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Rational inattention and the business cycle effects of productivity and news shocks

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Abstract

We solve a real business cycle model with rational inattention (an RI-RBC model). In the RI-RBC model, the growth rates of employment, investment, and output are about as persistent as in the data, with an amount of inattention consistent with survey data on expectations. Moreover, consumption, employment, and output move in the same direction in response to news about future productivity. By contrast, the baseline RBC model produces neither persistent growth rates nor business cycle comovement after news shocks.

Keywords: information choice, rational inattention, real business cycle model, productivity shocks, news shocks (*JEL*: D83, E32, E71).

NON-TECHNICAL SUMMARY

A lot of information is available in today's world. Rational inattention (RI) is the idea that individuals cannot absorb all available information but can decide what to focus on – information may be plentiful, but attention is scarce.

The literature on RI studies the consequences of a rational allocation of attention for economic outcomes. This paper introduces RI into a baseline model of the business cycle, the real business cycle (RBC) model. In the RBC model, macroeconomic fluctuations are caused by changes in aggregate productivity. The paper studies the effects of aggregate productivity changes in the model when the standard assumption of perfect information is replaced by the assumption that decision-makers in firms and households are rationally inattentive. More generally, the paper develops a methodology for solving business cycle models when agents are rationally inattentive.

A key challenge for business cycle models is to explain why macroeconomic fluctuations are persistent. In the data, the growth rates of aggregate employment, investment, and output are positively autocorrelated. The baseline RBC model fails to reproduce this feature of the data. The RI-RBC model in this paper introduces an amount of inattention consistent with survey data on expectations. The growth rates of aggregate employment, investment, and output in the model are then approximately in line with the data. At the same time, the expected loss to firms and households from being inattentive is small. Thus, in the RI-RBC model macroeconomic fluctuations are persistent because information disseminates gradually when individuals are rationally inattentive.

The propagation of news shocks about aggregate productivity is also altered under rational inattention. A "news shock" means that information about future changes in productivity becomes available while current productivity is unchanged. Rationally inattentive decision-makers absorb only a limited amount of the available information, however. Consequently, they are unsure about the precise timing of productivity changes, and they act as if productivity has already changed with some probability. This effect implies that in the RI-RBC model consumption, employment, and output move in the same direction after a news shock, as they do in the data in a business cycle. By contrast, the baseline RBC model fails to produce business cycle comovement after a news shock.

1 Introduction

Rational inattention is the idea that people cannot process all available information but can decide what to focus on. The literature on rational inattention, started by Sims (2003) and recently reviewed in Maćkowiak, Matějka, and Wiederholt (2023), is growing rapidly. However, even though Sims motivated rational inattention in the context of business cycle analysis, very few papers so far have solved a dynamic stochastic general equilibrium (DSGE) model with rational inattention.[1]

Solving a DSGE model with rational inattention is challenging. In a rational inattention model, a decision-maker chooses an optimal signal about the state of the economy, recognizing that a more informative signal requires more attention, which is costly. The agent takes actions based on the optimal signal, rather than based on perfect information or some exogenous incomplete information set. To solve a DSGE model with rational inattention, one needs to solve signal choice problems of individual agents in a dynamic environment. Moreover, one needs to find the fixed point of an economy in which the optimal signal of an agent depends on the signals chosen by other agents.

In this paper, we solve a baseline real business cycle model with rational inattention (an RI-RBC model).[2] Neoclassical firms produce homogeneous output with capital and labor. There are no adjustment costs. Households have standard preferences for consumption and leisure. Aggregate total factor productivity (TFP) follows an exogenous stochastic process. The physical environment is completely standard, and the perfect information equilibrium is familiar. We focus on the equilibrium when decision-makers in firms are subject to rational inattention and households have perfect information. We also solve for the equilibrium when all agents, firms and households, are subject to rational inattention. We show how rational inattention changes the propagation of productivity shocks and news about future productivity.

In the data, TFP approximately follows a random walk process, or the growth rate of TFP approximately follows a white noise process. A standard exercise in a business cycle model is to assume a productivity process like the one in the data and compare the second moments of endogenous variables in the model and in the data. It has been a challenge for the RBC model to match the persistence in the data. The first-order autocorrelations of employment, investment, and

[1] See Maćkowiak and Wiederholt (2015) and Afrouzi and Yang (2021).

[2] Kydland and Prescott (1982), Hansen (1985), Prescott (1986), and King, Plosser, and Rebelo (1988) are classic references on the RBC model.

output growth are positive in the data but approximately zero in the baseline model. In the baseline model, these variables inherit the autocorrelation of exogenous productivity growth.[3] We find that rational inattention improves the propagation mechanism. When firms are subject to rational inattention, the impulse responses of employment, investment, and output to a productivity shock become hump-shaped. Since the optimal signal contains noise, the firms' beliefs are anchored on the steady state and evolve gradually. As a result, employment, investment, and output respond with delay to a productivity shock. The first-order autocorrelations of employment, investment, and output growth in the model become positive. Remarkably, they are approximately in line with the data when we assume an amount of inattention by firms consistent with survey data on expectations. This quantitative finding holds true even though rational inattention is the only source of inertia and the marginal cost of attention is small.

Next, we turn to news shocks about future productivity ("news shocks" for short). In the RBC model, fluctuations in productivity generate business cycle comovement: employment, investment, output, and consumption move in the same direction in response to a productivity shock, as they do in the data in a business cycle expansion or contraction. However, this result is sensitive to the timing of information in the model. In the real world, information about changes in productivity may become available some time before they occur. In the model, it matters if agents can learn in advance about changes in productivity. If agents can learn in advance, variables respond in ways inconsistent with a business cycle. Suppose that productivity will rise in the future, while current productivity is unchanged. The news shock causes a wealth effect. Firms have no incentive to increase labor demand before productivity improves, while households reduce labor supply due to the wealth effect. As a result, hours worked fall. With capital predetermined and current productivity unchanged, output contracts. Consumption rises due to the wealth effect while investment falls. The model fails to produce comovement in response to news about future productivity. It predicts an output contraction after news that productivity will improve.[4]

To obtain comovement after a news shock, one can make particular assumptions about production and preferences in the neoclassical model. Alternatively, one can combine imperfect competi-

[3]This shortcoming of the model was first noted by Cogley and Nason (1995) and Rotemberg and Woodford (1996).

[4]With a high elasticity of intertemporal substitution, the model predicts a rise in employment and investment and a fall in consumption. The substitution effect due to an increase in the real interest rate dominates the wealth effect in this case, pushing consumption down and labor supply up.

tion and nominal stickiness with suboptimal monetary policy.[5] We think that how economic agents respond to news is fundamentally a question about expectation formation. Information about changes in productivity may be available in advance, but people may fail to absorb it completely. The assumption of rational inattention seems well suited to apply to the question if people have an incentive to be perfectly aware of the timing of aggregate productivity changes.

How does a news shock propagate when agents are subject to rational inattention? The main qualitative insight is that rational inattention causes an increase in the firms' demand for capital and labor on impact of a positive news shock. The reason is that the optimal signal confounds current with future productivity.[6] Thus, firms react on impact of a news shock as if productivity has already changed with some probability. The intuition for the optimality of a confounding signal is that, under rational inattention, it is optimal to choose low-dimensional signals and it is optimal to get shocks into beliefs early on. Choosing a signal with a dimension strictly smaller than the dimension of the state of the economy saves on attention. Getting shocks into beliefs early on smooths the resulting action which helps minimize the expected loss from inattention.

