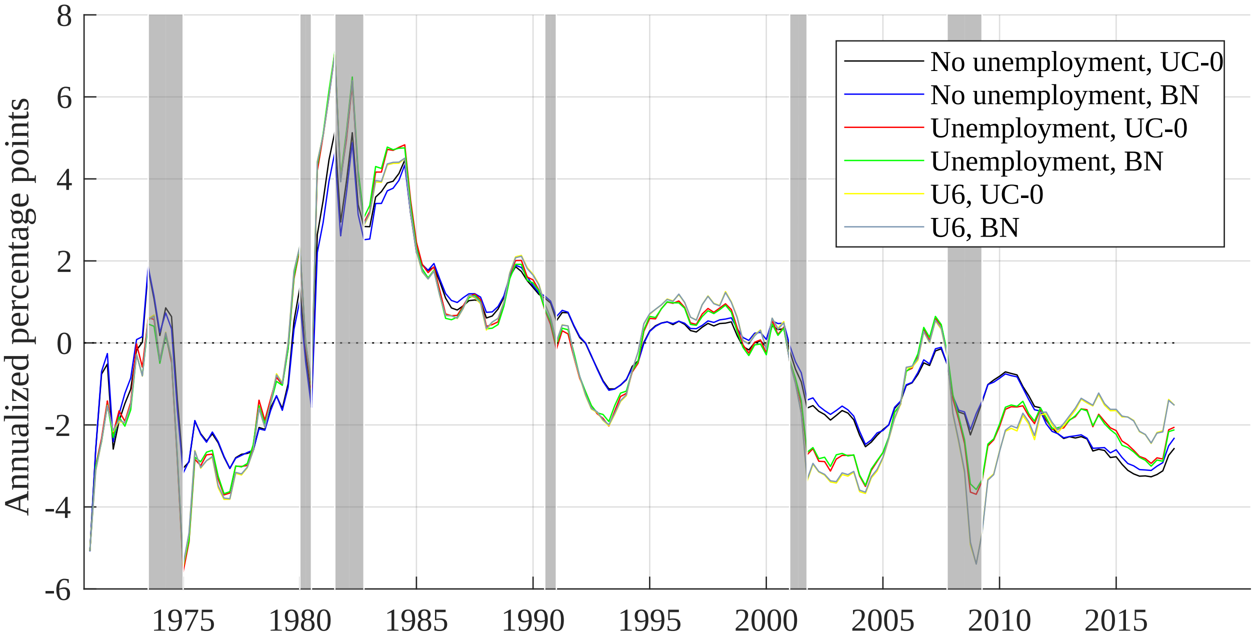
***Note**: UC-0 refers to the plain unobserved components model, while BN refers to the (Beveridge-Nelson) specification without restrictions of uncorrelated trend and cycle innovations. Unemployment refers to the specification including unemployment in the Phillips-Curve. U6 uses an unemployment rate that includes discouraged workers, all other marginally attached workers and those workers who are part-time purely for economic reasons.*

Figure 23: Comparison across growth rates for the US



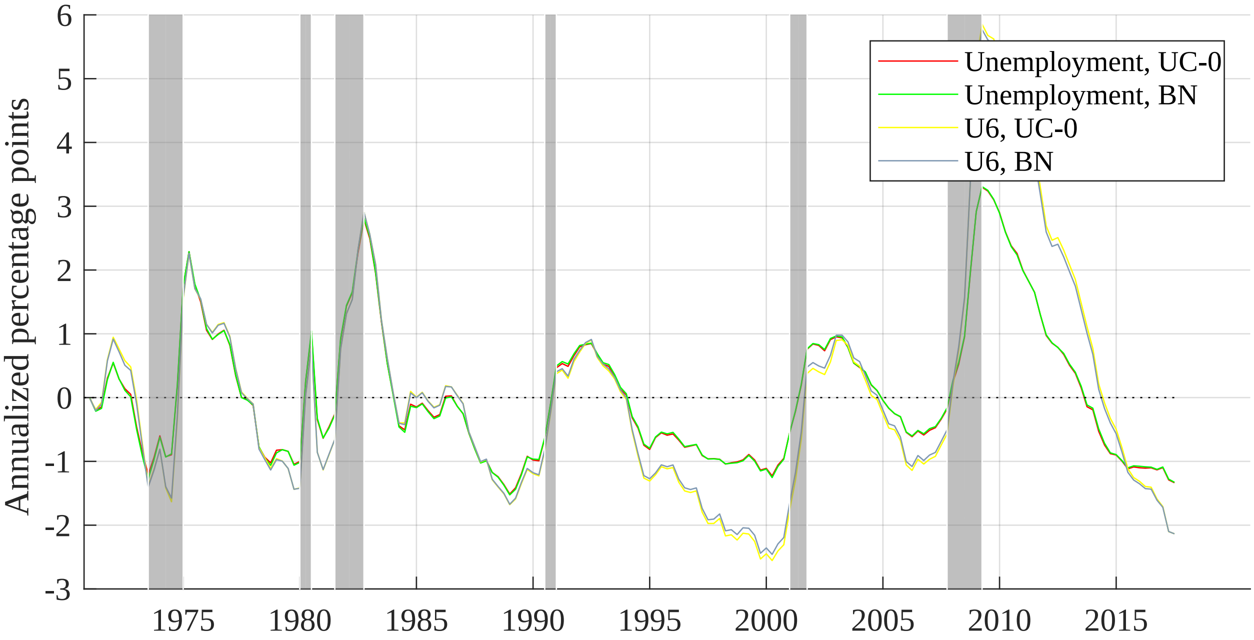
Note: *UC-0 refers to the plain unobserved components model, while BN refers to the (Beveridge-Nelson) specification without restrictions of uncorrelated trend and cycle innovations. Unemployment refers to the specification including unemployment in the Phillips-Curve. U6 uses an unemployment rate that includes discouraged workers, all other marginally attached workers and those workers who are part-time purely for economic reasons.*

