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Toward a green economy: the role of central bank's asset purchases

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Abstract

We use a DSGE model to study the effectiveness of green-asset purchases by the central bank (Green QE), along the transition to a carbon-free economy driven by an emission tax, abstracting from price stability considerations. We find that Green QE helps to further reduce emissions, especially in the early stage of the transition. We find that a crucial parameter to determine the effectiveness of Green QE is the elasticity of substitution between the brown and the green good: the higher the elasticity the stronger the impact of the policy on emissions.

Keywords: Central Bank, Monetary Policy, Quantitative Easing, Climate Change

JEL Codes: E52, E58, Q54

Non-Technical Summary

The European Union aims to be climate-neutral by 2050, consistent with the commitment to keeping the increase in global average temperature to well below 2°C above pre-industrial levels. While elected governments are primarily responsible for meeting this goal, central banks may also play a role. One option on the table is to design a programme of green asset purchases, the so called Green QE. We set up a model to study the potential effectiveness of Green QE alongside a transition to an economy with zero carbon emissions driven by a carbon tax. The carbon tax gradually increases over time and leads to progressive pollution abatement by firms. We adopt a purely positive perspective, focusing on the effects on emissions and ignoring welfare effects. We also ignore any legal considerations regarding the central bank's mandate or price stability considerations. In the case of the ECB, the mandate clearly establishes that this monetary policy measure should not prejudice or conflict with the primary objective of price stability, i.e. the ECB could consider a monetary policy easing via an increase of its green assets portfolio only in an environment of downward prices pressure.

Green QE works similarly to a capital subsidy – which can be financed using the revenues from the carbon tax – reducing the rental rate of capital for green firms, which can cut their price. The demand shifts from the brown to the green sector, and emissions decrease. We find that Green QE is more effective when the carbon tax is absent or low and so the timing of the purchases is crucial for the effectiveness of Green QE. As in the long run emissions go to zero anyway as a result of the carbon tax, so the programme is more useful to reduce emissions in the short/medium run, in order to get a larger effect on the pollution stock. For this reason, a temporary but more aggressive program where purchases are concentrated in the short run and then the portfolio is slowly run down is more effective than a gradual and long-lasting one of a similar size.

Second, we find that a crucial parameter to determine the effectiveness of the policy is the elasticity of substitution between the brown and the green good: in purchasing green bonds, central banks should determine what is green and what is brown in order to maximize this elasticity of substitution. This implies that the so called “best-in-class” seems to be more appropriate than the “best-in-universe” approach: instead of discriminating between sectors (best-in-universe approach) the policy should aim to discriminate between firms that produce substitute goods within the same sector, favoring those with the cleanest technology (best-in-class approach).

While in our model the overall impact of this policy is somewhat limited, it does not capture other channels that can play an important role (e.g. signalling effect and R&D investments).

1 Introduction

Limiting the escalation in global temperature is one of the big challenges of the 21^{th} century. According to the scientific community, the acceleration of temperature increase observed in the last decades is largely driven by an exponential rise in greenhouse gas emissions, as a result of the expansion in global production since the industrial revolution. As of June 2022, almost all countries in the world had ratified the Paris Agreement, which has the ambitious goal of keeping a global temperature rise throughout the current century well below 2 degrees Celsius above pre-industrial levels, and to pursue efforts to limit the rise to 1.5 degrees. In order to meet these goals, the European Union aims to be climate-neutral by 2050, by reaching net-zero greenhouse gas emissions.

Designing effective environmental policies is a task for elected governments, which have the most appropriate instruments to address the climate challenge. Several economists suggest that central banks may also play a role in mitigating the increase in global temperature: according to De Grauwe (2019), Schoenmaker (2019), Brunnermeier and Landau (2020), one option on the table is to design a programme of green asset purchases, the so called "Green QE". Central banks such as the ECB, the Bank of England, and the Sverige Riksbank have indeed started to study how to decarbonize their balance sheets and in particular their monetary policy portfolios.

Motivated by these facts, we ask whether Green QE is useful in further reducing the flow of emissions and the stock of atmospheric carbon along the transition, and how it could be better designed to maximize its effectiveness.[1] We answer this question through the lense of a DSGE model, calibrated on the euro area. We define Green QE as a purchase programme of green bonds by the central bank, financed with higher reserves. The model features two production sectors: a green sector, where firms do not pollute; a brown sector, where production generates CO2 emissions, which fuel the stock of atmospheric carbon. Brown firms are charged with a tax for each unit of emissions; in order to reduce tax payments, brown firms can cut emissions by increasing abatement

[1] In the paper we use the terms "atmospheric carbon" and "pollution" interchangeably.

spending. We model the attention paid by households to the environmental content of their investments by assuming that they enjoy utility from investing in green bonds and suffer disutility from investing in brown bonds. This assumption captures the taste for specific types of assets along the lines of Fama and French (2007) and it is consistent with the existence of a negative premium between green and brown bonds, the so called "greenium" (as in Zerbib, 2019, Fatica et al., 2021, and Liberati and Marinelli, 2021). Crucially, this assumption breaks the Wallace neutrality (Wallace, 1981), making green and brown bonds imperfect substitutes for households in the short and in the long run: if the central bank purchases green bonds by issuing reserves the greenium becomes even more negative.

We carry out the following two main experiments.

First, we simulate the transition to an emission-free economy. The government sets an emission tax that increases over time for 30 years, in line with the European Commission environmental targets, up to the point that brown firms fully abate emissions. By increasing production costs for the brown sector along the transition, resources shift from the brown to the green sector, which becomes bigger in relative terms in the new steady state. We show that in the new steady state the economic activity shrinks, compared to a scenario with no emission taxes.

Second, we simulate three different types of Green QE by the central bank along the transition, on top of the government's emission-tax policy. We model Green QE as an additional envelope of purchases by the central bank targeted only to green bonds. The expansion of the balance sheet is financed by issuing reserves. The three types of Green QE differ as to the timing of the purchases and the persistence of the policy: i) gradually increasing and permanent; ii) front loaded and permanent; iii) front-loaded and transitory, i.e. the central bank allows the stock of green bonds held to decline after some years. We show that Green QE is more effective in reducing the stock of pollution when purchases are concentrated in the first years of the transition (cases ii and iii), as the link between emission and brown production is still not weakened by abatement spending.

Instead, the effect on pollution is much smaller, when Green QE increases gradually (case i) because the bulk of purchases takes place at the end of the transition when high abatement spending weakens the link between emissions and brown production. However, from a quantitative point of view, the effect on the stock of pollution, either global and European, is very small in every scenario.

We also identify some parameters that are important for the effectiveness of Green QE along the transition. First, the higher the curvature of the bond utility function the more Green QE is effective, as in this case households change by less their asset composition, weakening the Wallace neutrality principle. Second the impact on emissions depends on the elasticity of substitution between the green and the brown goods. If the two goods are complements Green QE *increases* emissions, because the resulting expansion of green output implies a *larger* demand of brown output, brown production rises, and so emissions. At the opposite, the effectiveness of Green QE on emissions is increasing in the elasticity of substitution between the two goods because by altering the relative price of the two goods the policy is able to shift demand. Third, we show that the effectiveness of Green QE is convex in the size of purchases. Finally, a lower steady-state greenium reduces the relevance of the bond utility function, strengthening the Wallace neutrality principle and making green QE less effective.

Our paper fits in the stream of the literature that studies the transition to a carbon-free economy in general equilibrium models. William Nordhaus simulates the long-run effects of climate change studying different policy scenarios in several applications of his DICE model (Nordhaus, 2008, Nordhaus and Sztorc, 2013, Nordhaus, 2017). Diluiso et al. (2021) analyse financial and monetary policies along the transition to an economy with lower emissions and in response to negative shocks in the brown sector. Benmir and Roman (2020) study monetary and macroprudential policies that can attenuate the welfare losses driven by the introduction of a carbon tax. Carattini et al. (2021) assess the financial risk arising from climate policies and how it can be mitigated through macroprudential policy. Bartocci et al. (2022) introduce green subsidies and carbon taxes in a large-scale model, studying several policies. In a two-country model, Ferrari and Pagliari (2021) find that conventional monetary policy displays little effects in reducing emissions along the green transition, but it could partially shield households from the cost of the brown tax. With respect to these papers, we analyse the role of Green QE along the transition: we show that Green QE helps reduce emissions, but the effect is quantitatively small. In an independent work and using a different model, Abiry et al. (2022) show that a reallocation of the portfolio composition of the central bank toward the clean sector is much less effective than carbon taxation.

In a previous paper (Ferrari and Nispi Landi, 2021), we study the effects of a transitory Green QE that cannot have any effect in steady state; in the present paper we also consider a permanent Green QE that can be effective in the long run, and we analyse the effect of purchase programmes along the transition to an economy with zero emissions. This is possible as in the model used in this paper green and brown bonds are explicitly included in the households' utility function, making green and brown bonds imperfect substitutes also in the long run; in our previous paper we make the two bonds imperfect substitutes only in the short run, by modelling transitory transaction costs in the financial sector.

Finally, the paper is also related to Papoutsi et al. (2021), who make two important points. First, the Corporate Sector Purchase Program (CSPP) of the ECB is tilted toward more polluting sectors, beyond the capital shares of these sectors, given that more polluting firms typically issue relatively more bonds than green firms; second, in a theoretical model they show that, absent a carbon tax, tilting the central bank's portfolio towards green firms can be beneficial to address an environmental externality. Compared to Papoutsi et al. (2021), we perform dynamic simulations of different versions of Green QE, in a calibrated medium-scale model for the euro area. Even if we do not carry out a welfare analysis, we also show that Green QE becomes more useful without a carbon tax, which weakens the link between emissions and brown production. However, we argue that, in general, green purchases by the central bank have little impact on the stock of pollution.

2 Model

We set up a New Keynesian framework augmented with a green and a brown sector, as in Ferrari and Nispi Landi (2021). The green sector produces the green output, and it does not pollute. The brown sector produces the brown output and it generates emissions. The flow of emissions fuels the stock of atmospheric carbon. Brown firms decide how much to spend in abatement to limit emissions and thus to reduce carbon-tax spending. The green and the brown output are used as inputs by a continuum of intermediate firms, that act in monopolistic competition and are subject to nominal rigidities. A final good-firm combines the differentiated intermediate goods to produce a final good. The final good is bought by households for consumption and by capital producers, which transform it in physical capital. The model shares some similarities with the set up in Papoutsi et al. (2021), which also features polluting sectors. There are two main differences. First, we include typical New Keynesian features, such as monopolistic competition, price stickiness, and an elastic labor supply, in order to be closer to the monetary economics literature. Second, our model includes bonds in the utility function, to make green and brown bonds imperfect substitutes, giving Green QE a chance to work. Papoutsi et al. (2021) include bond-holding costs for the same purpose.

The main goal of the paper is not a welfare evaluation of Green QE: for simplicity, unlike most of the literature, in this model pollution is not detrimental for total factor productivity. This assumption allows to easily find a balance growth path of the model, with most variables that grow along this path at the exogenous growth rate of the labor augmenting productivity.[2] The goal of this paper is a positive analysis of Green QE along the transition to a zero-emission economy: in the model, the transition is not necessarily optimal, and we take it as given.

From now on we denote with G the green sector and with B the brown sector. We indicate with a "tilde" variables that are detrended, i.e. that are divided by their trend,

[2] We could assume that pollution yields disutility to households. As far as pollution enters "separable" in the utility function, our results would not change.

which is the non-stationary labor-augmenting productivity z_t for most variables; we indicate with a "hat" detrended variables in percentage deviations from the steady state; variables without a time index are meant to be in steady state.

In what follows, we lay out the optimization problems of all the agents of the model. We leave the full list of equations to the Appendix.

2.1 Households

The representative household maximizes the following utility function:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log\left(c_t - \varsigma c_{t-1}\right) - \frac{h_t^{1+\varphi}}{1+\varphi} + \frac{\nu_G}{1-\kappa_G}\left(\frac{B_{Ht}^G}{P_t z_t}\right)^{1-\kappa_G} - \frac{\nu_B}{1+\kappa_B}\left(\frac{B_{Ht}^B}{P_t z_t}\right)^{1+\kappa_B} \right],$$

subject to the budget constraint:

$$c_t + \frac{D_{Ht} + B_{Ht}^G + B_{Ht}^B}{P_t} = \frac{r_{t-1} D_{Ht-1} + R_t^G B_{Ht-1}^G + R_t^B B_{Ht-1}^B}{P_t} + w_t h_t - t_t + \Gamma_t. \quad (1)$$

The choice variables are consumption c_t, hours worked h_t, the nominal holding of green and brown one-period bonds B_{Ht}^G and B_{Ht}^B, and the nominal holding D_{Ht} of one-period public bonds plus central bank's reserves;[3] R_t^G, R_t^B, and r_t are the nominal interest rate on green, brown, and public bonds respectively; w_t is the hourly wage; t_t denotes lump-sum taxes; Γ_t denotes profits from ownership of firms; P_t is the CPI level; z_t is labor-augmenting TFP, which grows at rate θ.

Green bonds are issued by firms that do not pollute, while brown bonds are issued by firms that generate detrimental emissions. We are assuming that utility is increasing in the amount of real detrended green bonds, and decreasing in the amount of real detrended brown bonds. With this assumption we aim to capture the taste of investors for specific assets beyond the payoffs, in the spirit of Fama and French (2007) and Hartzmark and Sussman (2019). It turns out that green and brown bonds are not perfect substitutes, and in equilibrium a negative green-brown spread opens up: our model features the so

[3] Public bonds and reserves are perfect substitutes and pay the same interest rate r_t.

called “greenium”, in line with several empirical studies (Zerbib, 2019, Fatica et al., 2021, Liberati and Marinelli, 2021), which can be interpreted as a preference for green bonds.[4] This assumption also ensures that Wallace neutrality does not hold and Green QE is effective.[5] Unlike Ferrari and Nispi Landi (2021), the Wallace neutrality does not hold in the long-run either, making Green QE effective also in the long run.