The main quantitative insight is that the rational inattention effect on input demand is strong enough to change the responses of employment and output on impact of a news shock from negative to positive, and the response of investment from negative to approximately zero. The rational inattention effect on labor demand more than offsets the wealth effect on labor supply. Thus, employment and output increase on impact of a news shock. The rational inattention effect on investment demand approximately offsets the wealth effect on desired saving. As a result, the response of investment on impact of a news shock is approximately zero, as opposed to a sizable negative number in the standard model.

Hence, the single assumption of rational inattention by firms makes the model predict an output expansion after news that productivity will improve. By assuming that households have perfect information, we stack the deck against us, because in this case the standard wealth effect that reduces labor supply and desired saving is fully operating. When we solve for the equilibrium with both firms and households subject to rational inattention, we find that comovement strengthens.

[5]See the literature review later in this section.

[6]In Lucas (1972), firms are assumed to observe a one-dimensional signal about nominal aggregate and relative demand. In the RI-RBC model with news shocks, firms choose to observe a one-dimensional signal about current and future productivity.

The literature has explored the idea that moving away from full information rational expectations can improve the propagation mechanism relative to the baseline RBC model. Eusepi and Preston (2011) abandon rational expectations, replacing it by adaptive learning. They find that the first-order autocorrelations of employment, investment, and output growth in the model become positive. We add rational inattention, a form of incomplete information rational expectations, to the baseline RBC model. Surprisingly, the single assumption of rational inattention turns out to be sufficient to bring the first-order autocorrelations of employment, investment, and output growth in the model approximately into line with the data.[7]

The literature shows that one can construct a model in which business cycle comovement arises in response to news about future productivity, but this typically requires introducing multiple assumptions. To obtain comovement, Jaimovich and Rebelo (2009) add to the baseline RBC model an investment adjustment cost and variable capital utilization, which together produce a shift in the firms' demand for inputs in advance of an expected future productivity change, and a new class of preferences which controls the wealth effect on the households' side.[8] Den Haan and Kaltenbrunner (2009) add to the baseline model a search friction in the labor market, which gives firms an incentive to smooth employment analogously to a labor adjustment cost. Den Haan and Kaltenbrunner find that if vacancy creation is inefficiently low on average, then the model can generate comovement of employment, investment, consumption, and output conditional on a news shock.[9] Beaudry and Portier (2004) and (2007) introduce multiple sectors, with a productivity process that does not affect the production of investment goods and with a complementarity between inputs, and show that in this economy a positive news shock gives an incentive to produce more investment

[7] Business cycle models face the challenge of matching the persistence in the macroeconomic data more generally, not only conditional on a productivity shock. See Sims (1998) for a general discussion, Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007) in the context of New Keynesian models, and Auclert, Rognlie, and Straub (2020) in the context of a heterogeneous agent New Keynesian model. Our finding may therefore be helpful also for model builders who allow for sources of fluctuations other than productivity.

[8] Schmitt-Grohé and Uribe (2012) estimate a similar augmented RBC model.

[9] In this case, when good news about the future arrives, the output gains from vacancy creation more than offset the creation costs. See also Chahrour, Chugh, and Potter (2022). There are also related models without nominal rigidity that produce comovement conditional on "demand shocks" such as risk premium, noise or confidence shocks (e.g., Angeletos and La'O, 2010, 2013, Angeletos, Collard, and Dellas, 2018, Basu et al., 2021, Ilut and Saijo, 2021, and Di Tella and Hall, 2022).

goods and to hire more labor, which yields comovement. As an alternative, one can work with a New Keynesian model with imperfect competition and nominal stickiness. In that model, each firm commits to supply output at a fixed price, and therefore a rise in consumption boosts input demand. The response of the economy to a news shock depends on monetary policy. With optimal monetary policy the response is identical to the flexible-price neoclassical benchmark. Thus, to obtain comovement one needs to add the assumption of suboptimal monetary policy that tolerates inflation after a news shock. Lorenzoni (2009) analyzes a New Keynesian economy with a Taylor rule where noise in a public signal about productivity causes comovement.[10] By contrast, we explore how a single new assumption, rational inattention, changes the propagation of a news shock in the baseline RBC model.

A vast empirical literature finds that a sizable fraction of movements in TFP is forecastable.[11] Authors make different assumptions to identify a shock that moves TFP a lot in the future and little, or not at all, on impact. Beaudry and Portier (2006) show that two alternative identification assumptions in a vector autoregression (VAR) both yield the result that news shocks cause comovement. The subsequent research pursues three additional approaches to identification. Papers that use patent data, either as a variable in a VAR or as an external instrument, find that news shocks produce comovement.[12] Papers that identify news shocks using the max-share method of Francis et al. (2014) reach, to some extent, conflicting conclusions. The results depend on the sample period. Barsky and Sims (2011) and Kurmann and Sims (2021) do not find comovement after a news shock in the U.S. data since 1960, while Görtz, Tsoukalas, and Zanetti (2022b) who focus on the data since the onset of the Great Moderation do find comovement.[13] Finally, Chahrour

[10] With a Taylor rule a standard medium-size New Keynesian model (Smets and Wouters, 2007) produces comovement after news about future productivity. The same is true in a heterogeneous agent version of the model (we thank Christian Wolf for these observations). For a review of the literature on news shocks, see Lorenzoni (2011), Beaudry and Portier (2014), and Jaimovich (2017).

[11] Beaudry and Portier (2006), Barsky and Sims (2011), Barsky, Basu, and Lee (2014), Kurmann and Sims (2021), Cascaldi-Garcia and Vukotic (2022), Chahrour and Jurado (2022), Görtz, Tsoukalas, and Zanetti (2022b), Miranda-Agrippino, Hacioğlu Hoke, and Bluwstein (2022), and others.

[12] Cascaldi-Garcia and Vukotic (2022), Miranda-Agrippino, Hacioğlu Hoke, and Bluwstein (2022).

[13] The details of the identification assumptions are different in Barsky and Sims (2011), Kurmann and Sims (2021), and Görtz, Tsoukalas, and Zanetti (2022b). Görtz, Tsoukalas, and Zanetti (2022b), Appendix A.2, show that when they use the identification assumptions of Barsky and Sims (2011) or Kurmann and Sims (2021) and focus on the data since the onset of the Great Moderation, they find that news shocks produce comovement. See also Görtz,

and Jurado (2022) identify a fundamental shock to TFP and report that macroeconomic variables exhibit business cycle comovement in anticipation of that shock.

The RI-RBC model suggests that empirical researchers who study different sample periods can be expected to reach conflicting conclusions regarding comovement. Whether a news shock produces comovement in the model depends on macroeconomic volatility. In a low volatility environment (think of the Great Moderation) agents pay little attention to the macroeconomy, and news shocks cause positive comovement of consumption and labor input because the aforementioned rational inattention effects are strong. In a high volatility environment (think of the period before the Great Moderation) agents pay more attention, and news shocks cause negative comovement because the equilibrium is closer to the perfect information equilibrium. We illustrate this prediction of the model in an experiment in which we change the volatility of the productivity process. As we discuss, data from the U.S. Survey of Professional Forecasters support the view that agents pay less attention to the macroeconomy since the onset of the Great Moderation than before.