Figure 24: Comparison across other determinants (z) for the US



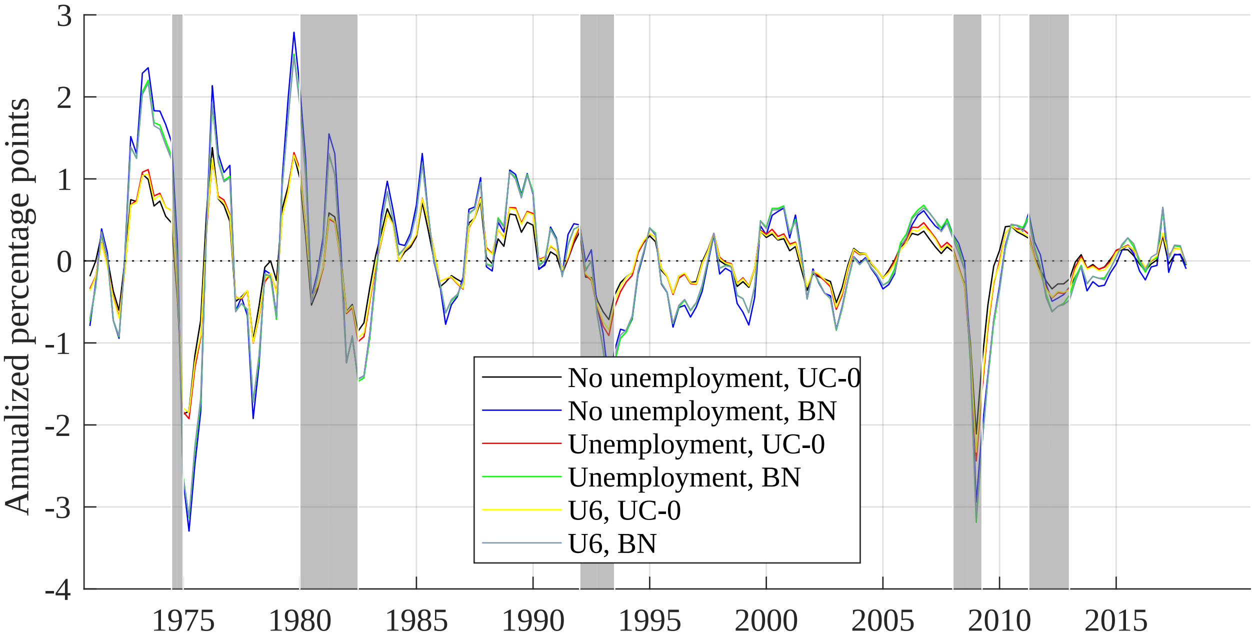
***Note**: UC-0 refers to the plain unobserved components model, while BN refers to the (Beveridge-Nelson) specification without restrictions of uncorrelated trend and cycle innovations. Unemployment refers to the specification including unemployment in the Phillips-Curve. U6 uses an unemployment rate that includes discouraged workers, all other marginally attached workers and those workers who are part-time purely for economic reasons.*

Figure 25: Comparison across unemployment gaps for the US



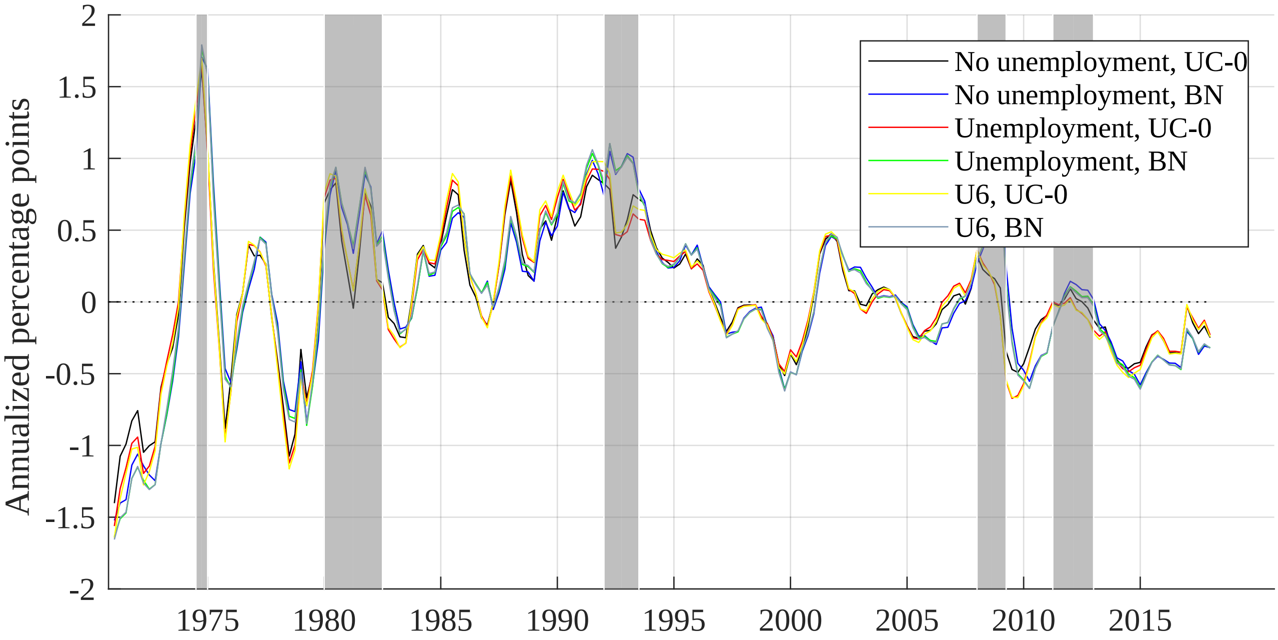
Note: *UC-0 refers to the plain unobserved components model, while BN refers to the (Beveridge-Nelson) specification without restrictions of uncorrelated trend and cycle innovations. Unemployment refers to the specification including unemployment in the Phillips-Curve. U6 uses an unemployment rate that includes discouraged workers, all other marginally attached workers and those workers who are part-time purely for economic reasons.*