Define the following real variables: $d_t \equiv \frac{D_t}{P_t}$, $b^G_{Ht} \equiv \frac{B^G_{Ht}}{P_t}$, $b^B_{Ht} \equiv \frac{B^B_{Ht}}{P_t}$, $r^G_t \equiv \frac{R^G_t}{\pi_t}$, $r^B_t \equiv \frac{R^B_t}{\pi_t}$, where π_t is the gross inflation rate. The first order conditions of the problem yield the following Euler equations:

$$1 = \beta \mathbb{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t} \frac{r_t}{\pi_{t+1}} \right) \tag{2}$$

$$1 = \beta \mathbb{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t} r^G_{t+1} \right) + \frac{\nu_G}{z_t \lambda_t} \left(\frac{b^G_{Ht}}{z_t} \right)^{-\kappa_G} \tag{3}$$

$$1 = \beta \mathbb{E}_t \left(\frac{\lambda_{t+1}}{\lambda_t} r^B_{t+1} \right) - \frac{\nu_B}{z_t \lambda_t} \left(\frac{b^B_{Ht}}{z_t} \right)^{\kappa_B}, \tag{4}$$

and an expression for the labor supply:

$$h_t^{\varphi} = w_t \lambda_t, \tag{5}$$

where λ_t is the marginal utility of consumption, which includes habits:

$$\lambda_t = \frac{1}{c_t - \varsigma c_{t-1}} - \beta \varsigma \mathbb{E}_t \left(\frac{1}{c_{t+1} - \varsigma c_t} \right). \tag{6}$$

Linearizing equations (3) and (4) around a steady state with constant productivity

[4]Without this assumption, the bond demand would be perfectly elastic and the greenium would be always zero (in steady state and in response to shocks), despite different weights of green and brown goods in the CES production bundle.

[5]Assets in the utility function is a common assumption in the DSGE literature. Recently, macroeconomists have used this assumption to make bonds imperfect substitutes thus breaking the Wallace neutrality (Alpanda and Kabaca, 2020), to better explain the data (Rannenberg, 2021) and to solve several puzzles of New Keynesian models (Michaillat and Saez, 2021).

growth, we get the following arbitrage conditions:

$$\hat{b}_{Ht}^{G} - \hat{b}_{Ht}^{B} = \eta \mathbb{E}_t \left(\hat{r}_{t+1}^{G} - \hat{r}_{t+1}^{B} \right) - \frac{\eta\theta}{\beta} \left(\frac{r^B - r^G}{r^G r^B} \right) \hat{\lambda}_t, \tag{7}$$

where we impose $\eta \equiv \frac{r^B}{\kappa_B (r^B - rr)} = \frac{r^G}{\kappa_G (rr - r^G)}$, and rr is the real interest rate on public bonds in steady state. The previous condition shows that a reduction in the green-brown spread induces households to replace green with brown bonds, other things equal.

2.2 Final-good firms

The representative final-good firm uses the following CES bundle to produce the final good y_t:

$$y_t = \left[\int_0^1 y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{8}$$

where $y_t(i)$ is an intermediate good produced by intermediate firm i, whose price is $P_t(i)$. The profit maximization problem yields the following demand function $\forall i$:

$$y_t(i) = y_t \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon}. \tag{9}$$

2.3 Intermediate-good firms

There is a continuum of firms indexed by i, producing a differentiated input and using the following function:

$$y_t(i) = y_t^I(i), \tag{10}$$

where y_t^I is a CES bundle of green production y_t^G and brown production y_t^B:

$$y_t^I(i) = \left[(1-\zeta)^{\frac{1}{\xi}} \left(y_t^G(i) \right)^{\frac{\xi-1}{\xi}} + \zeta^{\frac{1}{\xi}} \left(y_t^B(i) \right)^{\frac{\xi-1}{\xi}} \right]^{\frac{\xi}{\xi-1}}. \tag{11}$$

Firms operate in monopolistic competition and they set prices subject to the demand of the final-good firm (9). Firms pay quadratic adjustment costs $AC_t(i)$ in nominal terms:

$$AC_t(i) = \frac{\kappa_P}{2}\left(\frac{P_t(i)}{P_{t-1}(i)} - \overline{\pi}\right)^2 P_t y_t,$$

where $\overline{\pi}$ is the inflation target.

The intermediate firm i solves an intratemporal problem to choose the optimal input combination, and an intertemporal problem to set the price. The intratemporal problem, i.e. minimizing costs subject to a given level of production, reads:

$$\min_{y_t^B(i), y_t^G(i)} p_t^G y_t^G(i) + p_t^B y_t^B(i)$$

$$s.t. \left[(1-\zeta)^{\frac{1}{\xi}}\left(y_t^G(i)\right)^{\frac{\xi-1}{\xi}} + \zeta^{\frac{1}{\xi}}\left(y_t^B(i)\right)^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}} = y_t^I(i)$$

where p_t^G and p_t^B are the price of green and brown production respectively, expressed relatively to the CPI. The problem yields the following demand functions for the green and brown input:

$$y_t^G(i) = (1-\zeta)\left(\frac{p_t^G}{p_t^I}\right)^{-\xi} y_t^I(i) \tag{12}$$

$$y_t^B(i) = \zeta\left(\frac{p_t^B}{p_t^I}\right)^{-\xi} y_t^I(i), \tag{13}$$

where $p_t^I = \left[(1-\zeta)\left(p_t^G\right)^{1-\xi} + \zeta\left(p_t^B\right)^{1-\xi}\right]^{\frac{1}{1-\xi}}$ is the real marginal cost of the firm.

The intertemporal problem reads:

$$\max_{\{P_t(i)\}_{t=0}^{\infty}} \mathbb{E}_0\left\{\sum_{t=0}^{\infty}\beta^t\frac{\lambda_t}{\lambda_0}\left[\left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon}\left(\frac{P_t(i)}{P_t} - p_t^I\right)y_t - \frac{\kappa_P}{2}\left(\frac{P_t(i)}{P_{t-1}(i)} - \overline{\pi}\right)^2 y_t\right]\right\},$$

where firms use the same stochastic discount factor of households. In a symmetric equilibrium, the intertemporal problem yields a non-linear Phillips Curve:

$$\pi_t \left(\pi_t - \overline{\pi}\right) = \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{y_{t+1}}{y_t} \pi_{t+1} \left(\pi_{t+1} - \overline{\pi}\right) \right] + \frac{\varepsilon}{\kappa_P} \left(p_t^I - \frac{\varepsilon - 1}{\varepsilon} \right), \tag{14}$$

which links inflation to real marginal costs. If $\kappa_P > 0$, changing prices is costly and the classical dichotomy between nominal and real variables is broken.

2.4 Green and brown firms

Green and brown firms use the following production function, for $j = G, B$:

$$y_t^j = a_t \left(k_{t-1}^j\right)^{\alpha} \left(z_t h_t^j\right)^{1-\alpha}, \tag{15}$$

where k_t^j and h_t^j are capital and labor used in sector j, and a_t is total factor productivity, which follows an AR(1) process:

$$\log\left(a_t\right) = \log\left(\overline{a}\right) + \rho_a \log\left(a_{t-1}\right) + v_t^a \tag{16}$$

and $v_t^a \sim N\left(0, \sigma_a^2\right)$ is a technology shock. Green and brown firms issue bonds b_t^j to households and to the central bank. Bonds finance capital expenditure:

$$b_t^j = q_t k_t^j, \tag{17}$$

where q_t is the price of the capital good. The bond is expressed in real terms and pay a real interest rate r_t^j, for $j = G, B$. According to the International Capital Market Association (ICMA), green bonds enable capital-raising and investment for new and existing projects with environmental benefits. We are therefore using a much wider definition, because we consider as green those assets issued by non-polluting firms, consistently with the theoretical literature (Diluiso et al., 2021, Ferrari and Nispi Landi, 2021, Papoutsi et al., 2021). Given that the market for green bonds as defined by the ICMA is growing but still small, this assumption allows us to model Green QE on a larger scale.

Firms buy capital from capital producers, which in turn buy back non-depreciated capital from basic firms; hence, the effective cost of capital for brown firms reads:

$$r_{kt}^{B} \equiv r_t^B q_{t-1} - (1-\delta)\, q_t, \tag{18}$$

where δ is the depreciation rate of capital.

Firms pay a tax τ_t for each unit of emissions e_t. The tax is relevant only for brown firms, as green firms do not pollute; as in Nordhaus (2008), we assume that for each unit of brown output, brown firms release $\nu_E (1-\mu_t)$ carbon-model units in the atmosphere, as shown by the following emission function:

$$e_t = \nu_E (1-\mu_t)\, y_t^B, \tag{19}$$

where μ_t is the fraction of emissions that brown firms abate. The flow of emissions fuels the stock of atmospheric carbon x_t:

$$x_t = (1-\delta^x)\, x_{t-1} + e_t + e_t^{row}, \tag{20}$$

where e_t^{row} denote exogenous rest-of-the-world emissions, which grow at the same rate of labor-augmenting productivity z_t. Following Nordhaus (2008), we assume a convex abatement-cost function ABC_t:

$$ABC_t = \frac{\nu_M}{1+\chi} \mu_t^{1+\chi} y_t^B. \tag{21}$$

The profit function of brown firms reads:

$$\Gamma_t^B = p_t^{Bnet} \left(k_{t-1}^B\right)^{\alpha} \left(z_t h_t^B\right)^{1-\alpha} - w_t h_t^B - r_{kt}^B k_{t-1}^B, \tag{22}$$

where p_t^{Bnet} is the brown price net of taxes and abatement costs:

$$p_t^{Bnet} \equiv \left[p_t^B - \tau_t \left(1 - \mu_t\right) \nu_E - \frac{\nu_M}{1+\chi} \mu_t^{1+\chi}\right]. \tag{23}$$

The first order conditions describe the choice of capital, labor, and abatement:

$$w_t h_t^B = (1-\alpha) p_t^{Bnet} y_t^B \tag{24}$$

$$r_{kt}^B k_{t-1}^B = \alpha p_t^{Bnet} y_t^B \tag{25}$$

$$\mu_t = \left(\frac{\nu_E \tau_t}{\nu_M}\right)^{\frac{1}{\chi}}. \tag{26}$$

Equation (26) shows that abatement is an increasing function of the emission tax: if the tax is 0, brown firms do not have any incentive to abate emissions.

The problem of green firms is similar, with the only exception that green firms do not pollute, so they do not pay taxes and abatement costs.

2.5 Capital producers

Capital producers use the output produced by final-good firms and non-depreciated capital from intermediate firms, to produce physical capital. Capital is then sold to green and brown firms. Capital producers solve the following problem:

$$\max_{\{i_t, k_t\}_{t=0}^{\infty}} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \left[q_t k_t - (1-\delta) q_t k_{t-1} - i_t\right] \right\}$$

$$s.t.\ k_t = (1-\delta) k_{t-1} + \left[1 - \frac{\kappa_I}{2}\left(\frac{i_t}{i_{t-1}} - \theta\right)^2\right] i_t,$$

where k_t is aggregate capital in the economy and i_t denotes investment. The first order condition reads:

$$q_t \left\{ 1 - \frac{\kappa_I}{2} \left(\frac{i_t}{i_{t-1}} - \theta \right)^2 - \kappa_I \frac{i_t}{i_{t-1}} \left(\frac{i_t}{i_{t-1}} - \theta \right) \right\} + \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} q_{t+1} \left(\frac{i_{t+1}}{i_t} \right)^2 \kappa_I \left(\frac{i_{t+1}}{i_t} - \theta \right) \right] = 1. \tag{27}$$

2.6 Policy

The central bank invests in corporate bonds b^G_{Ct} and b^B_{Ct} and public bonds d_{Ct} issuing nominal reserves RE_t:

$$b^G_{Ct} + b^B_{Ct} + d_{Ct} = \frac{RE_t}{P_t}. \tag{28}$$

Reserves and public bonds are perfect substitutes, so they yield the same nominal interest rate r_t.[6] We define $re_t \equiv \frac{RE_t}{P_t}$ as the real reserve balances. The central bank's real profits Γ_{Ct} are the following:

$$\Gamma_{Ct} = \left(r^G_t - \frac{r_{t-1}}{\pi_t} \right) b^G_{Ct-1} + \left(r^B_t - \frac{r_{t-1}}{\pi_t} \right) b^B_{Ct-1}.$$

Our model is calibrated to the euro area, where fiscal policy is typically implemented at the country level. Following the DSGE literature that models the euro area (i.e. Christoffel et al., 2008 and Coenen et al., 2018, among others), we are considering the euro area as an individual large country with a shared fiscal policy. This assumption is fairly innocuous: the only relevant fiscal decision in our model is the setting of the carbon tax τ_t, which could be seen as a euro-area coordinated policy to address climate change. The other fiscal variables are assumed to be constant along the balanced-growth path (public spending g_t) or they are irrelevant as a result of the Ricardian equivalence (total

[6] This assumption is not crucial. We could make these assets imperfect substitutes by introducing public bonds in the utility function of households, but we would not gain much for the purpose of the analysis.

public bonds d_{Gt} and lump-sum taxes t_t). The government budget constraint reads:

$$g_t + \frac{r_{t-1}}{\pi_t} d_{Gt-1} = t_t + d_{Gt} + \tau_t e_t + \Gamma_{Ct}. \tag{29}$$

Given these assumptions, we need to specify a rule for the following policy variables:

$$POL \equiv \left\{ \tau_t, re_t, d_{Ct}, b^B_{Ct}, b^G_{Ct}, r_t \right\}. \tag{30}$$

We assume that public and brown bonds held by the central bank grow at the same rate of the labor-augmenting productivity (thus they are constant along a balanced-growth path). The emission tax τ_t is set such that emissions go linearly to 0 in 2050, in line with EU's commitment to global climate action under the Paris Agreement: the tax increases over time until 2050 and it remains constant afterward. In each policy scenario, we specify a path for for central bank's reserves re_t, with b^G_{Ct} being determined by (28), given d_{Ct} and b^B_{Ct}. The nominal interest rate follows a standard Taylor rule:

$$\frac{r_t}{r} = \left(\frac{r_{t-1}}{r} \right)^{\rho_r} \left(\frac{\pi_t}{\overline{\pi}} \right)^{\phi_\pi (1-\rho_r)}. \tag{31}$$

2.7 Market clearing

Clearing in the goods market implies:

$$y_t = c_t + i_t + g_t + y^B_t \frac{\nu_M}{1+\chi} \mu_t^{1+\chi} + \frac{\kappa_P}{2} \left(\pi_t - \overline{\pi} \right)^2 y_t. \tag{32}$$

Clearing in the corporate bond market:

$$b^G_t = b^G_{Ht} + b^G_{Ct} \tag{33}$$

$$b^B_t = b^B_{Ht} + b^B_{Ct}. \tag{34}$$

Market clearing for public bonds/reserves:

$$d_{Gt} + re_t - d_{Ct} = d_{Ht}. \tag{35}$$

Market clearing in labor and capital markets:

$$h_t = h_t^B + h_t^G \tag{36}$$

$$k_t = k_t^B + k_t^G. \tag{37}$$

2.8 Additional variables

2.8.1 Carbon price

The price of one ton of CO2 (the so called carbon price) is an important statistic in the environmental-macroeconomic literature. In our model, τ_t is the price of one carbon-model unit in terms of output-model units. Let p_t^C be the price of one ton of CO2 in Euro. We compute p_t^C as follows:

$$p_t^C = \frac{s_1 s_2}{s_3} \tau_t, \tag{38}$$

where s_1, s_2, and s_3 are conversion rates defined as follows. The conversion rate s_1 denotes Euro billions per one output-model unit:

$$s_1 = \frac{y^E}{\tilde{y}}, \tag{39}$$

where $y^E = 3022.4$ Euro bil. is the quarterly GDP in the euro area in 2019Q4, while $\tilde{y} = 2.2469$ denotes the initial steady-state detrended output; the conversion rate s_2 denotes Gigatons of Carbon (GtC) per one carbon-model unit:

$$s_2 = \frac{x^{GtC}}{\tilde{x}}, \tag{40}$$

where $x^{GtC} = 870.1476$ GtC is the stock of atmospheric carbon in 2019 and $\tilde{x} = 1947.9$ is the detrended atmospheric carbon in model units in the initial steady state; finally, one ton of carbon is equal to $s_3 = 3.67$ tons of CO2.