Several papers make progress solving attention problems of individual agents in a dynamic environment.[14] Maćkowiak and Wiederholt (2015) solve a DSGE model with rational inattention where the physical environment is similar to a simple New Keynesian model (there is no capital and one side of the market sets the price while the other side of the market chooses the quantity).[15] By contrast, here the physical environment is a standard neoclassical business cycle model (there is capital and prices, which all agents take as given, adjust to guarantee market clearing). We adopt a guess-and-verify method to find the fixed point, at each iteration using the results of Maćkowiak, Matějka, and Wiederholt (2018) to solve agents' attention problems. This is the first time in the literature that an RBC model with rational inattention is solved.

The next section defines the physical environment. Section 3 introduces rational inattention. Section 4 develops intuition for the effects of rational inattention, by considering special cases of the model. Section 5 studies the effects of productivity shocks and news about future productivity

Gunn, and Lubik (2022a).

[14] Sims (2003), Maćkowiak and Wiederholt (2009), Woodford (2009), Sims (2010), Steiner, Stewart, and Matějka (2017), Maćkowiak, Matějka, and Wiederholt (2018), Stevens (2020), Afrouzi and Yang (2021), Jurado (2022), Miao, Wu, and Young (2022). See the survey of rational inattention by Maćkowiak, Matějka, and Wiederholt (2023) for a summary of these papers.

[15] See also Afrouzi and Yang (2021).

in the complete model. Section 6 considers a version of the model in which all agents, firms and households, are subject to rational inattention. Section 7 concludes. An online Appendix contains supplementary material.

2 Model – physical environment

We consider a baseline RBC model that allows for an additional factor of production ("an entrepreneurial input") in fixed supply. The production function is Cobb-Douglas and exhibits decreasing returns to scale in the variable factors, capital and labor. We introduce a third factor in fixed supply because to formulate the attention problem of a firm we need the firm's choice of capital and labor under perfect information, not only the capital-labor ratio, to be determinate.

Time is discrete. There is a continuum of firms indexed by $i \in [0,1]$. All firms produce the same good using an identical technology represented by the production function

$$Y_{it} = e^{a_t} K_{it-1}^{\alpha} L_{it}^{\phi} N_i^{1-\alpha-\phi}$$

where Y_{it} is output of firm i in period t, K_{it-1} is capital input, L_{it} is labor input, and e^{a_t} is total factor productivity, common to all firms. N_i is an entrepreneurial input, specific to firm i, in fixed supply. The parameters α and ϕ satisfy $\alpha \geq 0$, $\phi \geq 0$, and $\alpha + \phi < 1$.

The capital stock of firm i follows the law of motion

$$K_{it} - K_{it-1} = I_{it} - \delta K_{it-1} \tag{1}$$

where $\delta \in (0,1]$. The firm maximizes the expected discounted sum of profits or dividends. The dividend of firm i in period t, D_{it}, is given by

$$D_{it} = Y_{it} - W_t L_{it} - I_{it} \tag{2}$$

where W_t is the wage rate. The dividends of all firms flow to a mutual fund. Households own and trade shares in the mutual fund.[16]

Total factor productivity is determined according to the law of motion

$$a_t = \rho a_{t-1} + \sigma \varepsilon_{t-h} \tag{3}$$

[16]When firm i was sold to the mutual fund, the entrepreneurial input was paid the present value of its future marginal products and in return committed to supply its service without additional payments.

where ε_t follows a Gaussian white noise process with unit variance, $\rho \in (0,1)$, $\sigma > 0$, and $h \geq 0$. A shock drawn by nature in period t affects productivity in period $t+h$. We solve the model either with $h = 0$ (a standard productivity shock) or with $h \geq 1$ (a news shock).[17]

There is a continuum of households indexed by $j \in [0,1]$. Each household j maximizes the expected discounted sum of utility. The discount factor is $\beta \in (0,1)$. The utility function is

$$U\left(C_{jt}, L_{jt}\right) = \frac{C_{jt}^{1-\gamma} - 1}{1-\gamma} - \frac{L_{jt}^{1+\eta}}{1+\eta}$$

where C_{jt} is consumption by household j in period t, L_{jt} is hours worked, $\gamma > 0$, and $\eta \geq 0$ (typically, we will set $\gamma = 1$ and $\eta = 0$). The budget constraint in period t is

$$V_t Q_{jt} - V_t Q_{jt-1} = W_t L_{jt} + D_t Q_{jt-1} - C_{jt}$$

where V_t is the price of a share in the mutual fund in period t, Q_{jt} is household j's share in the mutual fund, and $D_t \equiv \int_0^1 D_{it} di$ is the dividend from the mutual fund.

Aggregate output is $Y_t \equiv \int_0^1 Y_{it} di$. Aggregate capital and investment are defined analogously. Aggregate consumption is $C_t \equiv \int_0^1 C_{jt} dj$.

In equilibrium in every period the wage adjusts so that labor demand equals labor supply, $\int_0^1 L_{it} di = \int_0^1 L_{jt} dj$, and the price of a share in the mutual fund adjusts so that asset demand equals asset supply normalized to one, $\int_0^1 Q_{jt} dj = 1$.

The non-stochastic steady state of this economy is described in Appendix A. To solve the model when firms and households have perfect information, we log-linearize their first-order conditions and the other equilibrium conditions at the non-stochastic steady state. This yields the completely standard log-linear equilibrium conditions stated in Appendix B. We refer to the solution as the perfect information equilibrium.

3 Model – rational inattention by firms

A rationally inattentive individual cannot process all available information but can decide what to focus on. The decision-maker in firm i chooses an optimal signal about the state of the economy. They maximize the expected discounted sum of profits, recognizing that a more informative signal

[17] We will also consider the case when productivity is driven by two orthogonal shocks, a standard productivity shock and a news shock. For ease of exposition, we abstract from long-run growth.

requires more attention, which is costly. This section begins by deriving the agent's objective. We then state the agent's attention problem. Finally, we define the equilibrium in the economy in which firms are subject to rational inattention and households have perfect information.

3.1 Loss in profit from suboptimal actions

We derive an expression for the expected discounted sum of losses in profit when actions of firm i deviate from the profit-maximizing actions – the actions the firm would take if it had perfect information in every period. To obtain this expression, we compute the log-quadratic approximation to the expected discounted sum of profits at the non-stochastic steady state.

Recall that the profit of firm i in period t is given by $Y_{it} - W_t L_{it} + (1-\delta) K_{it-1} - K_{it}$. We assume that the mutual fund instructs each firm to value profits according to the marginal utility of consumption.[18] The profit function can be written in terms of log-deviations from the non-stochastic steady state:

$$C^{-\gamma} e^{-\gamma c_t} Y \left\{ e^{a_t + \alpha k_{it-1} + \phi l_{it}} - \phi e^{w_t + l_{it}} + \left(\frac{\alpha}{\beta^{-1} - 1 + \delta} \right) \left[(1-\delta) e^{k_{it-1}} - e^{k_{it}} \right] \right\}$$

where an upper-case letter without a time subscript denotes the value of a variable in the non-stochastic steady state, and a lower-case letter denotes the log-deviation of a variable from its value in the non-stochastic steady state. The term $C^{-\gamma} e^{-\gamma c_t}$ is the marginal utility of consumption.