Figure 26: Comparison across output gaps for the EA



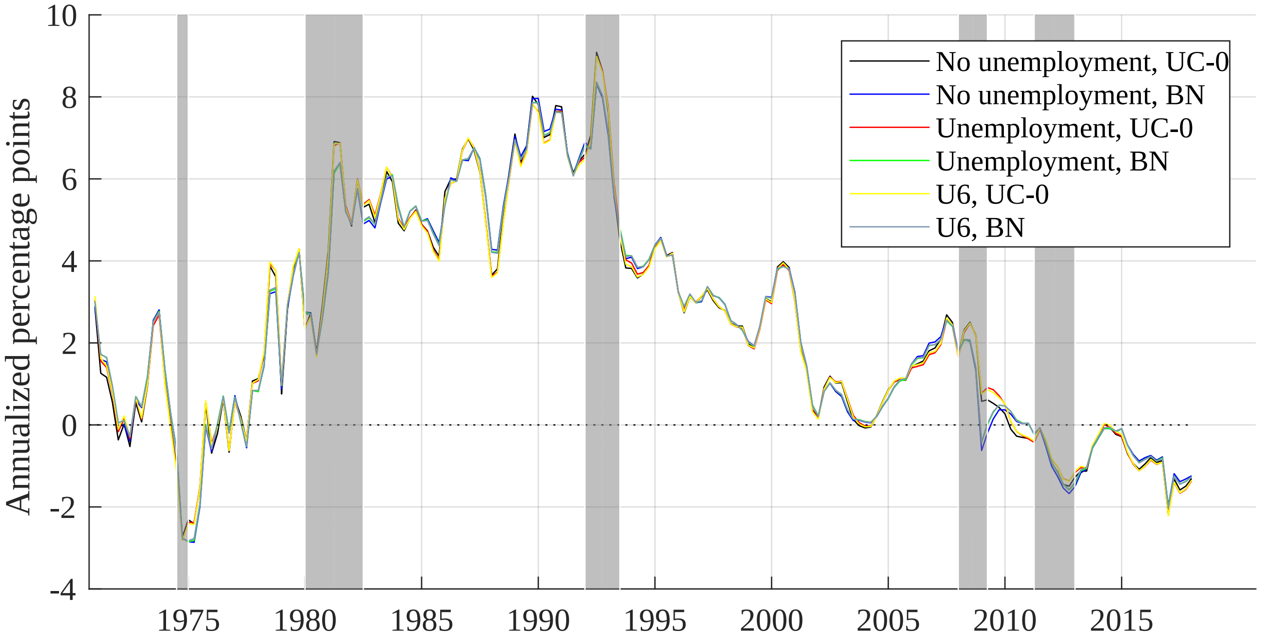
Note: *UC-0 refers to the plain unobserved components model, while BN refers to the (Beveridge-Nelson) specification without restrictions of uncorrelated trend and cycle innovations. Unemployment refers to the specification including unemployment in the Phillips-Curve. U6 uses an unemployment rate that includes discouraged workers, all other marginally attached workers and those workers who are part-time purely for economic reasons.*

Figure 27: Comparison across rate gaps for the EA



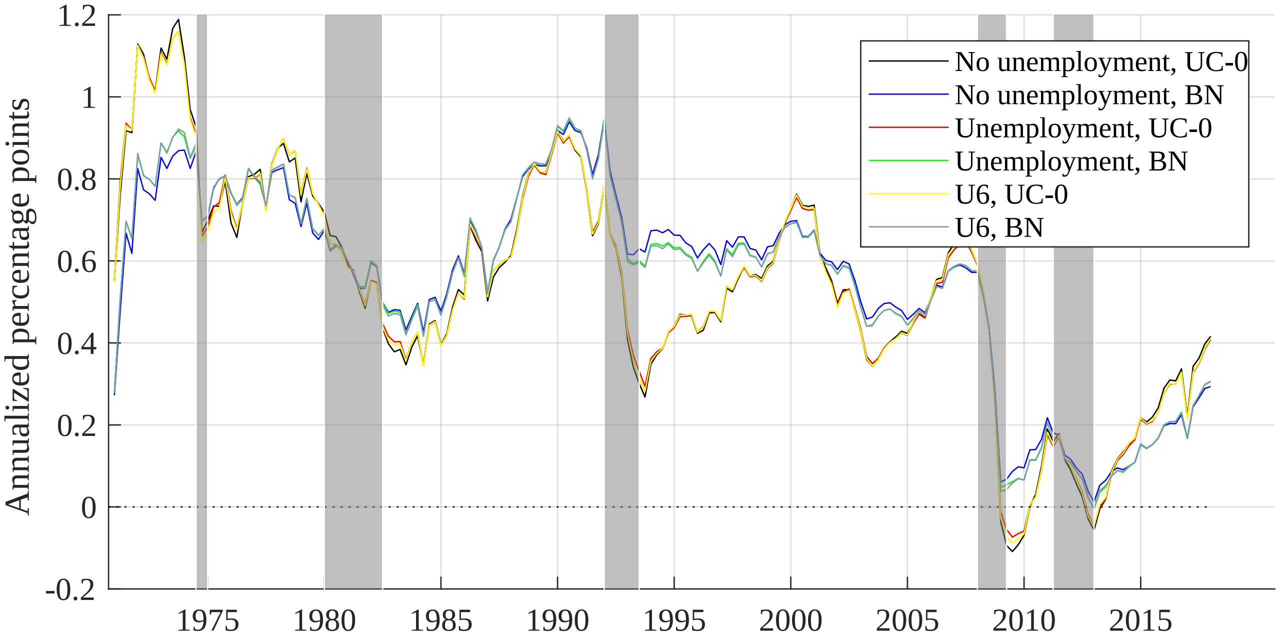
Note: UC-0 refers to the plain unobserved components model, while BN refers to the (Beveridge-Nelson) specification without restrictions of uncorrelated trend and cycle innovations. Unemployment refers to the specification including unemployment in the Phillips-Curve. U6 uses an unemployment rate that includes discouraged workers, all other marginally attached workers and those workers who are part-time purely for economic reasons.

Figure 28: Comparison across natural rates for the EA



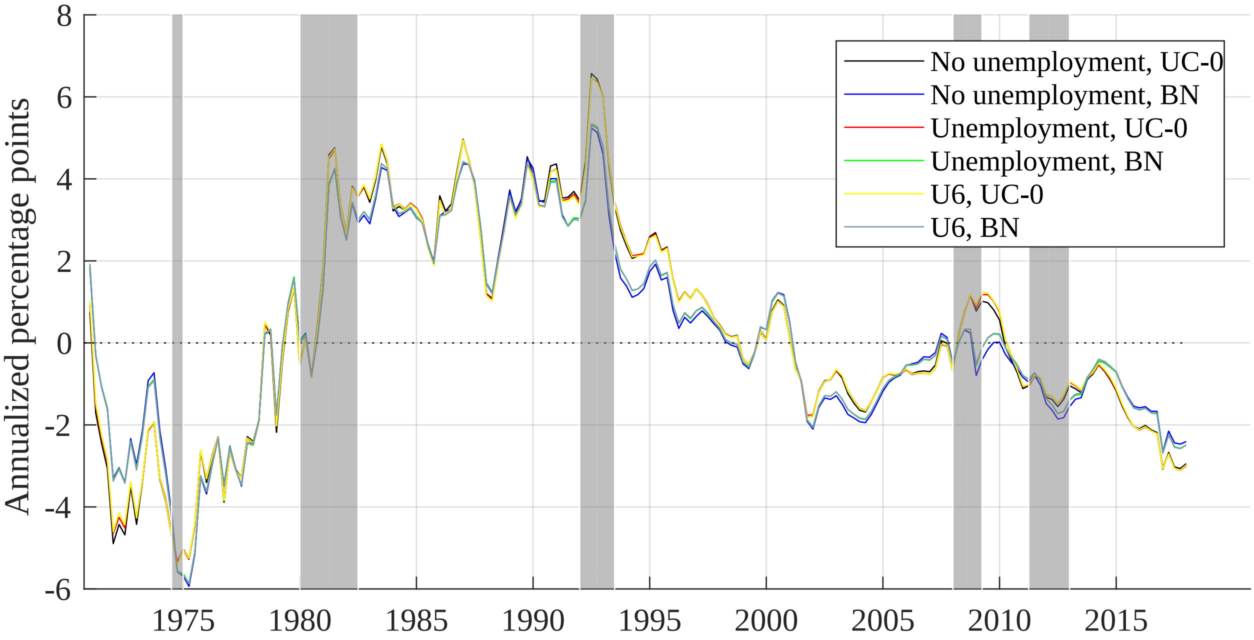
***Note**: UC-0 refers to the plain unobserved components model, while BN refers to the (Beveridge-Nelson) specification without restrictions of uncorrelated trend and cycle innovations. Unemployment refers to the specification including unemployment in the Phillips-Curve. U6 uses an unemployment rate that includes discouraged workers, all other marginally attached workers and those workers who are part-time purely for economic reasons.*