2.8.2 Euro-area Pollution

In our model, x_t is the stock of atmospheric carbon generated by world emissions. We also define a measure of euro-area atmospheric carbon, that is pollution generated only by euro-area emissions:

$$x_t^{ea} = (1 - \delta^x)\, x_{t-1}^{ea} + e_t. \tag{41}$$

2.9 Calibration

We calibrate the model to the euro area, at the quarterly frequency. We set most economic parameters following the new version of the New Area-Wide Model (NAWM-II) in Coenen et al. (2018) (Table 1).

Regarding the initial steady-state ratios, we follow the NAWM-II and target $\frac{c}{y}$, $\frac{i}{y}$, and $\frac{g}{y}$ equal to 57.5%, 21.0%, and 21.5%. To match these targets, we calibrate $\alpha = 0.30$ and $\bar{g} = 0.48$.

For the following environmental parameters, we use the calibration in Gibson and Heutel (2020), which update the estimates in Heutel (2012); we set the pollution depreciation δ_x to 0.0035; we calibrate the convexity χ of the abatement function to 1.6; the coefficient in the abatement function ν_M is set to $0.074\,(1+\chi)$. Moreover, we set the rest-of-the-world emissions to match a steady-state rest-of-the world/EA emission ratio of 15.31, the value observed in 2018: this implies $\tilde{e}^{row} = 13.30$. We set the coefficient in the emission function ν_E to 0.49, in order to target a price of 65 Euro per ton of CO2 under full abatement, a value in line with the literature.

In order to set the weight of the brown output ζ and the elasticity of substitution ξ between the green and the brown output we have to define what is green and what is brown. A first option is to interpret y^G and y^B as different energy sources, with a relatively high elasticity of substitution: this is what Carattini et al. (2021) and Giovanardi et al. (2021) do, in models similar to ours. A second option is to interpret the green as the service sector and the brown as the manufacturing sector, which is more polluting: in this case the elasticity of substitution between the two goods is relatively low. Our results show that Green QE is a limited tool to affect pollution; therefore, in order to be conservative, we choose the first option, given that the second option would magnify our findings: a low elasticity of substitution implies that the two goods are complements, thus a Green QE that boosts the green sector will end up to stimulate also the brown sector. Following Carattini et al. (2021), we set $\xi = 2$; following Giovanardi et al. (2021), who target the renewable energy share in Europe in 2018, we set the weight of the brown good ζ to 0.8. In Section 4, we explore what changes when we interpret the two sectors as services and manufacturing.

The parameters of the bonds' utility functions are specific to our model. Parameters κ_G and κ_B govern the concavity and the convexity of the green bond utility and the brown bond disutility function, respectively; these parameters are relevant for the elasticity of bond demands to the greenium: when κ_G and κ_B are higher, this elasticity is low and households are less willing to change their asset composition, making Green QE more effective. A first option is to set κ_G and κ_B following the studies that use assets in the utility function, where these parameters are calibrated around relatively low values (1 in Alpanda and Kabaca, 2020, 0.15 in Rannenberg, 2021), resulting in large elasticities. A second option is to calibrate directly the elasticity (parameter η in equation 7), in models where different bonds are not perfect substitutes; in an influential work based on a DSGE model, Chen et al. (2012) estimate the short-run elasticity of long-term bond holdings to the spread between long- and short-term rates at a number around 300.[7] Again, in order to give Green QE a chance to be relatively effective, we choose this second option and set $\eta = 300$, trying with the first option in Section 4. This assumption results in relatively

[7]The estimated median of the distribution parameter ζ' is 0.003274 in Table 2 of Chen et al. (2012); this parameter gives the inverse of the sensitivity of long-term bonds to the spread between long- and short-term rates (equation D23 in Chen et al., 2012's Appendix).

large κ_G and κ_B (8.93 and 8.94, respectively).

We also need to calibrate the parameters capturing the relative weight of green and brown bond utility, ν_G and ν_B. We set these parameters such that the annualized brown and green rates are 15 points respectively higher and lower than the real policy rate in the initial steady state. This implies that the annualized greenium is -30 basis points: this value is at the upper ends of estimates in the literature (see for instance Kapraun and Scheins, 2019), but in line with De Santis et al., 2018.[8] Other papers find a much smaller greenium (Liberati and Marinelli, 2021). We choose this relatively high value to be conservative: a lower spread would imply a smaller importance of bond utility functions, making Green QE less effective and strengthening our results, i.e. Green QE is a weak tool to address climate change, as it has little impact on pollution.[9]

We set the initial central-bank's reserves to GDP ratio to 40%, in order to target the ECB liability/GDP ratio in 2019. We assume that in the initial steady state, the central bank does not hold corporate bonds.

2.10 Model validity

We have made some changes to the standard DSGE framework, and it makes sense to verify that our model can replicate key moments in the data and yield reasonable impulse response functions to typical shocks.

First, we show that the model does a good job in matching standard deviations and correlations of key euro-area variables. To do that, we solve the model using a first-order approximation and we feed it with a very long random sequence of TFP shocks. As a benchmark, we use the HP cycle of GDP, consumption, investment, and inflation, along the horizon 1999Q1-2019Q4 (source Eurostat). We calibrate the standard deviation of TFP shocks to match the standard deviation of euro-area GDP (1.17%), and the habit

[8] In De Santis et al. (2018), the difference between the CSPP-eligible green industrial spread and the non-green counterpart is around -30 basis points, pre-CSPP announcement.

[9] This calibration implies $\nu_G = 15.16$ and $\nu_B = 3.07e-14$. The latter value is extremely low in order to offset an otherwise huge marginal disutilty of brown bonds $\nu_B b_{HB}^{\kappa_B}$, given $\kappa_B = 8.92$.

parameter ς to avoid a too high volatility of consumption and investment relatively to output: we use $\varsigma = 0.8$, a reasonable value.[10] Using only TFP shocks, the model is able to match key moments reasonably well (Table 2): consumption and inflation are less volatile than output, while investment is more volatile; consumption and investment are strongly correlated with output; output and consumption are strongly auto-correlated. By adding other shocks to the model, we could improve the fit: for instance, by adding demand-like shocks one could increase the correlation between inflation and output. However, our aim is not to provide a precise description of business cycle fluctuations in the euro area throughout the last two decades (for that goal we would need a detailed financial sector, sovereign risk, etc), but only to show that the model yields pretty standard predictions.

Second, we show that the impulse response functions to typical shocks, such as TFP and monetary shocks, are in line with the literature. After a standard-deviation increase in TFP, the economic activity expands, given the higher productivity of labor and capital (Figure A.1): consumption and investment rise. Inflation falls on impact, as TFP boosts supply, but then recovers quickly (generating a positive, yet small correlation with output). Given the increased supply of the brown input emissions rise, and so does pollution, even if much more slowly. The impulse responses are in line with the New Keynesian literature, both the standard one (Galí, 2015, Coenen et al., 2018) and the environmental one (Annicchiarico and Di Dio, 2015).

A 25 basis points rise in the interest rate drives an increase in aggregate demand, through the standard intertemporal substitution channel (Figure A.2): consumption and investment demand increase, prices go up, green and brown production rise, the latter generating higher emissions and pollution.

[10]The NAWM-II does not have internal habits (only external habits). In the DSGE model of Gertler and Karadi (2011), $\varsigma = 0.815$.

3 Analysis

In this section, we solve the model in perfect foresight: we start from a steady state where agents do not expect any shift in the environmental policies, then the whole set of policies is announced, and households can perfectly foresee the path of fiscal and monetary policies until the new steady state is reached. First, we simulate the transition from the initial steady state to an economy with zero emissions. Second, we study the effects of an increase in green bonds held by the central bank, throughout the transition. Third, with simulate different sizes of Green QE. Fourth, we analyze a green credit easing, i.e. an increase in green bonds held by the central bank financed with an equal sale of brown bonds.

3.1 The transition to a green economy

We assume that period 0 corresponds to 2019Q4, when the government introduces an emission tax that increases linearly for 120 quarters, such that from 2050 on all emissions are abated; in order to fully abate emissions, the carbon price is around 65 Euro per ton of CO2. In modeling the transition, we are assuming that the available technologies do not change.[11] In Figure 1 we plot the transition to the new steady state with zero emissions: the variables are in percentage deviations with respect to the value they would have had with no increase in the emission tax, unless otherwise stated.[12]

Pollution follows a slow law of motion and after one century has still not reached the new steady state; we are assuming that emissions in the rest of the world are not abated, so in the new steady state the reduction in global pollution is far from 100%.[13]

[11]While this assumption is fairly standard in the environmental DSGE literature (Diluiso et al., 2021, Carattini et al., 2021, Bartocci et al., 2022), it is also likely that the transition to a green economy would spur big transformations in the production technologies, driven by both the public and the private sector. For instance, these transformations may reduce the weight of the brown sector, abatement costs, and the elasticity of production with respect to emissions. For simplicity we abstract from these channels, but we acknowledge that these issues deserve further research.

[12]Given that TFP grows over time at a constant rate, consumption is much higher in 2050 with respect to 2020: the 10% fall in the new steady state is relative to the scenario with no environmental policy.

[13]If we assume that the rest of the world reduces emissions too, pollution would slowly go to zero.

The consumption fall depends on the higher abatement costs, which are around 5% of steady state GDP in case of full abatement. We highlight that the final reduction in consumption would be the same even under different speeds of the transition. However, in our model we are not factoring in the TFP costs of pollution, so we are somewhat over-estimating the consumption decrease.

The shift of resources toward abatement costs and the higher production costs drive a fall in investment too, which in turn reduces the stock of capital. The output fall is smaller than the consumption and investment decrease, given that we are including abatement costs in the definition of output: to accomplish that, in the new steady state households work more. The tax shifts resources from the brown to the green sector, which experiences a large expansion.

We find that the transition to a green economy is deflationary, despite the increases in the marginal costs of brown firms associated with the tax: inflation falls on impact, and then gradually comes back to the initial steady state. This result relies on the permanent nature of the tax, which increases over time. On the one hand, the increase in current emission tax raises firms' marginal costs, putting upward pressures on prices via the Phillips Curve; on the other hand, the expected further increase in future taxes reduces expected income, lowering aggregate demand via the Euler equation, putting downward pressures on prices; in a companion paper (Ferrari and Nispi Landi, 2022), we show analytically that the second effect always prevails, assuming rational expectations or perfect foresight. The fall in inflation induces the central bank to reduce the policy rate along the transition.[14]

The other variables would not be affected.

[14]Some variables display a small blip around quarter 120: this is the period when the tax stops increasing, and the economy starts adjusting to the final steady state. Assuming that the carbon tax is non-linear, i.e. grows more in the initial periods, would remove these blips: this simulation is available upon requests.

3.2 Green QE along the transition

We assume that Green QE consists of a very large increase in the stock of reserves (a 50% rise), which finances the purchase of green bonds only.

We simulate three different types of green purchases (Figure 2). In the blue solid line we consider a gradual permanent increase in the stock of green bonds until 2050 (GQE1). In the red dotted line, we consider a one-shot permanent increase (GQE2). In the black dashed line, we simulate a transitory increase, which gradually dies out over time until 2050, when the amount of green bonds comes back to the initial level (GQE3). During the transition, the central bank keeps using the Taylor rule for the nominal interest rate. The lines show the marginal impact of Green QE on top of the carbon tax. For most variables (like GDP), we plot the following object:

$$100 * \frac{X_t^{GQE,TAX} - X_t^{TAX}}{X_0}, \tag{42}$$

where $X_t^{GQE,TAX}$ is variable X in period t under Green QE and carbon tax, X_t^{TAX} is the variable under carbon tax only, X_0 is the variable in the initial steady state. Variables expressed in rates (like inflation, spread, and interest rate) are not divided by their initial level and they are reported at annual rates.