Taking the quadratic approximation to the expected discounted sum of profits, we obtain the following expression for the expected discounted sum of losses in profit from suboptimal actions:

$$\sum_{t=0}^{\infty} \beta^t E_{i,-1} \left[\frac{1}{2} (x_t - x_t^*)' \Theta_0 (x_t - x_t^*) + (x_t - x_t^*)' \Theta_1 \left(x_{t+1} - x_{t+1}^* \right) \right] \tag{4}$$

where $x_t \equiv (k_{it}, l_{it})'$, $x_t^* \equiv (k_{it}^*, l_{it}^*)'$, the matrices Θ_0 and Θ_1 are given by

$$\Theta_0 = -C^{-\gamma} Y \begin{bmatrix} \beta\alpha (1-\alpha) & 0 \\ 0 & \phi (1-\phi) \end{bmatrix}$$

$$\Theta_1 = C^{-\gamma} Y \begin{bmatrix} 0 & \beta\alpha\phi \\ 0 & 0 \end{bmatrix}$$

[18] All households have the same consumption level so long as households have perfect information.

and the stochastic process x_t^* satisfies the equations

$$E_t a_{t+1} - (1-\alpha) k_{it}^* + \phi E_t l_{it+1}^* = \frac{\gamma E_t (c_{t+1} - c_t)}{1 - \beta (1-\delta)} \tag{5}$$

$$a_t + \alpha k_{it-1}^* - (1-\phi) l_{it}^* = w_t \tag{6}$$

and the initial condition $k_{i,-1}^* = k_{i,-1}$. See Appendix C. The vector x_t^* is the *profit-maximizing* input choice when the decision-maker in the firm has perfect information in every period. Equations (5)-(6) are the usual optimality conditions for capital and labor, where firm i takes market prices and actions of all other agents as given and E_t denotes the expectation operator conditioned on the entire history up to and including period t. Equation (5) states that the profit-maximizing capital input equates the expected marginal product of capital to the cost of capital, where the latter is proportional to the expected consumption growth rate. Equation (6) states that the profit-maximizing labor input equates the marginal product of labor to the wage. The vector x_t is an *alternative* input choice. Expression (4) gives the expected discounted sum of losses in profit when the stochastic process for the firm's actions, x_t, differs – for whatever reason – from the stochastic process for the profit-maximizing actions, x_t^*. After the quadratic approximation this loss is quadratic in $x_t - x_t^*$. The interaction term $(x_t - x_t^*)' \Theta_1 \left(x_{t+1} - x_{t+1}^*\right)$ appears because bringing too much capital into a period raises the optimal labor input in that period.

Expression (4) does not have the standard form of the objective in a dynamic rational inattention problem because of the intertemporal interaction term $(x_t - x_t^*)' \Theta_1 \left(x_{t+1} - x_{t+1}^*\right)$. We therefore perform a change of variables to turn expression (4) into the standard form. We show in Appendix C that expression (4) is equivalent to

$$\sum_{t=0}^{\infty} \beta^t E_{i,-1} \left[\frac{1}{2} (x_t - x_t^*)' \Theta (x_t - x_t^*) \right] \tag{7}$$

where $x_t \equiv (k_{it}, l_{it} - \frac{\alpha}{1-\phi} k_{it-1})'$, $x_t^* \equiv (k_{it}^*, l_{it}^* - \frac{\alpha}{1-\phi} k_{it-1}^*)'$, the matrix Θ is given by

$$\Theta = -C^{-\gamma} Y \begin{bmatrix} \beta\alpha \left(1 - \alpha - \frac{\alpha\phi}{1-\phi}\right) & 0 \\ 0 & \phi(1-\phi) \end{bmatrix} \tag{8}$$

and the stochastic process x_t^* satisfies

$$x_t^* = \begin{pmatrix} \frac{1}{1-\alpha-\phi} \left[E_t a_{t+1} - \phi E_t w_{t+1} - (1-\phi) \frac{\gamma E_t (c_{t+1} - c_t)}{1-\beta(1-\delta)} \right] \\ \frac{1}{1-\phi} (a_t - w_t) \end{pmatrix}. \tag{9}$$

The first entry of the vector x_t is still the capital stock to be carried into period $t+1$, k_{it}. The second entry of the vector x_t is now the labor input *for a given capital stock*, $l_{it} - [\alpha/(1-\phi)]\, k_{it-1}$. The target vector x_t^* is given by equation (9). Its first entry – the profit-maximizing capital stock to be carried into period $t+1$ – is proportional to the difference between expected productivity and a weighted average of the expected wage and the cost of capital, where the expectation is conditioned on the entire history up to and including period t. Its second entry – the profit-maximizing labor input for a given capital stock – is proportional to the difference between productivity and the wage. Since the matrix Θ in objective (7) is diagonal, the best response of firm i in period t given any information set $\mathcal{I}_{it}$ is the conditional expectation of x_t^*, $x_t = E\left(x_t^* | \mathcal{I}_{it}\right)$. Moreover, assuming that the firm chooses $(k_{it}, l_{it} - \frac{\alpha}{1-\phi} k_{it-1})$ is equivalent to assuming that the firm chooses (k_{it}, l_{it}) so long as the firm knows its own past action k_{it-1}, which is the case if $\mathcal{I}_{it-1} \subset \mathcal{I}_{it}$.

3.2 The attention problem of a firm

In period $t = -1$, the decision-maker in firm i chooses the stochastic process for the signal to maximize the expected discounted sum of profits net of the cost of attention. In every period $t \geq 0$, the decision-maker observes a realization of the optimal signal and takes actions – chooses capital to be carried into the next period and labor input for a given capital stock.

The decision-maker in firm i solves:

$$\max_{\Gamma, \Sigma_\psi} \left\{ \sum_{t=0}^{\infty} \beta^t E_{i,-1} \left[\frac{1}{2} \left(x_t - x_t^*\right)' \Theta \left(x_t - x_t^*\right) \right] - \lambda \sum_{t=0}^{\infty} \beta^t I\left(\xi_t; S_{it} | \mathcal{I}_{it-1}\right) \right\} \tag{10}$$

subject to

$$x_t^* = G' \xi_t \tag{11}$$

$$\xi_{t+1} = F \xi_t + \mu_{t+1} \tag{12}$$

$$x_t = E\left(x_t^* | \mathcal{I}_{it}\right) \tag{13}$$

$$\mathcal{I}_{it} = \mathcal{I}_{i,-1} \cup \left\{S_{i0}, \ldots, S_{it}\right\} \tag{14}$$

$$S_{it} = \Gamma' \xi_t + \psi_{it} \tag{15}$$

$$I\left(\xi_t; S_{it} | \mathcal{I}_{it-1}\right) = H\left(\xi_t | \mathcal{I}_{it-1}\right) - H\left(\xi_t | \mathcal{I}_{it}\right). \tag{16}$$

The vector x_t is the action of the decision-maker in every period $t \geq 0$. In the RBC model, the manager of a firm chooses capital and labor input in every period. One can also think of the manager as choosing the capital stock to be carried into the next period and the labor input for a given capital stock, $x_t \equiv (k_{it}, l_{it} - \frac{\alpha}{1-\phi} k_{it-1})'$. These two formulations are equivalent so long as the manager remembers own past actions. See Section 3.1. Thinking of the manager as choosing the capital stock to be carried into the next period and the labor input for a given capital stock has the advantage of leading to a simpler expression for the expected discounted sum of losses in profit due to suboptimal actions. See again Section 3.1.