Figure 29: Comparison across growth rates for the EA



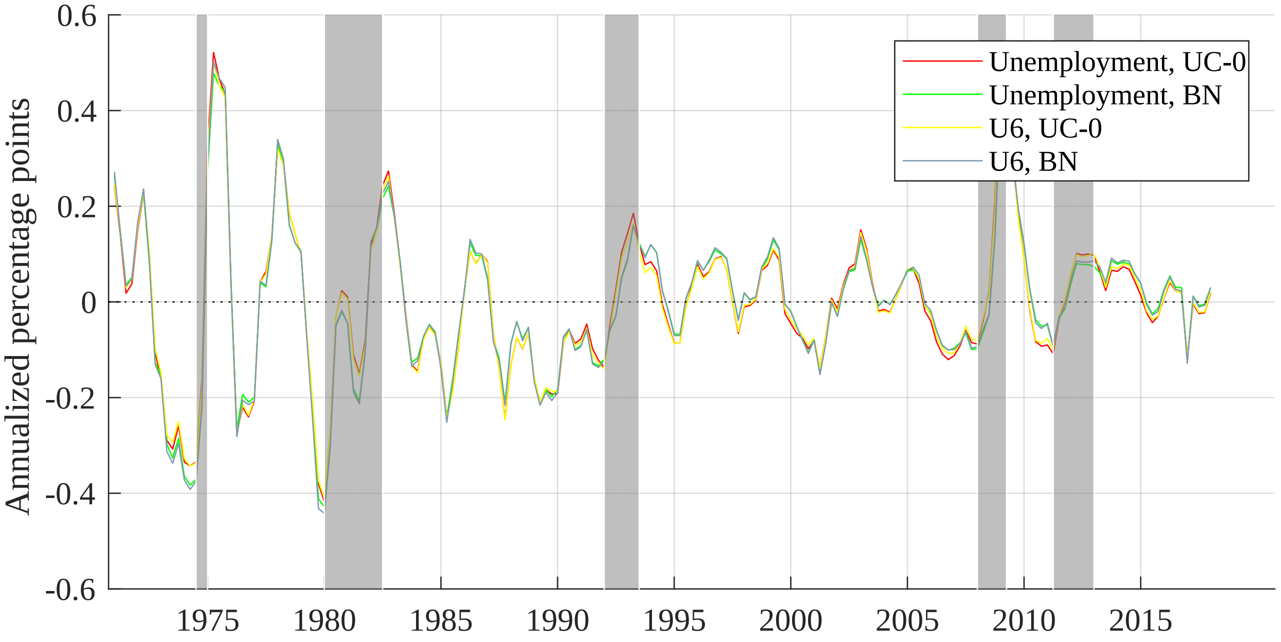
***Note**: UC-0 refers to the plain unobserved components model, while BN refers to the (Beveridge-Nelson) specification without restrictions of uncorrelated trend and cycle innovations. Unemployment refers to the specification including unemployment in the Phillips-Curve. U6 uses an unemployment rate that includes discouraged workers, all other marginally attached workers and those workers who are part-time purely for economic reasons.*

Figure 30: Comparison across other determinants (z) for the EA



Note: *UC-0 refers to the plain unobserved components model, while BN refers to the (Beveridge-Nelson) specification without restrictions of uncorrelated trend and cycle innovations. Unemployment refers to the specification including unemployment in the Phillips-Curve. U6 uses an unemployment rate that includes discouraged workers, all other marginally attached workers and those workers who are part-time purely for economic reasons.*

Figure 31: Comparison across unemployment gaps for the EA



Note: *UC-0 refers to the plain unobserved components model, while BN refers to the (Beveridge-Nelson) specification without restrictions of uncorrelated trend and cycle innovations. Unemployment refers to the specification including unemployment in the Phillips-Curve. U6 uses an unemployment rate that includes discouraged workers, all other marginally attached workers and those workers who are part-time purely for economic reasons.*

Appendix B Posterior Distributions

Posteriors were generated using a Metropolis-Hastings algorithm that generated two chains with 50.000 draws each. The initial 25.000 draws were discarded to account for a 50% burn-in period.

UC-0 refers to the plain unobserved components model, while BN refers to the (Beveridge-Nelson) specification without restrictions of uncorrelated trend and cycle innovations. Unemployment refers to the specification including unemployment in the Phillips-Curve. U6 uses an unemployment rate that includes discouraged workers, all other marginally attached workers and those workers who are part-time purely for economic reasons.

Densities of cross-correlation coefficients are centred around 0.5. Estimated cross-correlation coefficients can be inferred from multiplying values on the x-Axis by 2.