In all scenarios, when the central bank buys green bonds, households do not sell enough of them as a result of the incurred utility loss. This excess demand for green bonds pushes the green yield down, inducing green firms to issue bonds to finance more capital: green output rises.[15] The higher supply of green output reduces its price, thus intermediate-good firms replace brown output with green output, given an elasticity of substitution ξ greater than one: the more the green rate falls, the stronger this effect, and Green QE is more effective. The fall in brown production reduces emissions and pollution. Brown firms issue less bonds, which implies a decrease in the brown rate: if

[15] Absent the green preferences, households would sell the exact amount of green bonds purchased by the central bank, the green interest rate would not move and green firms would not have any incentive to issue more bonds.

the brown rate fall is large, the reduction in brown production is mitigated and Green QE is less effective.[16]

Given that the response of the green and the brown interest rate is crucial for the effectiveness of Green QE, it is instructive to look at the bond inverse demands by households in Figure 3; the red interval includes only values reached during the simulation of GQE1 and GQE2, while the blue line includes more values just for graphical purposes. We have derived the inverse demand functions using equations 3 and 4:[17]

$$r_{t+1}^{G} = \frac{\theta}{\beta}\left[1 - \frac{\nu_G}{\tilde{\lambda}}\left(\tilde{b}_{Ht}^{G}\right)^{-\kappa_G}\right] \tag{43}$$

$$r_{t+1}^{B} = \frac{\theta}{\beta}\left[1 + \frac{\nu_B}{\tilde{\lambda}}\left(\tilde{b}_{Ht}^{B}\right)^{\kappa_B}\right]. \tag{44}$$

Our assumptions about preferences have three main implications: i) the green bond inverse demand is increasing and concave; ii) the brown bond inverse demand is increasing and convex; iii) in the initial steady state (the right bound of the red line) the green and the brown inverse demand have same slope (i.e. $\frac{1}{\eta}$, see equation 7). When the central bank carries out Green QE, households reduce their green bond holding only if the green interest rate falls: the *concavity* of the green bond inverse demand magnifies the reduction in the green interest rate. On top of that, the *convexity* of the brown bond inverse demand ensures that the brown rate is almost constant in the red region: the lower demand of brown output resulting from Green QE, does not lead to a large reduction in the brown rate. Therefore our assumptions on preferences indeed favor the effectiveness of Green QE.

The timing of the purchases turns out to be crucial. Given that in the long run emissions go to zero anyway as a result of the tax, it is more useful to reduce emissions

[16]In fact, the economic mechanism of Green QE is very similar to that of a subsidy to green capital: such a subsidy would reduce the cost of green capital, boost the demand of green output and lower the demand of brown output, decreasing emissions. The main difference is that subsidies are under the control of fiscal policy, while QE policies are under the control of the central bank.

[17]We are keeping the marginal utility of consumption λ constant at the initial steady steady state, ignoring the expectation operator given the perfect foresight assumption, and dividing by the labor-augmenting productivity z_t.

in the short/medium run. The tax induces firms to spend in abatement, which in turn implies a lower reduction in emissions for any decrease in brown output (equation 19). This explains the greater effectiveness of earlier permanent and transitory purchases (GQE2 and GQE3), with respect to permanent gradual purchases (GQE1). Remarkably, the transitory purchase has an effect comparable to the permanent-one shot purchase, and it does not break the market-neutrality principle in the long-run.

Green QE also affects aggregate variables. In the short run, the expansion in the green sector is larger than the contraction in the brown sector, and output rises. Consumption initially falls, to finance higher investment in green capital, and then it increases. The rise in aggregate capital drives a positive response in labor, which is more productive. Under GQE2 and GQE3 the rise in aggregate demand boosts the inflation rate, which triggers a contractionary response of the central bank. As these impulse responses are in deviations compared to the scenario in Figure 1, this means that under Green QE the nominal interest rate and inflation fall by less compared to a scenario with carbon tax only. The dynamics are much slower for GQE1, which is more gradual relatively to the other two scenarios. In the long run, GQE3 is transitory and its effects die out; under GQE1 and GQE2, the economy reaches a new steady state with a higher level of economic activity, which is driven by a permanently lower interest rate in the green sector. From a quantitative point of view, the effects of Green QE on global and EA pollution are never larger than 0.012% and 0.2% respectively compared to the initial level. These small effects are in line with the results of Ferrari and Nispi Landi (2021) and hold even in a model where Green QE has a long-run impact, under favorable assumptions for the effectiveness of Green QE.

3.3 The size of purchases

The size of the policy is arbitrary, but this is an instrument that has not been used yet and we do not have a benchmark size of the purchase. Thus, we also study larger and smaller reserve increases, to find out whether the effects of Green QE are linear in the size of purchases: we consider GQE3 and simulate different sizes of the initial increase in reserves (Figure 4). We show that the effects of Green QE are convex in the size of Green QE: for instance, the reduction in pollution when the stock of reserves increases by 75% (Figure 4, black dashed line) is more than double compared to the reduction after a 50% increase (Figure 4, blue solid line). This non-linearity hinges on the shape of the bond demand in Figure 3. When the purchases are larger, the green bond demand by households becomes steeper (i.e. this is a movement along the demand curve to the left): households accept to sell green bonds only if the rate fall is sufficiently large.

3.4 Credit easing

Another option for the central bank is to finance the purchase of green bonds with an equal sale of brown bonds, such that the size of the balance sheets does not change (i.e. a credit easing). This policy may have two main advantages: i) it does not increase reserves; ii) it may reduce brown production directly by increasing the brown rate, as a result of a rightward movement along the household's brown bond demand (Figure 3, lower panel). Moreover, this policy can address the critique of Papoutsi et al. (2021), which argues that the euro-area CSPP is heavily tilted towards brown firms, beyond the share of these firms in euro area GDP.[18]

In the baseline calibration, for the sake of simplicity we have assumed that the central bank does not hold corporate bonds. We now assume that the central bank holds corporate bonds for 1.5% of euro-area GDP, as the ECB did at the end of 2019. We also assume that the central bank is market neutral and in the initial steady state:

$$\frac{b_C^B}{b_C^G + b_C^B} = \frac{k^B}{k^G + k^B} = 0.8. \tag{45}$$

In Figure 5, we simulate a permanent credit easing in the initial period such that all

[18]This occurs because brown firms are typically larger bond issuers than green firms; by following a market-neutrality principle, the ECB buys brown bonds beyond the share of these firms in euro-area GDP.

brown bonds are sold for ever and replaced by green bonds. The credit easing is much less effective: by selling all the brown bonds, without increasing reserves, the central bank can purchase green bonds only by 1.2% of the initial GDP: this implies a very mild reduction in emissions.[19] In GQE2, we have assumed that the central bank increases reserves by 50% (from 40% to 60% of GDP), which implies purchases of green bonds by 20% of GDP.[20]

4 Additional exercises

In this section we carry out four additional exercises. First, we compare a permanent Green QE with a permanent market-neutral QE. Second, we simulate a Green QE without the presence of the carbon tax. Third, we modify the weight of the brown good and the elasticity of substitution between the green and the brown good in the final-good bundle. Fourth, we change the parameter of the bond utility functions. The figures of this section are in the Appendix.

4.1 A market-neutral QE

It seems instructive to compare our baseline Green QE with a permanent market-neutral QE (Figure A.3). The blue solid line is the benchmark GQE2; the red dotted line simulates an increase in reserves that finance higher green and brown bonds, following the capital shares of the two sectors in the economy.

We highlight two main findings. First, a market-neutral QE raises emissions, given its positive effect both on green and brown output, in line with Ferrari and Nispi Landi (2021). Papoutsi et al. (2021) also show that the ECB market-neutral QE is more biased toward brown firms, which are larger bond issuers than green firms. For simplicity, in

[19]The emission reduction remains negligible even if we assume that in the initial steady state the central bank holds only brown bonds.

[20]These results are in line with the temporary credit easing simulated in Ferrari and Nispi Landi (2021), under the assumption of an intermediate value for the bank's adjustment costs. In particular, aggregate variables are barely affected by this policy.

our model all firms issue bonds: a market-neutral QE implies that the portfolio of the central bank replicates the capital/GDP shares of the two sectors. Nevertheless, the QE expansionary effects raise emissions.

Second, in our model a market-neutral QE is less effective than a Green QE. The reason is the slope of the household's bond demand functions (Figure 3). Compared to a Green QE, a market neutral QE implies the purchase of more brown bonds and less green bonds: given the concavity of the green and the convexity of the brown demand function, a market neutral QE implies a much smaller reduction in the brown rate, compared to the fall in the green rate obtained under a Green QE. For the same logic, a pure "Brown QE" would be much less effective than a Green QE. However, we warn that strong caveats apply for this result. Our model is mainly designed to give Green QE the chance to work, hence we have made relatively favorable assumptions for its effectiveness in reducing the green-brown spread and thus emissions. The model however lacks other features that could make a market-neutral QE more effective, such as financial market segmentation (as in Chen et al., 2012 and Burlon et al., 2018), or banking frictions (as in Gertler and Karadi, 2011 and Gertler and Karadi, 2013); these features would make a temporary market-neutral QE more effective in the short term, by reducing *both* green and brown rates, but they would not cause reduction in the green-brown spread.

4.2 Can Green QE lead the transition?

The marginal effects of Green QE on top of a carbon tax are small on emissions and pollution. The carbon tax is effective in driving the euro area to a zero-emission economy and Green QE can provide only a small additional contribution. Can Green QE alone lead the transition, without the introduction of carbon tax? We simulate the effects of a one shot permanent increase in Green QE (GQE2), keeping the carbon tax to 0 (Figure A.4, red dotted line line). We compare this exercise with the blue solid line in Figure 2 (also reported in Figure A.4), which shows the marginal contribution of Green QE on top of the carbon tax. The effectiveness of Green QE in reducing emissions is decreasing in the level of the carbon tax: as already observed in the previous section, the tax induces firms to spend in abatement, which in turn partially reduces the link between emissions and brown output (equation 19). Green QE can reduce emissions only by its impact on brown interest rates, and thus on brown production: if the link between brown production and emissions is stronger (for instance when $\tau = \mu = 0$), Green QE gets more effective. A larger reduction in emissions also drives a larger, but still small, decrease in the stock of euro-area and global pollution.

4.3 Brown sector's size and elasticity of substitution

In the baseline scenario, we interpret the green and the brown good as different sources of energy, in line with Carattini et al. (2021) and Giovanardi et al. (2021). In this section, we interpret the green as the service sector and the brown as the manufacturing sector. As shown by Papoutsi et al. (2021), in the euro area emissions are generated mostly by the secondary sector, whose capital income accounts approximately for 35% of total capital income in the euro area: we calibrate $\zeta = 0.35$. The elasticity of substitution between services and manufacturing goods is relatively low: we follow Gomes et al. (2012), a DSGE model of the euro area, and calibrate this elasticity of substitution to 0.5.[21] Under the new calibration (Figure A.5, red dotted line) Green QE *increases* emissions. This policy drives a reduction in the green rate that boosts the green sector. As the green and the brown goods are complements, the demand for the brown good rises, brown firms increase production and emissions rise.

4.4 Greenium and bond elasticities

In the baseline calibration, we have set a steady-state greenium in the upper end of estimates found in the literature; moreover, we have calibrated the curvature of the bond utility functions to relatively high values, in order to give a change to Green QE to be powerful. In this section, we calibrate these parameters to lower values. Specifically, we

[21] In changing ζ and ξ, we also modify the parameters that are set to match some steady-state targets.

set the annualized greenium to 5 basis points (as found in Liberati and Marinelli, 2021),[22] and we calibrate $\kappa_G = \kappa_B = 1$, which means a log green bond utility and a quadratic brown bond disutility, which are values more in line with the literature.[23] We simulate the three types of Green QE (Figure A.6): not surprisingly, we find that Green QE has much smaller effects, given its limited ability to affect the green and brown interest rates. In particular the effect on emission is two orders of magnitude smaller, compared to the baseline scenario.

5 Discussion and concluding remarks

The main goal of this paper is to analyze whether green asset purchases by the central bank can contribute to reduce CO2 emissions, when the government gradually introduces a carbon tax, and to understand how this measures can be tailored in order to maximize their effectiveness. Our simulations show that a Green QE is able to curb emissions by shifting demand from the brown to the green sector. However, we also show that the effect on the stock of euro-area and global pollution (i.e. the net cumulative sum of emissions) is small. These results are obtained in a model with generous assumptions in favor of the effectiveness of Green QE: a concave inverse demand of green bonds by households, a large size of green asset purchases, an elasticity of substitution between green and brown goods higher than unity.

Our results complement the findings in Papoutsi et al. (2021), Ferrari and Nispi Landi (2021) and Abiry et al. (2022). In the theoretical section, Papoutsi et al. (2021) show how tilting the portfolio of the central bank toward green assets can increase the yield on brown assets, thus reducing emissions: however, they do not provide a quantitative evaluation of this effect; they also show that this policy is desirable if a carbon tax is absent. Ferrari and Nispi Landi (2021) find that various forms of temporary Green QE along the business cycle have a limited effect on pollution. Abiry et al. (2022) show that

[22] Specifically, we set $400\left(r^B - rr\right) = 0.00025 = 400\left(rr - r^G\right)$.

[23] This calibration implies $\nu_G = 1.5901e - 04$ and $\nu_B = 3.7210e - 06$.

tilting the portfolio of all the central banks in the world toward green assets (without changing the size of the balance sheets) yields small effects when this policy is combined with a global carbon tax; absent a carbon tax, the effects are larger. Remarkably, Abiry et al. (2022) use different assumptions compared to those made in the current paper. We focus instead on a central bank's purchase of green bonds financed with an increase in reserves, on top of a carbon tax, finding small effects on cumulative emissions. Without a carbon tax, we also find that Green QE becomes slightly more effective, as the carbon tax weakens the link between brown production and emissions. Taken together, these findings suggest that Green QE has only limited environmental benefits, thus it is a weak tool to address the climate challenge; Green QE becomes more useful should the government fail or delay the introduction of an appropriate carbon tax.

While our results point out to limited effectiveness of Green QE, they do not suggest that this policy is necessarily in contrast with the primary objective of the central bank and therefore it could be considered suitable to fulfill secondary objectives if within their mandate.[24] On the one hand, the gradual introduction of a permanent carbon tax reduces both production, as brown firms face higher costs, and prices, as households cut demand expecting a lower future income. On the other hand, Green QE stimulates production and prices, partially offsetting the economic impact of the tax. This coincidence crucially hinges on the deflationary effects of the carbon tax: as shown in Ferrari and Nispi Landi (2022), a carbon tax that increases over time is deflationary if households are rational and have perfect foresight.

Should a central bank decide to implement Green QE, our findings also have some relevant implications for the design of the policy. First, we find that Green QE is more effective on climate in the short run, while their effectiveness decreases over time as the carbon tax kicks in. As there can be costs associated to the deviations from the

[24]Among major central banks, the Bank of England was the first one to tilt its monetary policy portfolio toward greener assets under its secondary objective, i.e. support the economic policy of the government (for further details see https://www.bankofengland.co.uk/markets/greening-the-corporate-bond-purchase-scheme). ECB is now considering to adopt the same policy (https://www.ecb.europa.eu/press/pr/date/2021/html/ecb.pr210708$_1$ $f104919225.en.html$).

market neutrality principle, it is better to act more aggressively immediately and then progressively reach the market neutrality, as opposed to a gradual implementation of the measure. Second, we find that a crucial parameter to determine the effectiveness of the policy is the elasticity of substitution between the brown and the green good: in purchasing green bonds, central banks should determine what is green and what is brown in order to maximize this elasticity of substitution. This implies that the so called "best-in-class" seems to be more appropriate than the "best-in-universe" approach: instead of discriminating between sectors (best-in-universe approach) the policy should aim to discriminate between firms that produce substitute goods within the same sector, favoring those with the cleanest technology (best-in-class approach).