The vector x_t^* is the action that the decision-maker would take if they had perfect information in period t. Equations (11)-(12) posit that this optimal action can be written as a linear function of some state vector ξ_t that follows a first-order VAR process with an innovation μ_t that follows a Gausssian vector white noise process with covariance matrix Σ_μ. In the RBC model, x_t^* is given by equation (9). We explain below how we go from equation (9) to equations (11)-(12).

The expected discounted sum of losses in profit due to suboptimal actions is given by equation (7) and reappears as the first term in objective (10). Given that the loss in profit due to suboptimal actions has this quadratic form with a diagonal matrix Θ, the best action in period t for any period t information set is given by equation (13).

The information set of the manager in period t is given by equation (14). It consists of the initial information and all signals that the manager has received up to and including period t.

The signal that the manager receives in period t is given by equation (15). This equation posits that the period t signal is on the period t state vector. Maćkowiak, Matějka, and Wiederholt (2018) show that signals on state vectors are optimal. The reason is that the purpose of acquiring information is to improve current and future own actions and the state vector ξ_t summarizes all available information about current and future optimal actions. Γ is an $n_\xi \times n_s$ matrix, where n_ξ is the dimension of the state vector and n_s is the dimension of the signal vector, and ψ_{it} follows a Gaussian vector white noise process with covariance matrix Σ_ψ.

The decision-maker chooses the signal process (the matrices Γ and Σ_ψ) so as to maximize the expected discounted sum of profits (the first term in objective (10)) minus the discounted sum of information costs (the second term in objective (10)). The fundamental trade-off is that receiving more informative signals makes the first term in the objective less negative but also raises the information costs. The decision-maker chooses Γ and Σ_ψ in period $t = -1$, anticipating that in periods $t = 0, 1, 2, \ldots$ the information set will evolve according to equations (14)-(15), the action will evolve according to equation (13), and the optimal action will evolve according to equations (11)-(12).

The per period information cost is linear in Shannon's mutual information, $\lambda I\left(\xi_t; S_{it}|\mathcal{I}_{it-1}\right)$, where $\lambda > 0$ is the marginal cost of attention and $I\left(\xi_t; S_{it}|\mathcal{I}_{it-1}\right)$ is given by equation (16). Here $H\left(\xi_t|\mathcal{I}_{it-1}\right)$ denotes the conditional entropy of ξ_t given $\mathcal{I}_{it-1}$ and $H\left(\xi_t|\mathcal{I}_{it}\right)$ denotes the conditional entropy of ξ_t given $\mathcal{I}_{it}$. Entropy is a measure of uncertainty. Thus, $I\left(\xi_t; S_{it}|\mathcal{I}_{it-1}\right)$ simply measures the uncertainty reduction about the period t state vector due to the period t signal.

The decision problem (10)-(16) is a standard linear quadratic Gaussian (LQG) dynamic rational inattention problem. The objective function is quadratic in the agent's action and the state vector, the state vector follows linear dynamics with Gaussian innovations, and the information cost is linear in Shannon's mutual information. Afrouzi and Yang (2021) and Miao, Wu, and Young (2022) provide efficient code for solving this problem numerically, while Maćkowiak, Matějka, and Wiederholt (2018) and Jurado (2022) focus on analytical results for this kind of problem. Our formulation of the problem resembles the formulations in Afrouzi and Yang (2021) and Miao, Wu, and Young (2022). There are three small differences. First, there are small differences in notation.[19] Second, Afrouzi and Yang (2021) and Miao, Wu, and Young (2022) allow for the possibility that Γ and Σ_ψ are different in every period $t \geq 0$, i.e., the decision-maker chooses a sequence $\{\Gamma_t, \Sigma_{\psi,t}\}_{t=0}^{\infty}$. We assume that the decision-maker chooses a Γ and a Σ_ψ that are constant over time. Furthermore, we assume that the decision-maker receives after the choice of Γ and Σ_ψ a long sequence of signals that places the agent immediately at the steady state of the Kalman filter. We also computed the optimal sequence $\{\Gamma_t, \Sigma_{\psi,t}\}_{t=0}^{\infty}$ and focused on the limit of this sequence and the corresponding steady-state Kalman filter; for $\beta = 0.99$, the difference is negligible (see Appendix D).[20] Third, Miao, Wu, and Young (2022) allow for the possibility that x_t appears on the right-hand side of equation (12), i.e., the state vector in period $t+1$ is affected by the own action in period t. We show that for the firm and the household in the RBC model this is not the case. For the firm, this

[19]Following Sims (2010), we denote the action by x_t, while Afrouzi and Yang (2021) and Miao, Wu, and Young (2022) denote the action by a_t and u_t, respectively. We denote the state vector by ξ_t, while Afrouzi and Yang (2021) and Miao, Wu, and Young (2022) denote the state vector by x_t.

[20]Miao, Wu, and Young (2022) call our approach the golden-rule information structure and the limit of the optimal sequence $\{\Gamma_t, \Sigma_{\psi,t}\}_{t=0}^{\infty}$ the steady-state information structure. They show that these two concepts coincide in the limit as β approaches 1.

can be seen from equation (9). The optimal action of the firm in period t does not depend on any past actions of the firm.[21]

It is worthwhile to point out that none of the restrictions on the signal process in equation (15) is binding. We already mentioned that a signal on the period t state vector is optimal.[22] Furthermore, one can show that a Gaussian signal is optimal.[23]

In the rational inattention literature, it is typically assumed that all noise in signals is idiosyncratic, because this assumption accords well with the idea that the source of the noise is the decision-makers' limited attention. Hence, we assume that the noise ψ_{it} is independent across firms.

Each manager chooses Γ and Σ_ψ taking as given Θ, λ, G, F, and Σ_μ. However, the law of motion for the state vector, given by F and Σ_μ, will be an equilibrium outcome. We will compute a rational expectations equilibrium in which all agents hold the correct belief about F and Σ_μ.

3.3 Definition and computation of equilibrium

For the moment, we focus on the equilibrium when decision-makers in firms are subject to rational inattention and households have perfect information. For simplicity, until Section 6 we refer to this equilibrium as the rational inattention equilibrium. In Section 6, we add rational inattention on the side of households.

The rational inattention equilibrium can be defined as follows. In period -1, each firm solves problem (10)-(16) given the correct belief about F and Σ_μ. In every period $t \geq 0$, nature draws a realization of ε_t and ψ_{it} for each i, firms and households maximize given their information sets, and markets clear: the wage w_t adjusts so that labor demand equals labor supply, $\int_0^1 l_{it} di = \int_0^1 l_{jt} dj$, and the price of a mutual fund share v_t adjusts so that asset demand equals asset supply, $\int_0^1 q_{jt} dj = 0$.