Figure 32: Posterior Distributions for US

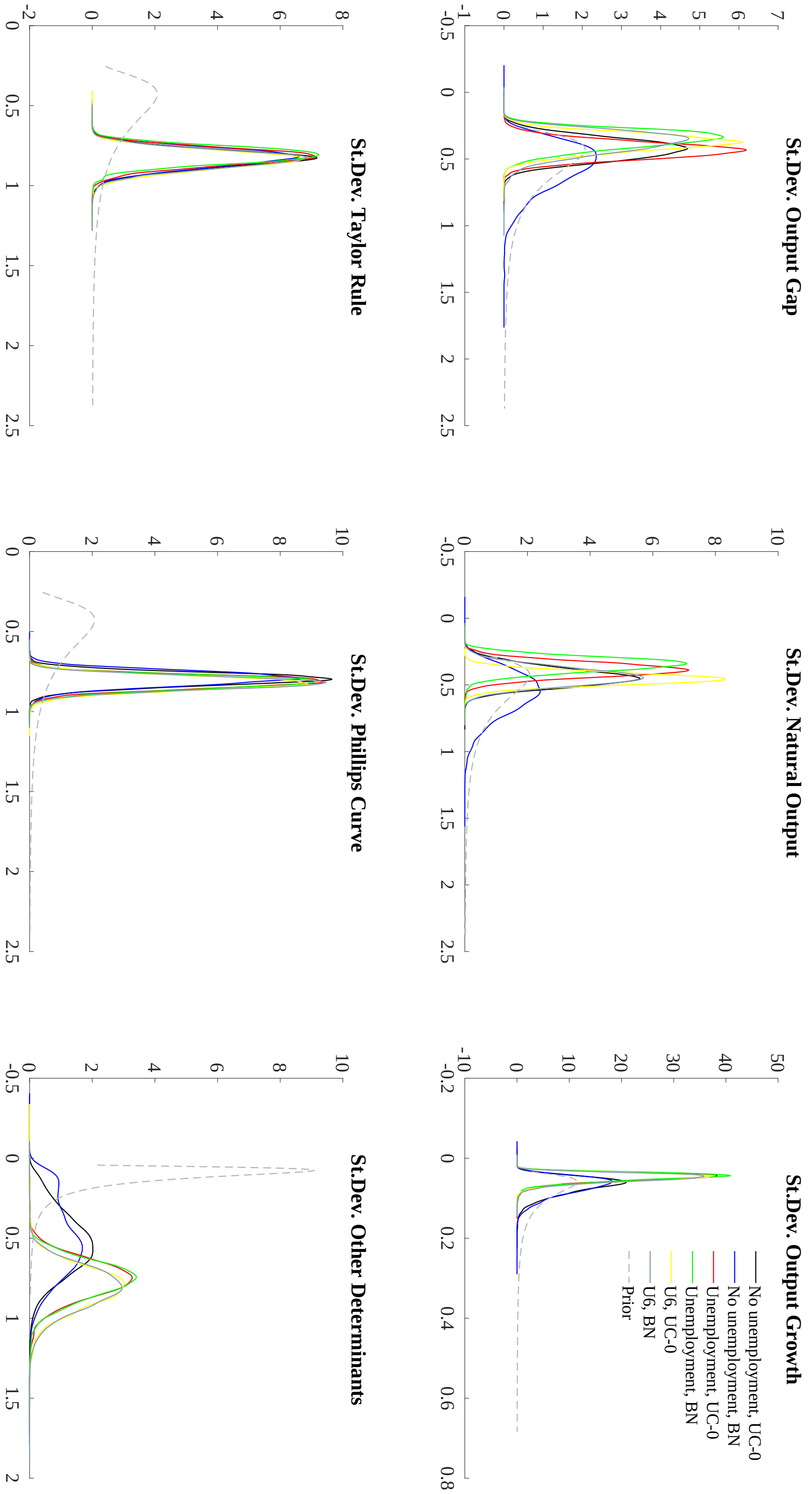


Figure 33: Posterior Distributions for US

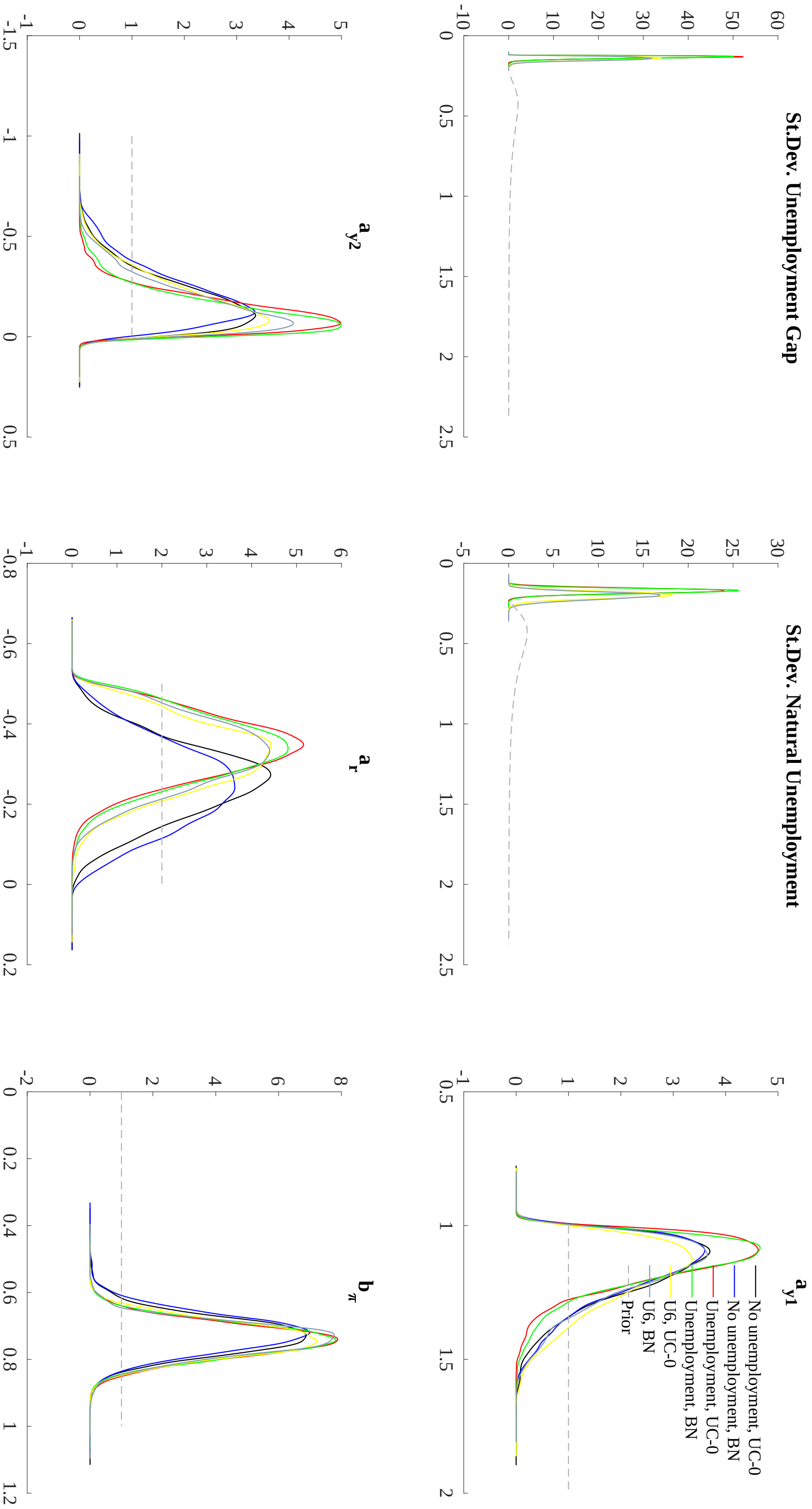