We have derived our findings using a baseline medium-scale New Keynesian model, augmented with features typical in the environmental literature (a brown sector, an emission function, a law of motion for pollution, abatement costs) and bond in the utility function to break the neutrality of central bank's purchases. Our calibration is fairly standard and the model yields reasonable second moments and impulse responses. However, we could have missed potentially relevant channels and some caveats are in order. First, in our model pollution does not affect TFP, hence we are ignoring feedback effects from the environment to the economic activity: in our model, the negative effects of climate change are underestimated. Second, the model could be enriched with an R&D sector, which can produce innovative green technologies that do not pollute or that reduce abatement spending, other things equal: Green QE can be targeted to bonds that finance the R&D sector, potentially stimulating a permanent expansion in green output. Third, we are not considering that Green QE may have a catalytic effect, spurring investment in the green sector by private agents. Fourth, the transition to a green economy could induce big transformations in the production technologies, and Green QE may accelerate these transformations. We leave these issues to feature research.

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Figures and Tables

Calibration

Parameter	Description	Value	Notes
β	Discount factor	0.9988	Real rate of 2% annually (NAWM-II)
φ	Inverse of Frisch elasticity	2	NAWM-II
ς	Habits	0.80	To match moments
ε	Elas. of subst. differentiated goods	3.8571	NAWM-II
α	Share of capital in production	0.2975	$\frac{i}{y} = 0.21$ (NAWM-II)
κ_P	Price adjustment costs	71.2043	NAWM-II (Calvo parameter)
δ	Depreciation rate	2.5%	NAWM-II
θ	Growth rate of trend variables	1.0038	NAWM-II
κ_I	Investment adjustment cost	10.78	NAWM-II
π	SS inflation	1.005	ECB target
$\tilde{g}$	Public spending	0.4831	$g/y = 0.215$ (NAWM-II)
ϕ_π	Taylor rule coefficient	2.74	NAWM-II
ρ_r	Inertia of Taylor rule	0.93	NAWM-II
ζ	Weight of brown good	0.8	Giovanardi et al. (2021)
ξ	Elas. of subst. brown-green good	2	Carattini et al. (2021)
δ_x	Pollution depreciation	0.0035	Gibson and Heutel (2020)
$\tilde{e}^{row}$	Emissions in the rest of the world	13.2974	$\frac{e^{row}}{e} = 15.31$
χ	Convexity of abatement function	1.6	Gibson and Heutel (2020)
ν_M	Coefficient in the abatement function	0.1924	Gibson and Heutel (2020)
ν_E	Coefficient in the emission function	0.4846	$p^C = 65$ under $\mu = 1$
ν_G	Bond utility parameter	15.1576	$400\left(r^G - rr\right) = -0.0015$
ν_B	Bond utility parameter	3.0663×10^{-14}	$400\left(r^B - rr\right) = 0.0015$
κ_G, κ_B	Bond elasticity	8.9300, 8.9367	$\eta = 300$
$\tilde{re}$	CB reserves	3.5983	$\frac{re}{4y} = 0.4$
$\tilde{b}_C^G, \tilde{b}_C^B$	CB corporate bonds	0, 0	NAWM-II
ρ_a	AR(1) TFP parameter	0.92	NAWM II
σ_a	Stand. dev. of TFP shock	1.17%	To match the stand. dev. of EA output

Table 1: Calibrated parameters.

Data vs Model

		$\tilde{y}_t$	$\tilde{c}_t$	$\tilde{i}_t$	$\tilde{\pi}_t$
Standard deviation (in %)	Data	1.19	0.66	2.85	0.28
	Model	1.19	1.05	2.85	0.47
Standard deviation relative to y	Data	1	0.55	2.39	0.23
	Model	1	0.88	2.40	0.40
Correlation with output	Data	1	0.83	0.84	0.42
	Model	1	0.99	0.99	0.14
Autocorrelation	Data	0.90	0.90	0.68	0.22
	Model	0.94	0.93	0.95	0.49

Table 2: The data refer to the HP filter of deseasonalized GDP, consumption, investment, and CPI inflation in the euro area, with a smoothing parameter of 1600, along the horizon 1999Q1-2019Q4. Source: Eurostat. The model refers to a first-order approximation of the equations, hit by TFP shocks.

The transition to a green economy

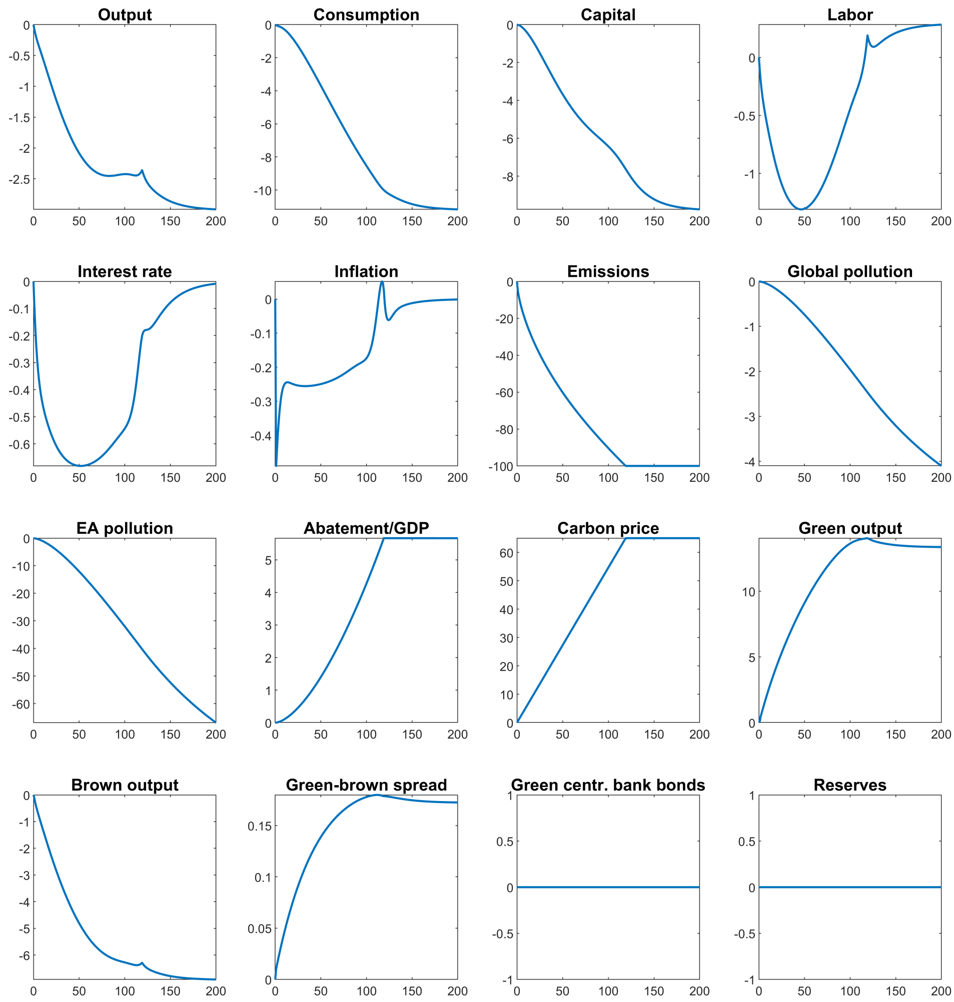

Figure 1: Transition to a zero-emission economy, driven by an emission tax. Most variables are in percentage deviations from the path they would have followed with no environmental policy; inflation, interest rate, and spread, whose responses are in level deviations reported at annual rates; carbon price is expressed in level deviations. The path for the emission tax is announced in period 0.

The impact of Green QE

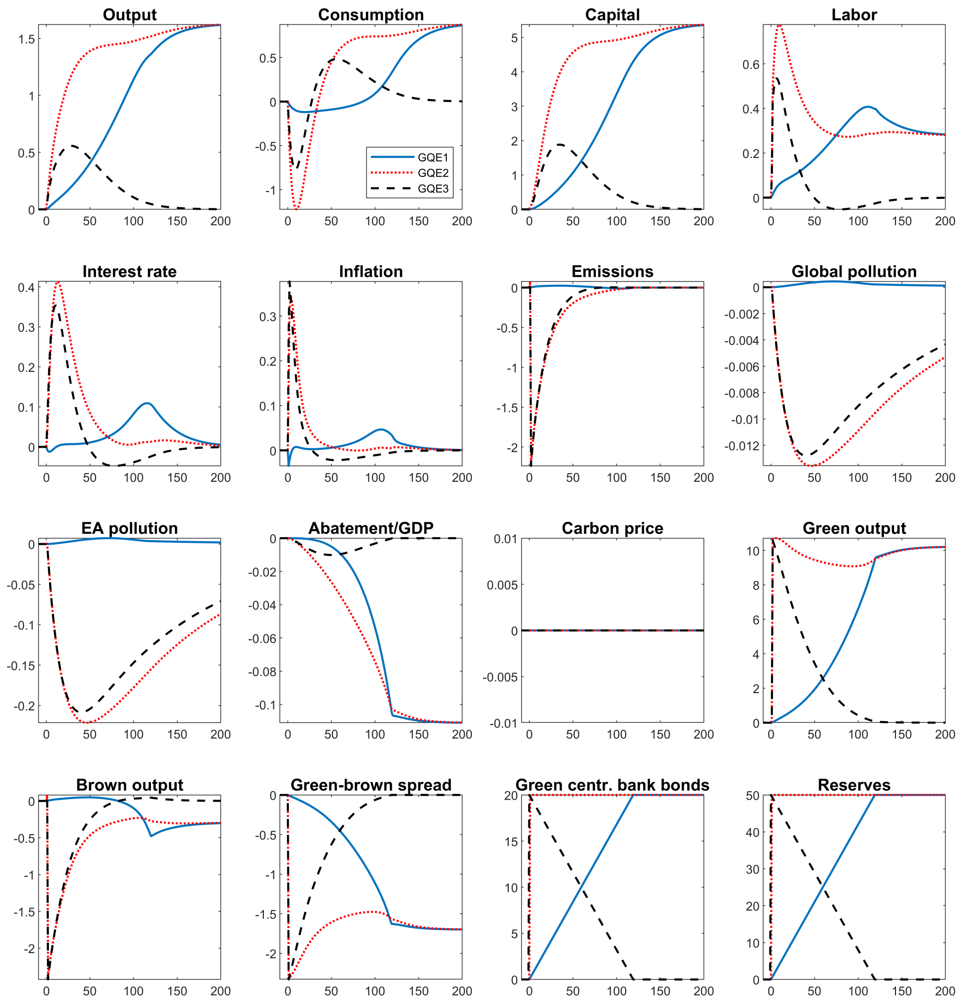

Figure 2: The lines show the marginal impact of Green QE on top of the carbon tax. Most variables are expressed in deviations compared to a scenario with no Green QE, divided by the initial level of the variable. Inflation, interest rate, and spread are not divided by their initial level. Green bonds held by the central banks are divided by annual steady-state GDP. Blue solid line: Green QE is gradual and permanent. Red dotted line: Green QE immediately jumps to the new steady state in period 1. Black dashed line: Green QE is transitory. In all scenarios, Green QE is announced in period 0.

Bond inverse demands

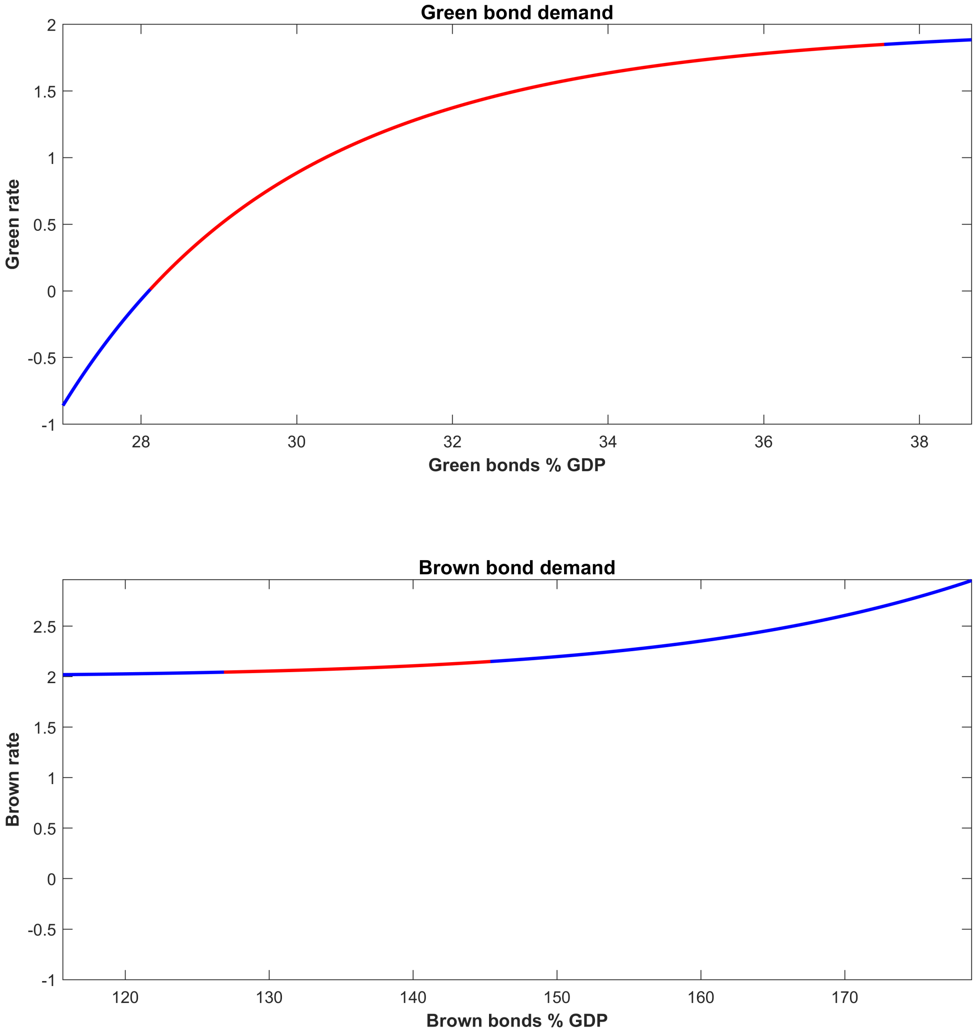

Figure 3: Bond inverse demands by households in % of initial annual GDP. Rates are annualized. The red line includes the interval of values that green and brown bonds held by households reach during the baseline GQE1 and GQE2.