We solve for equilibrium using a guess-and-verify method consisting of three steps. First, we guess the matrices F and Σ_μ that appear in equation (12) for the state vector ξ_t. Recall that by the

[21] In the language of the rational inattention literature, the decision problem (10)-(16) is a pure tracking problem, because from the point of view of the decision-maker the state vector ξ_t is exogenous and the decision-maker wants to keep the action x_t close to the target $G'\xi_t$ with a quadratic loss.

[22] See Proposition 1 in Maćkowiak, Matějka, and Wiederholt (2018) or Lemma 2.2 in Afrouzi and Yang (2021). The action x_t in Maćkowiak, Matějka, and Wiederholt (2018) is one-dimensional, but the proof of Proposition 1 in Maćkowiak, Matějka, and Wiederholt (2018) extends in a straightforward way from a one-dimensional action (x_t is a scalar) to a multi-dimensional action (x_t is a vector).

[23] See Proposition 1 in Jurado (2022) or Lemma 2.3 in Afrouzi and Yang (2021).

state vector we mean the vector from which one can compute the period t profit-maximizing action of the firm, x_t^* (equation (11)). As an example, consider the special case of the model in which labor is the only variable input ($\alpha = 0$) so that $x_t^* = l_{it}^*$. The period t profit-maximizing labor input, l_{it}^*, depends on exogenous productivity, a_t, and the equilibrium wage, w_t (the second line in equation (9)). We guess that in equilibrium w_t follows a Gaussian finite-order ARMA process, and we define ξ_t as the vector containing a_t, w_t, and, if appropriate, lags of w_t and current and lagged ε_t. Thus, ξ_t follows equation (12) and equation (9) maps ξ_t to l_{it}^* (i.e., the second line in equation (9) can be written as equation (11)). In general, when both capital and labor are variable inputs, x_t^* is two-dimensional and depends on two endogenous variables, the wage w_t and consumption c_t (equation (9)). We guess that in equilibrium w_t and c_t each follows a Gaussian finite-order ARMA process, and we define ξ_t as the vector containing a_t, w_t, c_t, and, if appropriate, lags of w_t and c_t and current and lagged ε_t. Again, ξ_t follows equation (12) and equation (9) maps ξ_t to x_t^* (i.e., equation (9) can be written as equation (11)).[24]

Second, we solve the attention problem of firm i, (10)-(16), and use the solution to compute the process followed by the firm's action, x_t, or equivalently k_{it} and l_{it}. The solution for Γ and Σ_ψ together with equations (11)-(15) and the definition of the vector x_t below objective (7) yield the law of motion for k_{it} and l_{it}. Firm-level output, investment, and profit follow from $y_{it} = a_t + \alpha k_{it-1} + \phi l_{it}$, $\delta i_{it} = k_{it} - (1-\delta) k_{it-1}$, and $(D/Y) d_{it} = y_{it} - (WL/Y)(w_t + l_{it}) - (I/Y) i_{it}$, and aggregate variables from $y_t = \int_0^1 y_{it} di$, $k_t = \int_0^1 k_{it} di$, $i_t = \int_0^1 i_{it} di$, and $d_t = \int_0^1 d_{it} di$.[25]

Third, we verify the guess about F and Σ_μ. When households have perfect information, they satisfy the usual first-order conditions[26]

$$\gamma E_t (c_{t+1} - c_t) = \beta E_t v_{t+1} - v_t + (1-\beta) E_t d_{t+1} \tag{17}$$

$$w_t - \gamma c_t = \eta l_t. \tag{18}$$

[24] That an endogenous variable in the model like w_t or c_t follows an ARMA process of some finite order p, q is a numerical approximation, but its accuracy can be checked by increasing the order to $p+1, q+1$, resolving for the equilibrium, and verifying that the equilibrium changes by a numerically negligible amount.

[25] These equations result from log-linearization of the production function, the law of motion of capital, the definition of profit, and the definitions of the aggregate variables. All relevant steady-state ratios appear in Appendix A.

[26] Households are identical, implying that $c_{jt} = c_t$ and $l_{jt} = l_t$ for each j.

The resource constraint reads[27]

$$y_t = (C/Y)\, c_t + (I/Y)\, i_t. \tag{19}$$

We verify the guess for the equilibrium wage process w_t and the equilibrium consumption process c_t using equations (18)-(19). The market-clearing mutual fund share price v_t follows from equation (17) and the solution for d_t and c_t.[28]

3.4 What can decision-makers pay attention to?

In this model, decision-makers *can* pay attention to *any* variable. Equation (15) specifies that the decision-maker can pay attention to any variable in the state vector ξ_t.[29] It is important to remember that here we have already used an optimality result. One can extend the signal vector from $S_{it} = \Gamma'\xi_t + \psi_{it}$ to $S_{it} = \Gamma'\xi_t + \Psi'\varsigma_t + \psi_{it}$, where the vector ξ_t is the state vector from equations (11)-(12) and the vector ς_t includes any other variable that is not in ξ_t. The equilibrium of the model is identical under these two specifications of the signal vector, because the decision-makers will set all the entries of the matrix Ψ equal to zero. It is optimal to focus attention on variables that determine current or future optimal actions (Maćkowiak, Matějka, and Wiederholt, 2018). Furthermore, the state vector ξ_t will typically contain exogenous fundamentals (e.g., productivity) and endogenous prices and quantities (e.g., the wage and consumption). Hence, in the equilibrium of this model, decision-makers will *choose* to learn from prices. Importantly, they will choose which prices to learn from and how to learn from them. Finally, we will set the marginal cost of attention, λ, so that profit losses due to less-than-perfect attention by decision-makers will be small and beliefs will be consistent with survey data.[30]

4 Developing intuition

How does rational inattention affect the propagation of productivity shocks and news about future productivity? To develop intuition this section studies special cases of the RI-RBC model. In the

[27]To obtain the resource constraint, we log-linearize the flow budget constraint of household j and we aggregate, imposing market clearing and plugging in the equation for the dividend from the mutual fund.

[28]When $\eta = 0$, the first-order condition (18) reads $w_t = \gamma c_t$ which implies that a guess about w_t amounts to a guess about c_t and vice versa, a property which makes it easier to find the fixed point of the model in Section 5.

[29]"Paying attention to" is formalized as "reducing uncertainty about," or equivalently, "getting a signal about."

[30]See Footnotes 36 and 42.

first case, labor is the only variable input. In the second case, capital is the only variable input. Section 5 analyzes the rational inattention equilibrium of the complete model.

4.1 The case with labor only

Suppose that labor is the only variable input, $\alpha = 0$. The attention problem of a firm simplifies. The firm's action is one-dimensional with $x_t = l_{it}$, $x_t^* = l_{it}^* = [1/(1-\phi)](a_t - w_t)$, and $\Theta = -C^{-\gamma} Y \phi (1-\phi)$. Labor supply is governed by equation (18). Households live hand-to-mouth because there is no capital and all households are identical.