Figure 34: Posterior Distributions for US

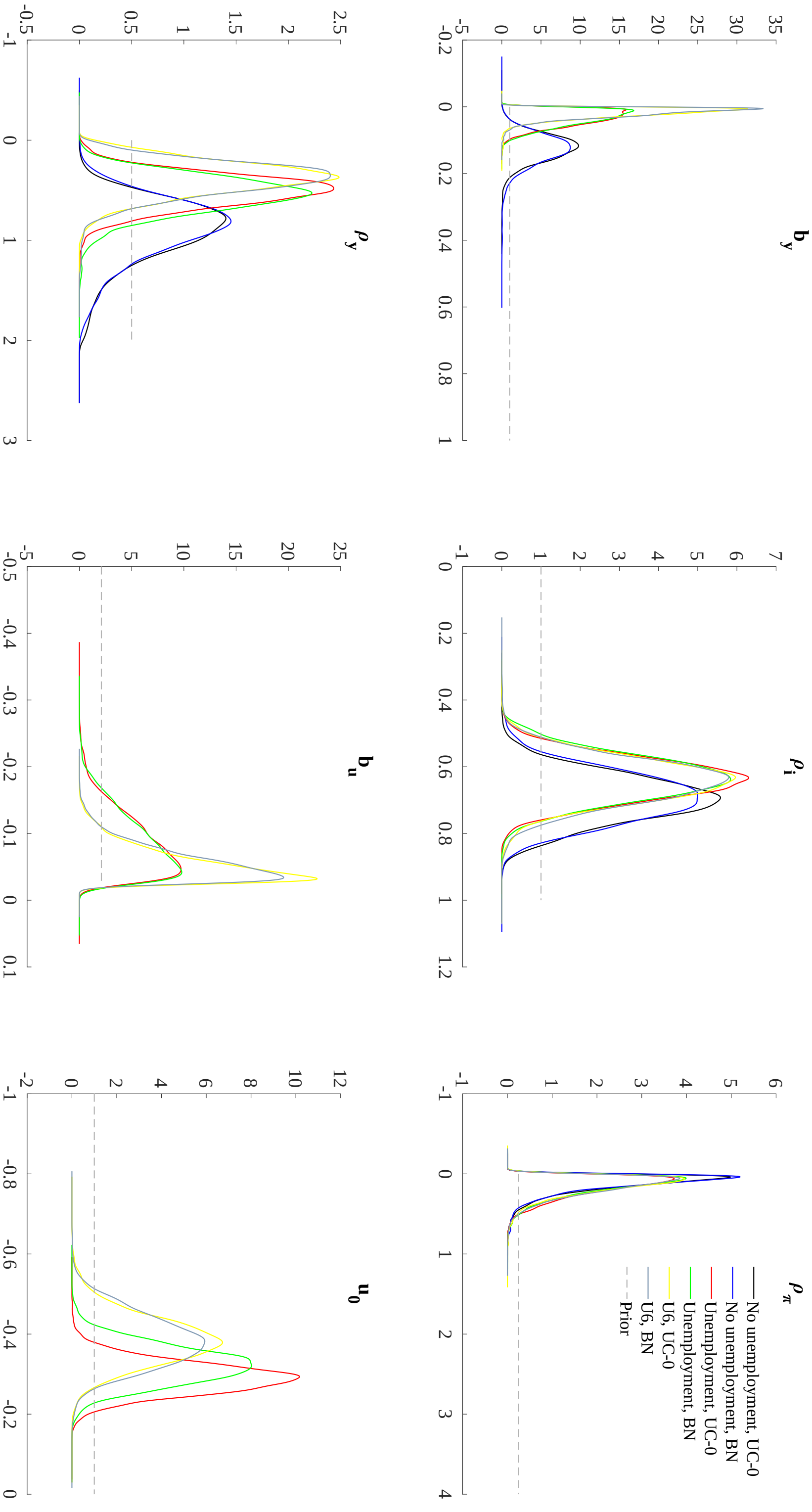


Figure 35: Posterior Distributions for US

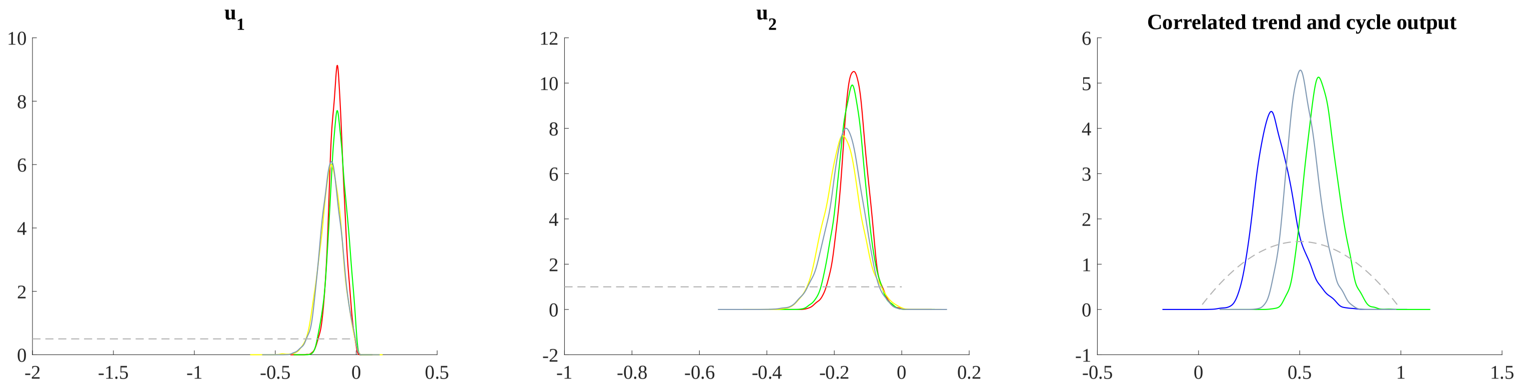


Figure 36: Posterior Distributions for EA