Different Green QE sizes

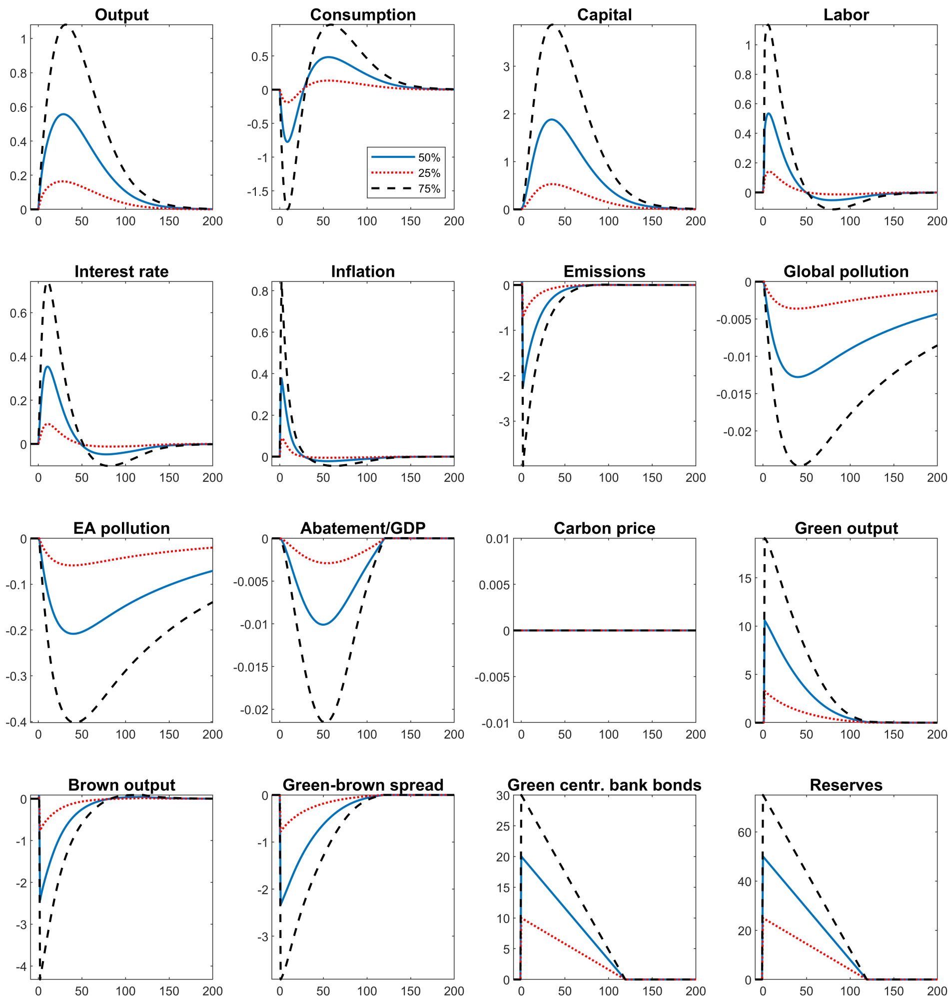

Figure 4: The lines show the marginal impact of Green QE on top of the carbon tax. Most variables are expressed in deviations compared to a scenario with no Green QE, divided by the initial level of the variable. Inflation, interest rate, and spread are not divided by their initial level. Green bonds held by the central bank are divided by annual steady-state GDP. Blue solid line: reserves increase by 50%. Red dotted line: reserves increase by 25%. Black dashed line: reserves increase by 75%. In all scenarios, Green QE is announced in period 0.

Credit easing

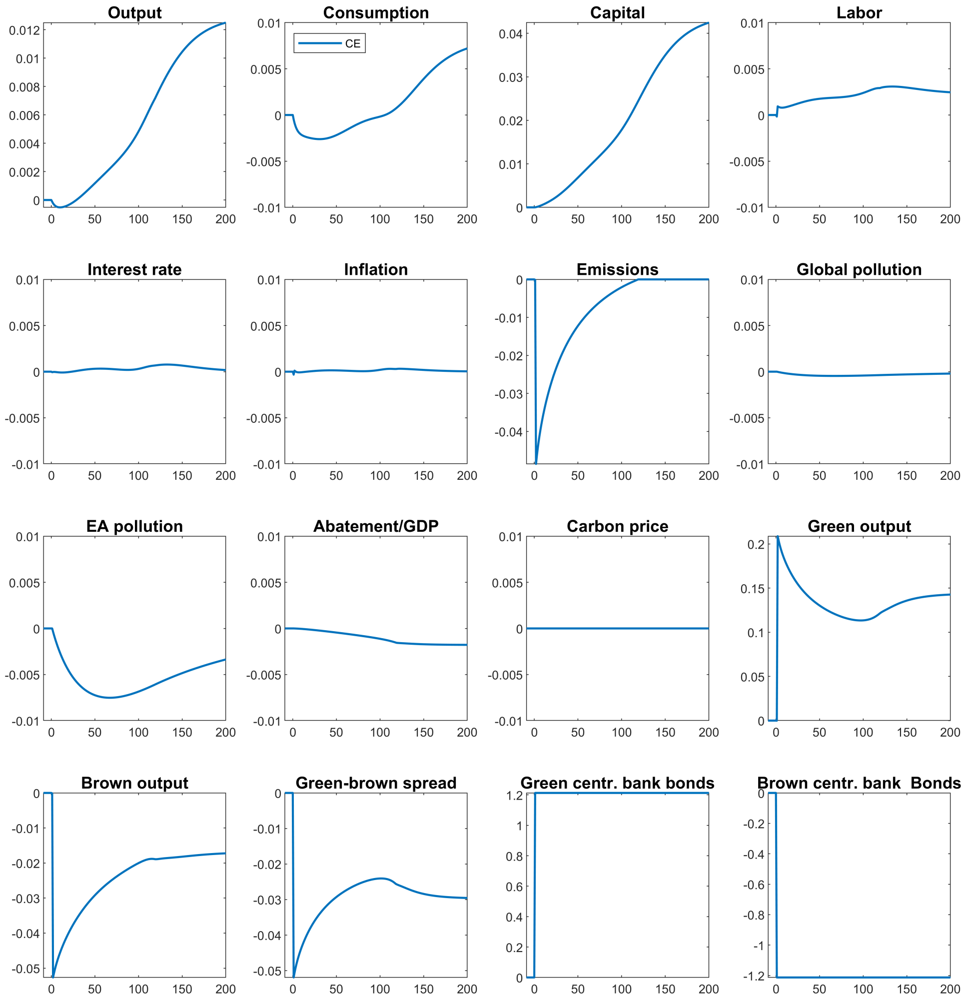

Figure 5: The lines show the marginal impact of credit easing on top of the carbon tax. Most variables are expressed in deviations compared to a scenario with no credit easing, divided by the initial level of the variable. Inflation, interest rate, and spread are not divided by their initial level. Green and brown central bank's bonds are divided by annual steady-state GDP. Blue solid line: credit easing, announced in period 0.

Online Appendix

A Model Equations

In this section, we list the full set of the model equations. We have detrended the non-stationary variables, by dividing them by the labor-augmenting productivity z_t, which grows at gross rate θ: these variables are denoted with a tilde. Within the trending variables, the only exception is λ_t, which grows at rate $\frac{1}{z_t}$. There are 29 equations for the following 29 endogenous variables:

$$X_t^{end} \equiv \Big[\tilde{c}_t, \tilde{i}_t, \tilde{y}_t, \tilde{k}_t, h_t, \tilde{w}_t, q_t, p_t^I, \pi_t, r_t, r_t^B, r_t^G, \tilde{b}_{Ht}^G, \tilde{b}_{Ht}^B,$$

$$\tilde{b}_{Ct}^G, \mu_t, p_t^G, p_t^B, \tilde{k}_t^G, \tilde{k}_t^B, h_t^G, h_t^B, r_{kt}^G, r_{kt}^B, \tilde{e}_t, \tilde{x}_t, \tilde{y}_t^G, \tilde{y}_t^B, \tilde{\lambda}_t\Big].$$

There are 2 exogenous variables:

$$X_t^{exo} \equiv \left[\tau_t, \tilde{re}_t\right].$$

The 29 equations are the following. Labor supply condition:

$$h_t^{\varphi} = \tilde{w}_t \tilde{\lambda}_t. \tag{A.1}$$

Euler equation for public bonds:

$$1 = \beta \mathbb{E}_t \left(\frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t \theta} \frac{r_t}{\pi_{t+1}} \right). \tag{A.2}$$

Euler equation for green and brown bonds:

$$1 = \beta \mathbb{E}_t \left[\frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t \theta} r_{t+1}^G \right] + \frac{\nu_G}{\tilde{\lambda}_t} \left(\tilde{b}_{Ht}^G \right)^{-\kappa_G} \tag{A.3}$$

$$1 = \beta \mathbb{E}_t \left[\frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t \theta} r_{t+1}^B \right] - \frac{\nu_B}{\tilde{\lambda}_t} \left(\tilde{b}_{Ht}^B \right)^{\kappa_B}. \tag{A.4}$$

Production function of intermediate firms:

$$\tilde{y}_t = \left[(1-\zeta)^{\frac{1}{\xi}} \left(\tilde{y}_t^G \right)^{\frac{\xi-1}{\xi}} + \zeta^{\frac{1}{\xi}} \left(\tilde{y}_t^B \right)^{\frac{\xi-1}{\xi}} \right]^{\frac{\xi}{\xi-1}}. \tag{A.5}$$

Demand of green and brown output:

$$\tilde{y}_t^G = (1-\zeta) \left(\frac{p_t^G}{p_t^I} \right)^{-\xi} \tilde{y}_t \tag{A.6}$$

$$\tilde{y}_t^B = \zeta \left(\frac{p_t^B}{p_t^I} \right)^{-\xi} \tilde{y}_t. \tag{A.7}$$

Phillips curve:

$$\pi_t (\pi_t - \overline{\pi}) = \beta \mathbb{E}_t \left[\frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t} \frac{\tilde{y}_{t+1}}{\tilde{y}_t} \pi_{t+1} (\pi_{t+1} - \overline{\pi}) \right] + \frac{\varepsilon}{\kappa_P} \left(p_t^I - \frac{\varepsilon - 1}{\varepsilon} \right). \tag{A.8}$$

Green and brown production functions:

$$\tilde{y}_t^G = \left(\frac{\tilde{k}_{t-1}^G}{\theta} \right)^{\alpha} \left(h_t^G \right)^{1-\alpha} \tag{A.9}$$

$$\tilde{y}_t^B = \left(\frac{\tilde{k}_{t-1}^B}{\theta} \right)^{\alpha} \left(h_t^B \right)^{1-\alpha}. \tag{A.10}$$

Green and brown labor demands:

$$\tilde{w}_t h_t^G = (1-\alpha) p_t^G \tilde{y}_t^G \tag{A.11}$$

$$\tilde{w}_t h_t^B = (1-\alpha) \left[p_t^B - \tau_t (1-\mu_t) \nu_E - \frac{\nu_M}{1+\chi} \mu_t^{1+\chi} \right] \tilde{y}_t^B. \tag{A.12}$$

Green and brown capital demand:

$$r_{kt}^{G}\frac{\tilde{k}_{t-1}^{G}}{\theta} = \alpha p_t^G \tilde{y}_t^G \tag{A.13}$$

$$r_{kt}^{B}\frac{\tilde{k}_{t-1}^{B}}{\theta} = \alpha \left[p_t^B - \tau_t \left(1-\mu_t\right)\nu_E - \frac{\nu_M}{1+\chi}\mu_t^{1+\chi}\right]\tilde{y}_t^B. \tag{A.14}$$

Definition of rental rates of capital:

$$r_{kt}^{G} = r_t^G q_{t-1} - \left(1-\delta\right) q_t \tag{A.15}$$

$$r_{kt}^{B} = r_t^B q_{t-1} - \left(1-\delta\right) q_t. \tag{A.16}$$

Optimal abatement effort:

$$\mu_t = \left(\frac{\nu_E \tau_t}{\nu_M}\right)^{\frac{1}{\chi}}. \tag{A.17}$$

Emission function:

$$\tilde{e}_t = \left(1-\mu_t\right)\nu_E \tilde{y}_t^B. \tag{A.18}$$

Law of motion of pollution:

$$\tilde{x}_t = \left(1-\delta^x\right)\frac{\tilde{x}_{t-1}}{\theta} + \tilde{e}_t + \tilde{e}^{row}. \tag{A.19}$$

Tobin Q evolution:

$$\begin{aligned}1 =& q_t\left[1 - \frac{\kappa_I}{2}\left(\frac{\tilde{i}_t}{\tilde{i}_{t-1}}\theta - \theta\right)^2 - \kappa_I\frac{\tilde{i}_t}{\tilde{i}_{t-1}}\theta\left(\frac{\tilde{i}_t}{\tilde{i}_{t-1}}\theta - \theta\right)\right] + \\ &+ \beta\mathbb{E}_t\left[\frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t\theta}q_{t+1}\left(\frac{\tilde{i}_{t+1}}{\tilde{i}_t}\theta\right)^2\kappa_I\left(\frac{\tilde{i}_{t+1}}{\tilde{i}_t}\theta - \theta\right)\right].\end{aligned} \tag{A.20}$$

Law of motion of capital:

$$\tilde{k}_t = \left(1-\delta\right)\frac{\tilde{k}_{t-1}}{\theta} + \left[1 - \frac{\kappa_I}{2}\left(\frac{\tilde{i}_t}{\tilde{i}_{t-1}}\theta - \theta\right)^2\right]\tilde{i}_t. \tag{A.21}$$