The perfect information equilibrium ("the PI equilibrium") can be solved for analytically. Equating labor demand, $\int_0^1 l_{it} di = [1/(1-\phi)](a_t - w_t)$, and labor supply, which follows from equations (18) and $c_t = y_t = a_t + \phi l_t$, yields the solution for aggregate labor input

$$l_t = \left(\frac{1-\gamma}{1-\phi+\gamma\phi+\eta} \right) a_t. \tag{20}$$

Labor input is proportional to productivity. Hence, the impulse response of labor input to a productivity shock is a scaled version of the impulse response of productivity, and the impulse response of labor input to a news shock is zero until productivity changes. Firms have no incentive to change labor demand until productivity changes. Similarly, households have no incentive to change labor supply in this special case of the model. The wealth effect on labor supply vanishes because hand-to-mouth households cannot vary saving and consumption in response to a news shock.

Consider the rational inattention equilibrium ("the RI equilibrium"). A model period is a quarter. As an example, we assume $\gamma = 0.5$, $\eta = 0$, $\phi = 0.6$, $\beta = 0.99$, $\rho = 0.9$, $\sigma = 0.01$, and $\lambda = (4/100,000) C^{-\gamma} Y$, which means that the per period marginal cost of attention is equal to $4/100,000$ of steady-state output.[31] The upper-left panel in Figure 1 shows the impulse response of aggregate labor input l_t to a productivity shock ($h = 0$).[32] In the PI equilibrium, labor input is proportional to productivity and thus the impulse response peaks on impact and declines monotonically (line with points). In the RI equilibrium, the impulse response is weaker and hump-shaped (line with circles). This is the usual result that rational inattention causes dampening and delay because of

[31] Section 5 discusses the choice of the value for the marginal cost of attention λ.

[32] In all figures, an impulse response of 1 is a 1 percent deviation from the non-stochastic steady state.

the noise in the signal which implies a positive weight on the prior. For a similar figure, see for instance Figure 1 in Sims (2003).

The upper-right panel in Figure 1 shows the impulse response of l_t to a news shock ($h = 4$). The shock is drawn in period 0 while productivity changes in period $h = 4$. In the PI equilibrium, labor input is proportional to productivity (equation (20)) and thus the impulse response is zero until productivity changes (line with points). Under rational inattention labor demand rises on impact of a news shock. The reason is that the optimal signal of firms confounds current with expected future productivity. The increase in labor demand puts upward pressure on the wage. Labor supply is still governed by equations (18) and $c_t = y_t = a_t + \phi l_t$. We find that in the RI equilibrium labor input is positive on impact of a news shock (line with circles) and keeps rising thereafter.

To see analytically that a confounding signal is optimal, consider the following special case. Suppose that a measure zero of firms are subject to rational inattention. Since a measure one of firms have perfect information, the equilibrium employment is given by equation (20) and the equilibrium wage is $w_t = [(\gamma + \eta)/(1 - \gamma)]l_t$, implying that the profit-maximizing labor input of an individual firm is proportional to productivity: $l_{it}^* = [1/(1-\phi)](a_t - w_t) = [(1-\gamma)/(1-\phi+\gamma\phi+\eta)]a_t$. Suppose that $a_t = \rho a_{t-1} + \sigma\varepsilon_{t-1}$ ($h = 1$). The state vector in equation (12) is then $\xi_t = (a_t, \varepsilon_t)'$. The optimal signal follows from Propositions 1, 2, and 5 in Maćkowiak, Matějka, and Wiederholt (2018). Proposition 1 states that the optimal signal is about the state vector, $S_{it} = \Gamma'\xi_t + \psi_{it}$. Proposition 2 states that with a one-dimensional action (here, labor input), the optimal signal is a *one-dimensional* signal about the state vector, $S_{it} = a_t + g\varepsilon_t + \psi_{it}$. Proposition 5 states that $g \neq 0$. Hence the optimal signal confounds current with expected future productivity. It turns out that this result still holds when all firms are subject to rational inattention and $h > 1$.

Our intuition for the optimality of a confounding signal consists of two parts. First, it is a well-known result in the rational inattention literature that the dimension of the optimal signal can be strictly smaller than the dimension of the state of the economy.[33] The disadvantage of a lower-dimensional signal is that it provides a less complete view of the economy. The advantage of

[33] For this result in the static case, see Proposition 1 in Kőszegi and Matějka (2020). For this result in the dynamic case, see Proposition 2 in Maćkowiak, Matějka, and Wiederholt (2018), Proposition 5 in Miao, Wu, and Young (2022), Theorem 2.2 in Afrouzi and Yang (2021), and Online Appendix C to Jurado (2022).

a lower-dimensional signal is that it saves on attention. In the special case discussed in the previous paragraph, Proposition 2 in Maćkowiak, Matějka, and Wiederholt (2018) implies that the optimal signal is one-dimensional.

Second, given the optimality of a one-dimensional signal (more generally, given the optimality of a lower-dimensional signal), the question is: what is this signal about? To gain intuition for the answer, consider the lower-left panel of Figure 1. Here we continue to assume that a measure zero of firms are subject to rational inattention and $h = 1$, implying that the profit-maximizing labor input of a firm is given by $l_{it}^{*} = [(1-\gamma)/(1-\phi+\gamma\phi+\eta)]a_t$ with $a_t = \rho a_{t-1} + \sigma\varepsilon_{t-1}$. The line with points depicts the impulse response of the profit-maximizing labor input to a news shock. The profit-maximizing labor input perfectly tracks the evolution of productivity. The line with circles shows the impulse response of the labor input to a news shock under the optimal rational inattention signal which has the form $S_{it} = a_t + g\varepsilon_t + \psi_{it}$ with $g \neq 0$. The line with squares shows the impulse response of the labor input to a news shock in the case of a noisy signal on current productivity, $S_{it} = a_t + \psi_{it}$. To make the line with circles and the line with squares comparable, the optimal signal under rational inattention and the noisy signal on current productivity are constrained to have the same signal-to-noise ratio, $var\left(\Gamma'\xi_t|\mathcal{I}_{it-1}\right)/var\left(\psi_{it}\right)$, which implies that the two signals are associated with the same information flow, $I\left(\xi_t; S_{it}|\mathcal{I}_{it-1}\right)$, and the same information cost. The signal on current productivity generates no overreaction of the labor input on impact of the news shock, but it generates underreaction of the labor input to the increase in productivity in all following periods, due to the noise in the signal. The optimal signal, by contrast, generates overreaction of the labor input on impact of the news shock and less underreaction of the labor input once productivity actually increases, by pushing the entire impulse response function up. With a quadratic objective, the overreaction on impact of the news shock causes only a small profit loss, because it introduces a small mistake, while the reduction in the underreaction in the following periods causes large gains in profit, because it reduces large mistakes. In addition, the overreaction occurs for one period, while the reduction in the underreaction is present for many periods. As a result, the optimal rational inattention signal generates smaller profit losses due to suboptimal actions than the signal on current productivity. The overreaction early on reduces the underreaction later on.[34]

[34]The profit loss due to suboptimal action depends also on the noise in the action, but it turns out that the response

Jurado (2022) offers a complementary intuition for why rationally inattentive agents respond too early to future innovations in the profit-maximizing actions. Formulating a dynamic rational inattention problem in the frequency domain, he argues that rationally inattentive agents care more about how their attention is allocated across frequencies than across time, and by spreading their attention across many periods, they are better able to reduce the uncertainty about the frequencies they are most uncertain about. See his Section 4 for the details.