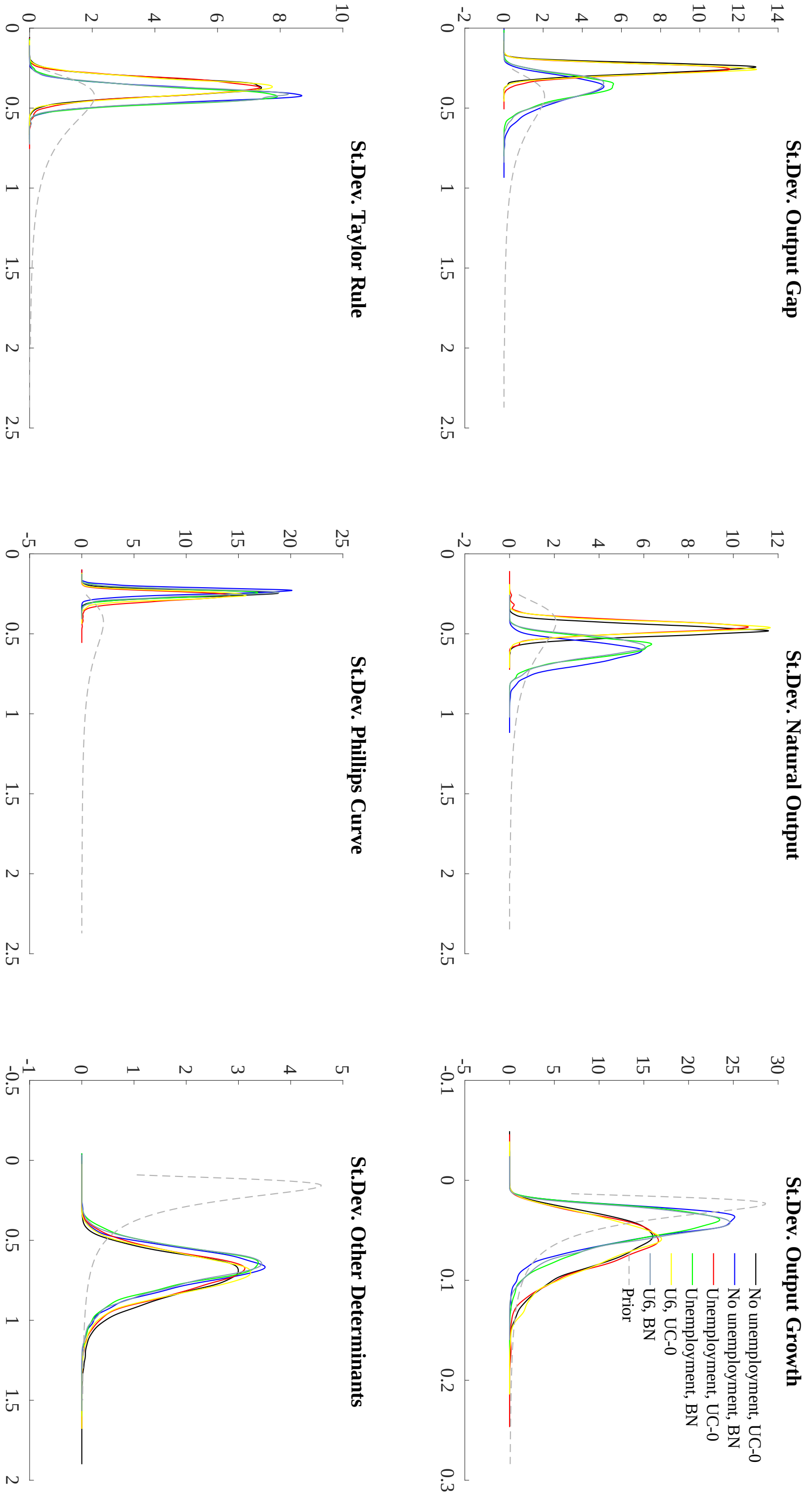


Figure 37: Posterior Distributions for EA

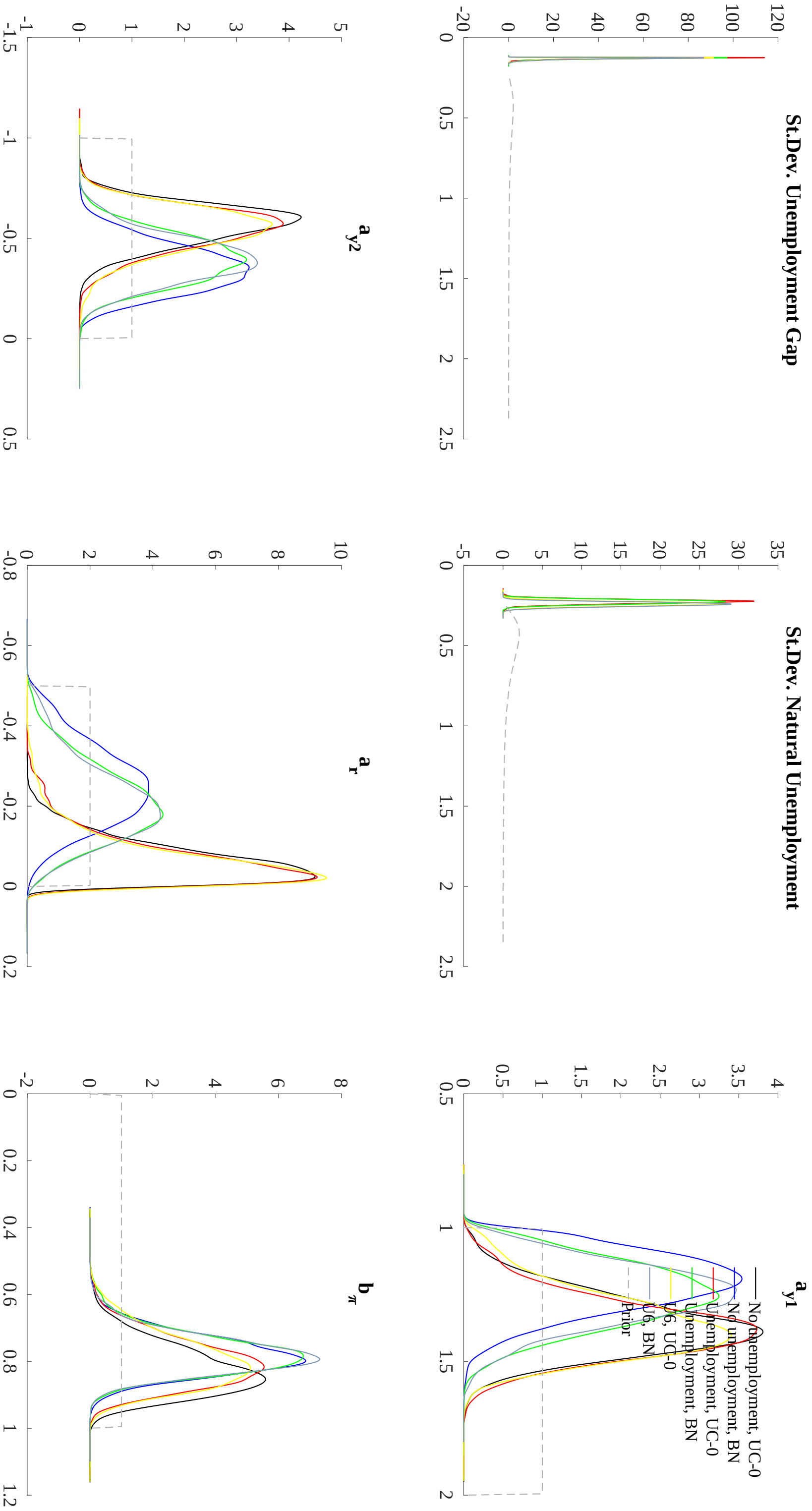


Figure 38: Posterior Distributions for EA