Resource constraint:

$$\tilde{y}_t = \tilde{c}_t + \tilde{i}_t + \tilde{g} + \tilde{y}_t^B \frac{\nu_M}{1+\chi} \mu_t^{1+\chi} + \frac{\kappa_P}{2} \left(\pi_t - \overline{\pi}\right)^2 \tilde{y}_t. \tag{A.22}$$

Market clearing for labor and capital:

$$h_t = h_t^B + h_t^G \tag{A.23}$$

$$\tilde{k}_t = \tilde{k}_t^B + \tilde{k}_t^G. \tag{A.24}$$

Market clearing for green and brown bonds:

$$q_t \tilde{k}_t^G = \tilde{b}_{Ht}^G + \tilde{b}_{Ct}^G \tag{A.25}$$

$$q_t \tilde{k}_t^B = \tilde{b}_{Ht}^B + \tilde{b}_C^B. \tag{A.26}$$

Taylor rule:

$$\frac{r_t}{r} = \left(\frac{r_{t-1}}{r}\right)^{\rho_r} \left(\frac{\pi_t}{\overline{\pi}}\right)^{\phi_\pi (1-\rho_r)}. \tag{A.27}$$

Balance sheets of the central bank:

$$\tilde{b}_{Ct}^G + \tilde{b}_C^B + \tilde{d}_C = r \tilde{e}_t. \tag{A.28}$$

Marginal utility of consumption:

$$\tilde{\lambda}_t = \frac{\theta}{\theta \tilde{c}_t - \varsigma \tilde{c}_{t-1}} - \beta \varsigma \mathbb{E}_t \left(\frac{1}{\theta \tilde{c}_{t+1} - \varsigma \tilde{c}_t} \right). \tag{A.29}$$

We also define the price of carbon and the EA pollution as follows:

$$p_t^C = \frac{s_1 s_2}{s_3} \tau_t \tag{A.30}$$

$$\tilde{x}_t^{ea} = \left(1 - \delta^x\right) \frac{\tilde{x}_{t-1}^{ea}}{\theta} + \tilde{e}_t. \tag{A.31}$$

B Initial Steady State

We compute the initial steady state using the following strategy. We simplify the model in a system of three equations and three variables (y, p^B,e). We set $\gamma^G \equiv r^G - rr$ and $\gamma^B \equiv r^B - rr$ ex ante and compute ν_G and ν_B ex post. We calibrate ex ante the real interest rate $rr = \frac{r}{\pi}$ and compute β ex post. We set $I \equiv \frac{i}{y}$ and $G \equiv \frac{g}{y}$ ex ante and compute α and $\tilde{g}$ ex post. We set $p^C = 65$ when $\tau = 1$, computing ν_E ex post. We set ex ante $\eta \equiv \frac{r^B}{\kappa_B(rr - r^B)} = \frac{r^G}{\kappa_G(r^G - rr)}$. We set $RoW \equiv \frac{\tilde{e}^{row}}{\tilde{e}}$ and compute $\tilde{e}^{row}$ ex post. In the initial steady state, $\tau = 0$, which implies $\mu = 0$.

Using the Euler equation for bonds,

$$\beta = \frac{\theta}{rr}.$$

Using the Phillips Curve and the Euler equations, we get:

$$\begin{aligned}
\pi &= \bar{\pi} \\
r &= \frac{\bar{\pi}\theta}{\beta} \\
r^G &= rr + \gamma^G \\
r^B &= rr + \gamma^B \\
r_k^B &= r^B - (1 - \delta) \\
r_k^G &= r^G - (1 - \delta) \\
q &= 1 \\
p_I &= \frac{\varepsilon - 1}{\varepsilon}.
\end{aligned}$$

Use the definition of p_I to find p^G:

$$\left(p^I\right)^{1-\xi} = \left[(1-\zeta)\left(p^G\right)^{1-\xi} + \zeta\left(p^B\right)^{1-\xi}\right]$$

$$\left(p^G\right)^{1-\xi} = \frac{1}{1-\zeta}\left[\left(p^I\right)^{1-\xi} - \zeta\left(p^B\right)^{1-\xi}\right]$$

$$p^G = \left\{\frac{1}{1-\zeta}\left[\left(p^I\right)^{1-\xi} - \zeta\left(p^B\right)^{1-\xi}\right]\right\}^{\frac{1}{1-\xi}}.$$

Use the input demands to find $\tilde{y}^B$ and $\tilde{y}^G$:

$$\tilde{y}^B = \zeta\left(\frac{p^B}{p^I}\right)^{-\xi}\tilde{y}$$

$$\tilde{y}^G = (1-\zeta)\left(\frac{p^G}{p^I}\right)^{-\xi}\tilde{y}.$$

Given $\tilde{y}$, we find s_1:

$$s_1 = \frac{y^E}{\tilde{y}}.$$

Find $\tilde{e}^{row}$ using RoW:

$$\tilde{e}^{row} = RoW \cdot \tilde{e}.$$

Given $\tilde{e}$, we find $\tilde{x}$ using the law of motion of atmospheric carbon:

$$\tilde{x} = \frac{\tilde{e} + e^{row}}{1 - \frac{1-\delta^x}{\theta}}.$$

Given $\tilde{x}$, we can find s_2:

$$s_2 = \frac{x^{GtC}}{\tilde{x}}.$$

When $\mu = 1$, $\tau^{full} = \frac{\nu_M}{\nu_E}$; hence, under full abatement it holds:

$$p^{Cfull} = \frac{s_1 s_2}{s_3} \tau^{full}$$

$$p^{Cfull} = \frac{s_1 s_2}{s_3} \frac{\nu_M}{\nu_E}$$

$$\nu_E = \frac{s_1 s_2}{s_3} \frac{\nu_M}{p^{Cfull}}$$

and we get ν_E. Given the investment ratio, we find $\tilde{i}$ and $\tilde{k}$:

$$\tilde{i} = I \tilde{y}$$

$$\tilde{k} = \frac{\tilde{i}}{\left(1 - \frac{1-\delta}{\theta}\right)}.$$

By the capital demands we know that:

$$\tilde{k}^G = \alpha \theta \frac{p^G \tilde{y}^G}{r_k^G}$$

$$\tilde{k}^B = \alpha \theta \frac{\tilde{y}^B}{r_k^B} \left[p^B - \tau \left(1 - \mu_t\right) \nu_E - \frac{\nu_M}{1+\chi} \mu^{1+\chi} \right].$$

Sum the capital demands:

$$\tilde{k} = \alpha \theta \left\{ \frac{p^G \tilde{y}^G}{r_k^G} + \frac{\tilde{y}^B}{r_k^B} \left[p^B - \tau \left(1 - \mu\right) \nu_E - \frac{\nu_M}{1+\chi} \mu^{1+\chi} \right] \right\},$$

and find α:

$$\alpha = \frac{\tilde{k}}{\theta \left\{ \frac{p^G \tilde{y}^G}{r_k^G} + \frac{\tilde{y}^B}{r_k^B} \left[p^B - \tau \left(1 - \mu_t\right) \nu_E - \frac{\nu_M}{1+\chi} \mu^{1+\chi} \right] \right\}},$$

and then use the capital demands to find $\tilde{k}^G$ and $\tilde{k}^B$. Use the production function to find h^B and h^B:

$$h^B = \left[\frac{\tilde{y}^B}{a\left(\frac{\tilde{k}^B}{\theta}\right)^\alpha}\right]^{\frac{1}{1-\alpha}}$$

$$h^G = \left[\frac{\tilde{y}^G}{a\left(\frac{\tilde{k}^G}{\theta}\right)^\alpha}\right]^{\frac{1}{1-\alpha}}.$$

Bonds held by households:

$$\tilde{b}_H^G = \tilde{k}^G - \tilde{b}_C^G$$

$$\tilde{b}_H^B = \tilde{k}^B - \tilde{b}_C^B,$$

given that in the initial steady state we know $\tilde{b}_C^G = \tilde{b}_C^B$ (they are both 0). Using the labor demand in the green sector we can find w:

$$w = \frac{(1-\alpha)\,\tilde{p}^G\tilde{y}^G}{h^G}.$$

Given the public spending ratio, we find $\tilde{g}$:

$$\tilde{g} = G\tilde{y}.$$

We find consumption by the resource constraint:

$$\tilde{c} = \tilde{y} - \tilde{i} - \tilde{g} - \tilde{y}^B\frac{\nu_M}{1+\chi}\mu^{1+\chi}.$$

Aggregate labor is given by:

$$h = h^B + h^G.$$

Using the definition of η, we find κ_G and κ_B:

$$\kappa_B = \frac{r^B}{\eta\left(rr - r^B\right)}$$
$$\kappa_G = \frac{r^G}{\eta\left(r^G - rr\right)}.$$

Marginal utility of consumption:

$$\tilde{\lambda}_t = \frac{\theta - \beta\varsigma}{\tilde{c}\left(\theta - \varsigma\right)}.$$

Using the bond Euler equations, we find the utility parameters:

$$1 = \frac{\beta}{\theta} r^G + \frac{\nu_G}{\tilde{\lambda}} \left(\tilde{b}_H^G\right)^{-\kappa_G}$$
$$\nu_G = \tilde{\lambda} \frac{\left(1 - \frac{\beta}{\theta} r^G\right)}{\left(\tilde{b}_H^G\right)^{-\kappa_G}}$$

$$1 = \frac{\beta}{\theta} r^B - \frac{\nu_B}{\tilde{\lambda}} \left(\tilde{b}_H^B\right)^{\kappa_B}$$
$$\nu_B = \tilde{\lambda} \frac{\frac{\beta}{\theta} r^B - 1}{\left(\tilde{b}_H^B\right)^{\kappa_B}}$$

We are left with three equations in three unknowns:

$$\tilde{w} h^B = (1-\alpha)\left[p^B - \tau\left(1-\mu\right)\nu_E - \frac{\nu_M}{1+\chi}\mu^{1+\chi}\right]\tilde{y}^B$$

$$\tilde{\lambda}\tilde{w} = h^{\varphi}$$

$$\tilde{e} = \left(1-\mu\right)\nu_E \tilde{y}_B.$$

C Final Steady State

In the final steady state, we set $\mu = 1$, which implies $\tilde{e} = 0$. Compared to the procedure for the initial steady state, we let $\tilde{r^G}$, and $\tilde{r^B}$ to be determined ex post. We simplify the model to a system of four equations and four variables: $\left\{y, p^B, r^G, r^B\right\}$. Following the same steps to compute the initial steady state, we end up with the following system of equations:

$$
\begin{aligned}
\tilde{w}h^B &= (1-\alpha)\left[p^B - \tau\left(1-\mu\right)\nu_E - \frac{\nu_M}{1+\chi}\mu^{1+\chi}\right]\tilde{y}^B \\
\tilde{\lambda}\tilde{w} &= h^{\varphi} \\
1 &= \frac{\beta}{\theta}r^G + \frac{\nu_G}{\tilde{\lambda}}\left(\tilde{b}_H^G\right)^{-\kappa_G} \\
1 &= \frac{\beta}{\theta}r^B - \frac{\nu_B}{\tilde{\lambda}}\left(\tilde{b}_H^B\right)^{\kappa_B}.
\end{aligned}
$$

D Additional figures and table

IRF to a positive TFP shock

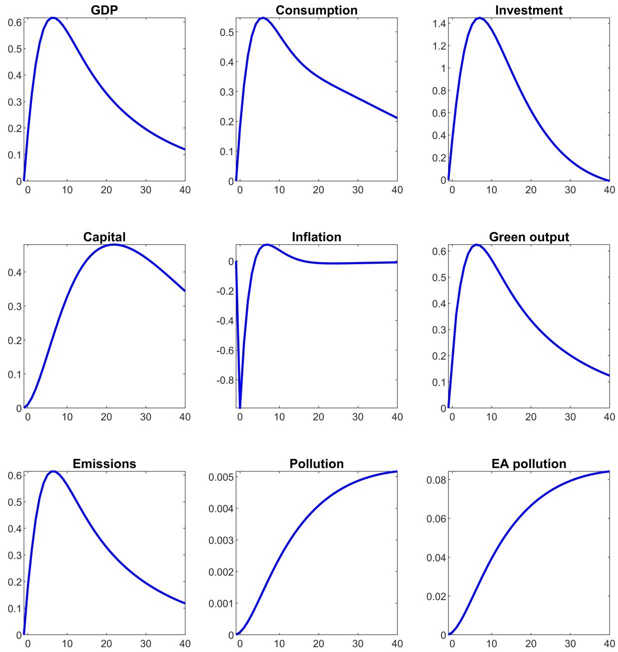

Figure A.1: IRF to a transitory TFP shock. Variables are potted as percentage deviations from the steady state, except inflation, which is plotted in annualized level deviations.

IRF to an expansionary monetary shock

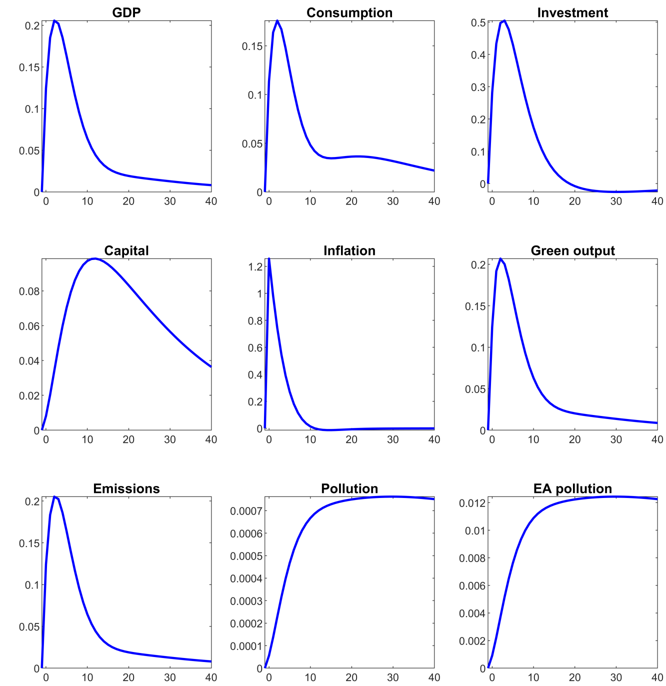

Figure A.2: IRF to an exogenous reduction in the policy rate. Variables are potted as percentage deviations from the steady state, except inflation, which is plotted in annualized level deviations.