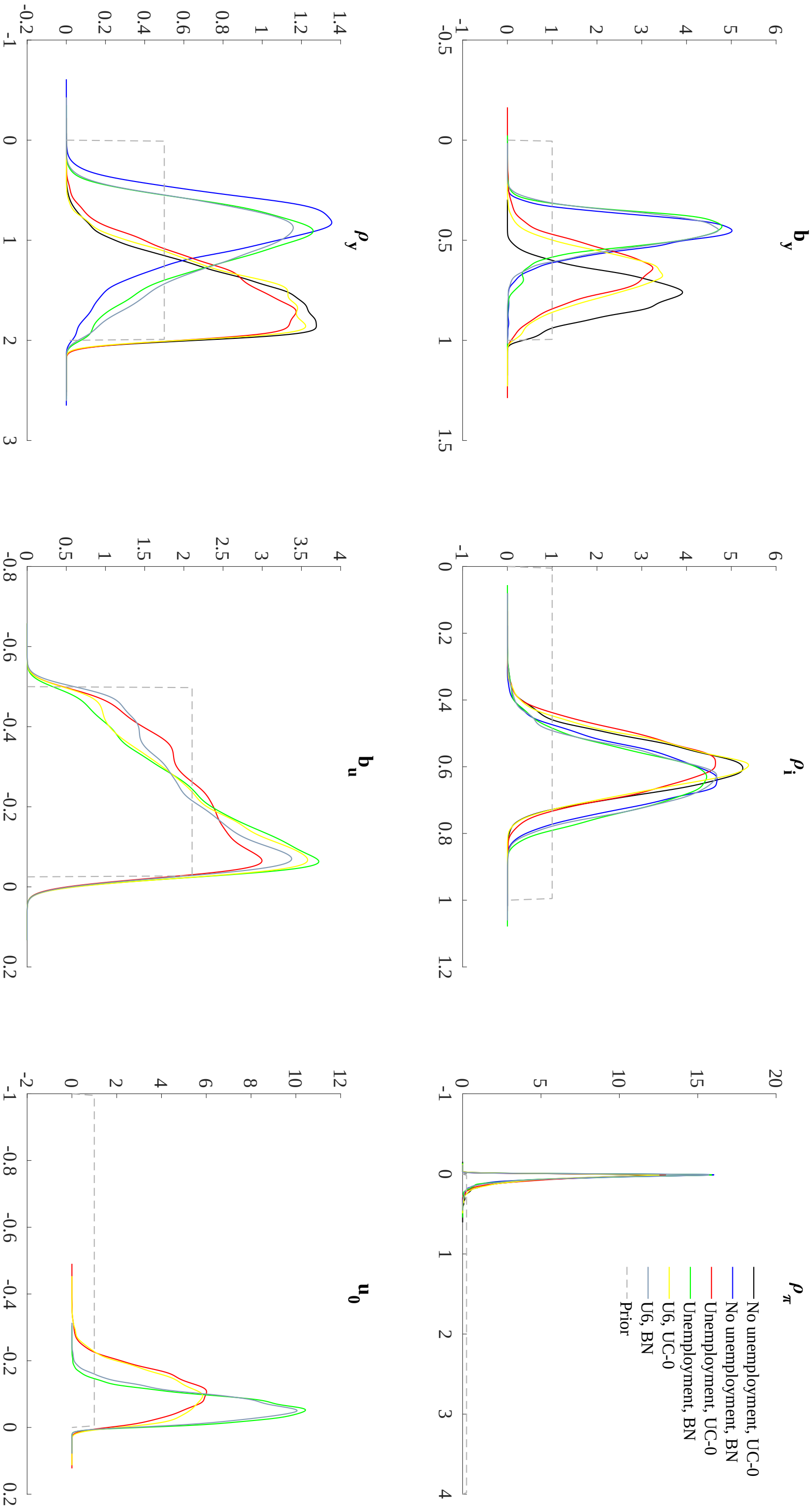
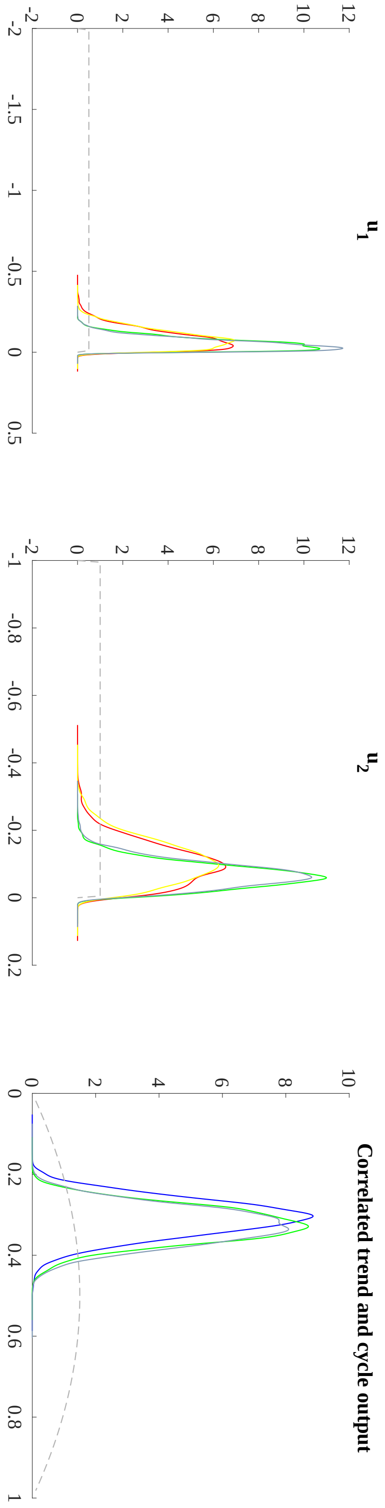


Figure 39: Posterior Distributions for EA



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