GQE vs QE

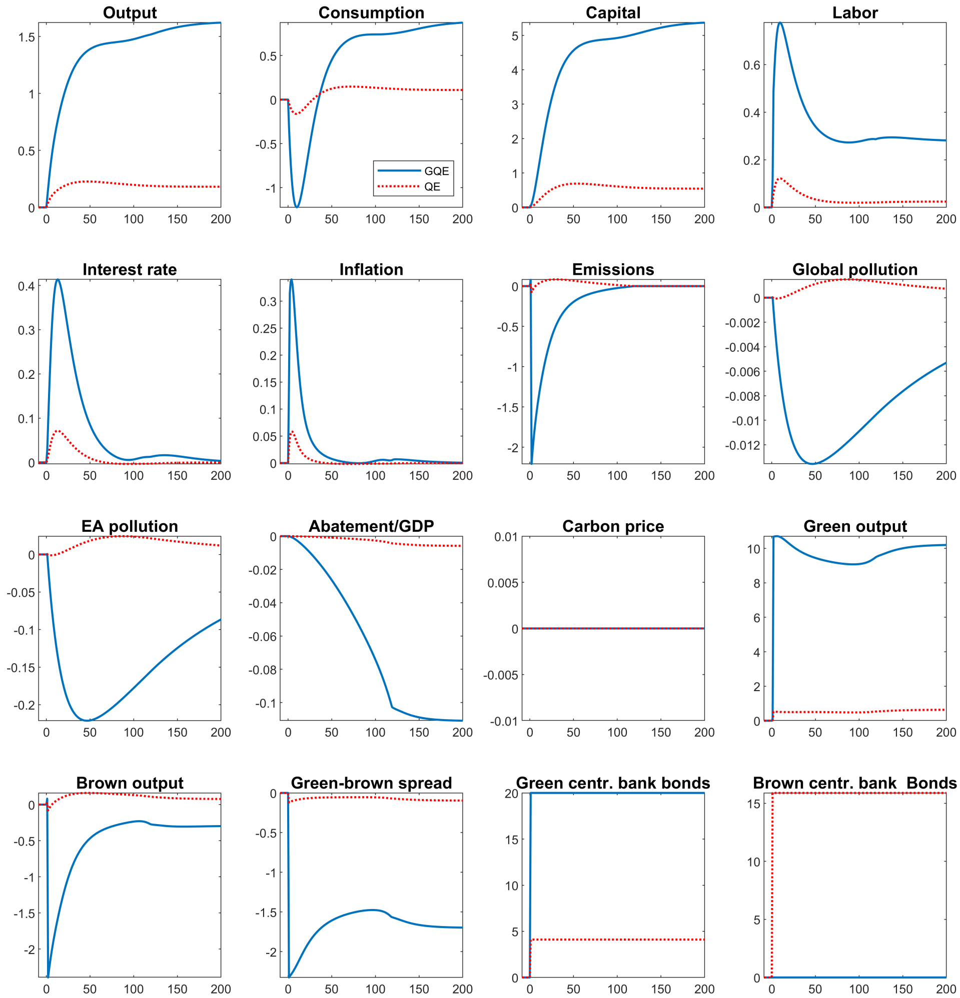

Figure A.3: The lines show the marginal impact of Green QE (or QE) on top of the carbon tax. Most variables are expressed in deviations compared to a scenario with no Green QE (QE), divided by the initial level of the variable. Inflation, interest rate, and spread are not divided by their initial level. Green and brown bonds held by the central bank are divided by annual steady-state GDP. Blue solid line: GQE2 baseline. Red dotted line: market-neutral QE. In all scenarios, Green QE is announced in period 0.

Green QE without fiscal policy

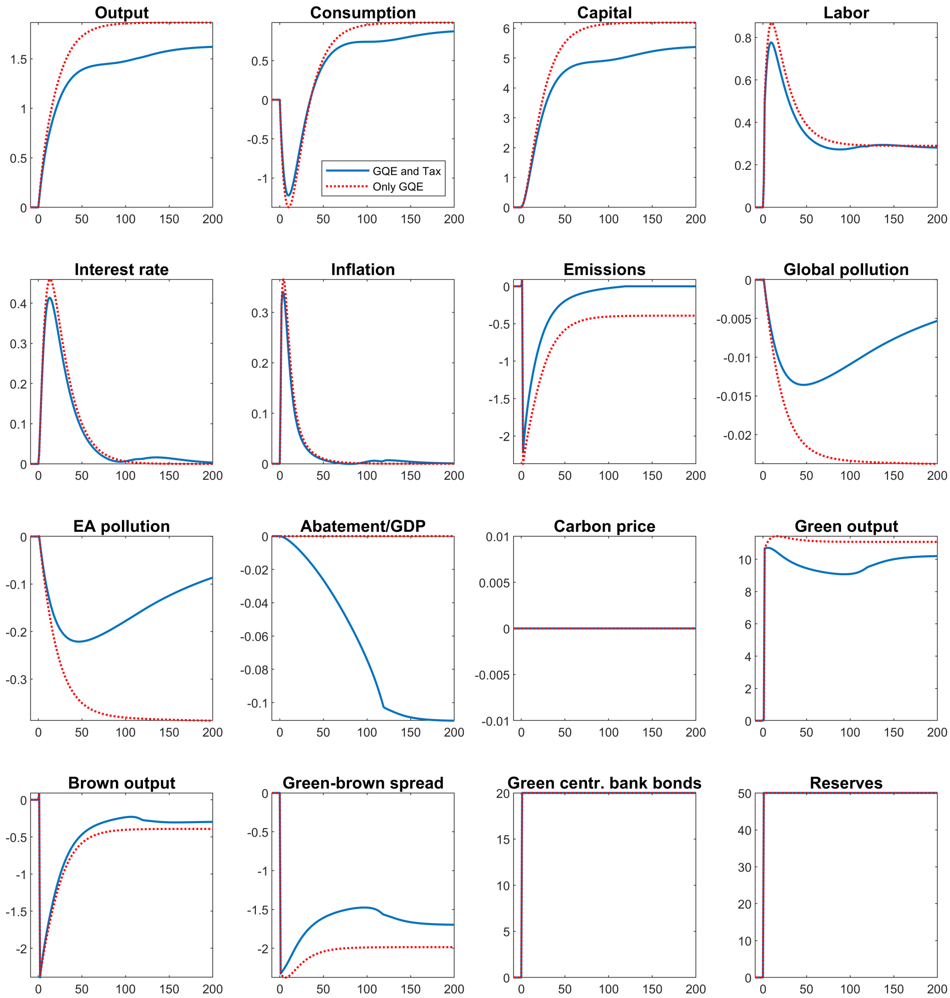

Figure A.4: The lines show the marginal impact of Green QE on top of the carbon tax (blue solid line) or compared to a no-policy scenario (red dotted line). Most variables are expressed in deviations compared to a scenario with no Green QE, divided by the initial level of the variable. Inflation, interest rate, and spread are not divided by their initial level. Green bonds are divided by annual steady-state GDP.

Green QE: Changing ζ and ξ

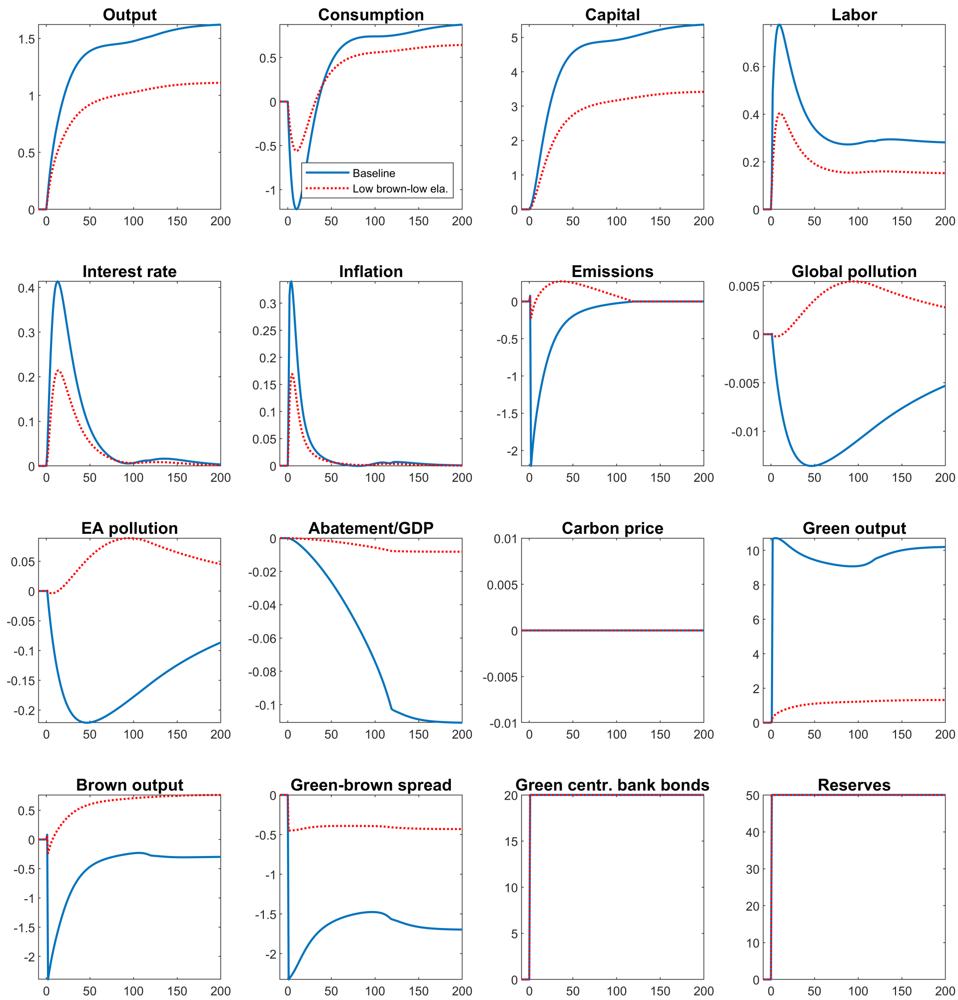

Figure A.5: The lines show the marginal impact of Green QE on top of the carbon tax. Most variables are expressed in deviations compared to a scenario with no Green QE, divided by the initial level of the variable. Inflation, interest rate, and spread are not divided by their initial level. Green bonds are divided by annual steady-state GDP. Blue solid line: $\zeta = 0.8$, $\xi = 2$. Red dotted line: $\zeta = 0.35$, $\xi = 0.5$.

Green QE: changing the bond utility function

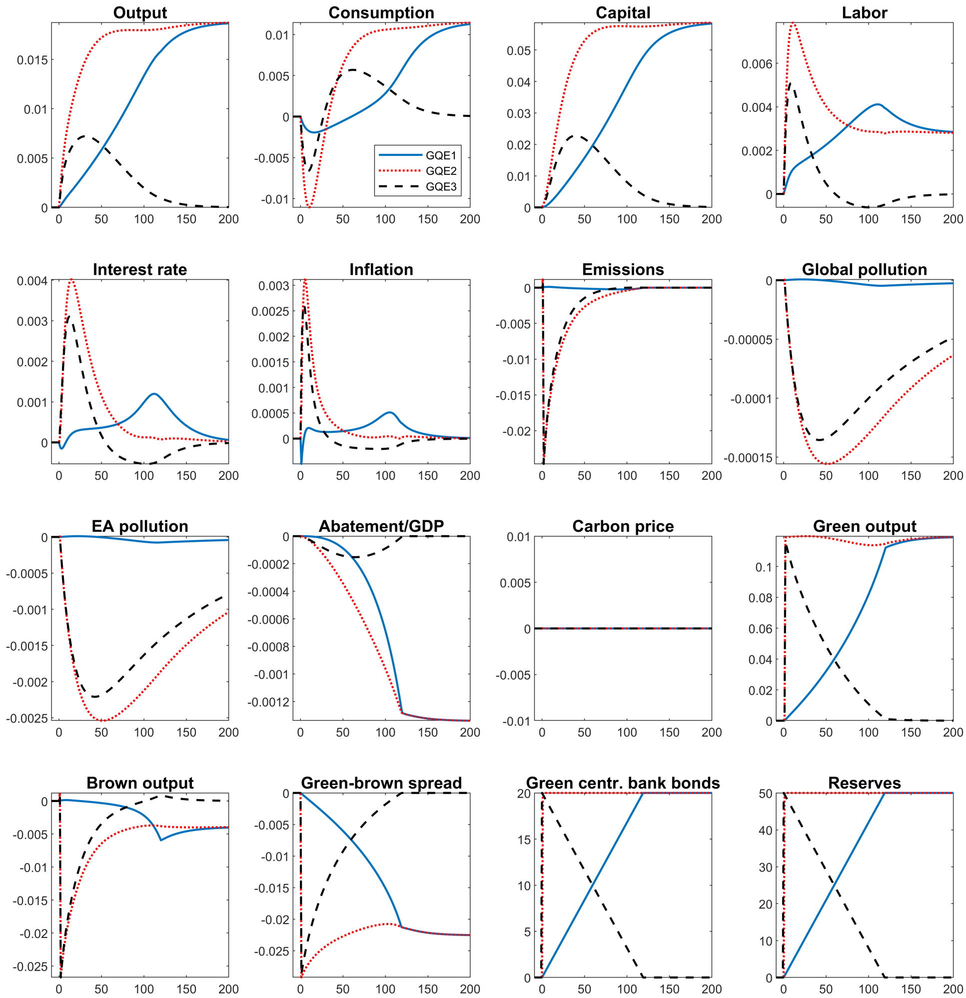

Figure A.6: We set $400\left(rr - r^B\right) = 0.00025 = -400\left(r^G - rr\right)$ and $\kappa_B = \kappa_G = 1$. The lines show the marginal impact of Green QE on top of the carbon tax. Most variables are expressed in deviations compared to a scenario with no Green QE, divided by the initial level of the variable. Inflation, interest rate, and spread are not divided by their initial level. Green bonds are divided by annual steady-state GDP. Blue solid line: Green QE is gradual and permanent. Red dotted line: Green QE immediately jumps to the new steady state in period 1. Black dashed line: Green QE is transitory. In all scenarios, Green QE is announced in period 0.

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