In the same setting as in the lower-left panel in Figure 1, let us vary the marginal cost of attention λ. The impulse response of the action (here, a firm's labor input) on impact of a news shock, in period 0, is non-monotonic in λ. With a λ near zero (Figure 1, lower-right panel, line with asterisks), the rational inattention solution is close to the perfect information case in which the impulse response on impact is zero. With a high λ (line with diamonds), the solution is close to a "no information" model in which the impulse response in all periods is zero.

Appendix D reports additional numerical results. One result is that the more distant is the change in productivity, the weaker is the response of the action on impact of a news shock. If productivity will change in the near future, a rationally inattentive agent believes that productivity has already changed with a non-trivial probability. The short-run response of the action can then be strong (even though the perfect information response is zero). If productivity will change only in a longer run, the agent is fairly confident that productivity has not yet changed. The short-run response of the action approaches the perfect information response. Appendix D also considers a version of the model in which productivity is driven by two orthogonal shocks, a standard productivity shock and a news shock. The result that labor input rises on impact of a positive news shock is unchanged (and the impulse response of labor input to a news shock is very similar to the one reported in this subsection). See Appendix D for the details and other findings.

of labor input to a noise shock is very similar under the optimal signal and under the signal on current productivity. One can also consider a signal on the news shock, $S_{it} = \varepsilon_t + \psi_{it}$. The action based on this signal is qualitatively the same as the action based on the signal $S_{it} = a_t + \psi_{it}$: no mistake on impact of the news shock, followed by underreaction. For a given amount of information flow, the firm observing a signal about ε_t does worse than the firm observing a signal about a_t. Given a choice between three options, a signal on ε_t, a signal on a_t, and both signals, the firm chooses the signal on a_t.

4.2 The case with capital only

Suppose that capital is the only variable input, $\phi = 0$. The attention problem of a firm is analogous to Section 4.1 but with $x_t = k_{it}$, $x_t^* = k_{it}^* = \frac{1}{1-\alpha}\left[E_t a_{t+1} - \frac{\gamma E_t(c_{t+1}-c_t)}{1-\beta(1-\delta)}\right]$, and $\Theta = -C^{-\gamma} Y \beta\alpha (1-\alpha)$. The behavior of households is governed by equation (17).

Assume log utility from consumption, $\gamma = 1$, and full capital depreciation, $\delta = 1$. The PI equilibrium can be solved for analytically: $k_t = \alpha k_{t-1} + a_t$, $k_t = i_t = y_t = c_t = d_t = v_t$. In this special case, the model can produce some positive autocorrelation in investment and output growth. However, the impulse response of each variable to a news shock is zero until productivity changes. An increase in expected productivity creates an incentive to invest in the period before productivity improves, but this incentive is completely offset by a rise in the cost of capital.

Consider the RI equilibrium. As an example, we set $\gamma = 1$, $\alpha = 0.33$, $\beta = 0.99$, $\delta = 1$, $\rho = 0.9$, $\sigma = 0.01$, and $\lambda = (4/100,000)C^{-\gamma}Y$. The top panel in Figure 2 displays the impulse response of aggregate investment i_t to a productivity shock ($h = 0$). The RI equilibrium (line with circles) features more first-order autocorrelation in the growth rate of investment compared with the PI equilibrium (line with points). The middle panel in Figure 2 shows the impulse response of i_t to a news shock ($h = 4$). In the RI equilibrium investment is positive in period 0 (line with circles) and keeps rising thereafter, while in the PI equilibrium investment is zero until productivity changes in period 4 (line with points). Rational inattention induces an increase in investment demand on impact of a news shock and, as a result, investment rises in equilibrium. Since the attention problem of a firm is analogous to Section 4.1, the intuition for what happens to investment demand is the same as the intuition given there. Similarly to Section 4.1, the bottom panel in Figure 2 compares the rational inattention model with the alternative model (the model with the signal on current productivity) with $h = 4$. The alternative model yields no capital input mistakes conditional on a news shock from period 0 through period $h - 1$, followed by larger mistakes than in the rational inattention model. By smoothing the action, the signal in the rational inattention model lowers the overall expected profit loss.

Let us summarize Section 4. The impulse responses to a productivity shock and a news shock change significantly when firms are subject to rational inattention. Employment and investment react with *delay* to a productivity shock, because the noise in the signal implies a positive weight on the prior. Moreover, employment and investment *rise* in response to news that productivity will improve in the future, because the optimal signal confounds current with expected future productivity. Agents choose a low-dimensional representation of the state, and getting a news shock into the beliefs early on reduces the underreaction later on.

5 Predictions of the model

What does rational inattention imply about the business cycle effects of productivity and news shocks? We return to the complete model with variable capital and labor, $\alpha > 0$ and $\phi > 0$, and compare the RI equilibrium with the PI equilibrium. In the case of news shocks, we focus on $h = 2$ and $h = 4$ following the key papers on news shocks which also focus on changes in productivity a few quarters ahead ($h = 3$ in Beaudry and Portier, 2004, $h = 2$ in Jaimovich and Rebelo, 2009).

We set $\gamma = 1$, $\eta = 0$, $\alpha = 0.33$, $\phi = 0.65$, $\beta = 0.99$, $\delta = 0.025$, $\rho = 0.9$, and $\sigma = 0.008$. Thus, we assume log utility from consumption and linear disutility from work, $\alpha+\phi$ close to 1, a depreciation rate of 2.5 percent per quarter, and a persistent productivity process with an innovation that has a standard deviation of 0.8 percent.[35] We know that the model with perfect information ($\lambda = 0$) produces neither persistent growth rates of employment, investment, and output nor comovement after a news shock, and we conjecture based on Section 4 that the model with rational inattention ($\lambda > 0$) may be more successful. We set $\lambda = (6/100,000)C^{-\gamma}Y$, which means that the per period marginal cost of attention is equal to $6/100,000$ of steady-state output, because this is the lowest value of λ for which the model produces comovement with $h = 2$. It turns out, as we show below, that with this value of λ the model is also consistent with survey data on expectations.[36]

[35] Fernald (2014) constructs a quarterly series on the growth rate of TFP adjusted for capacity utilization. Regressing Fernald's series on its own lag in the sample 1955Q1-2007Q4 yields a point estimate of -0.08, which would imply a coefficient of $1 - 0.08 = 0.92$ in equation (3). The estimated standard deviation of the error term is 0.0083. Rounding off these estimates, we arrive at $\rho = 0.9$ and $\sigma = 0.008$. One can also convert Fernald's series into a series on the log level of TFP and fit an AR(1) to that series after detrending, but the estimated coefficient depends on the detrending method.

[36] Increasing λ somewhat would strengthen comovement. Only the ratio λ/σ^2 matters for the equilibrium impulse responses, because the first term in objective (10) is linear in σ^2 and the second is linear in λ. In the RI equilibrium we can compute the expected profit loss of firm i from suboptimal actions. Here this is equal per period to $3/100,000$ ($h = 0$) and $4/100,000$ ($h = 2$) of steady-state output, even smaller than the marginal cost of attention λ.

5.1 The effects of productivity shocks

Consider the PI equilibrium with $h = 0$. Figure 3 shows the impulse responses to a productivity shock (lines with points). Aggregate labor input, investment, output, and consumption move in the same direction, consistent with a business cycle. The impulse responses of hours worked, investment, and output peak on impact and decline monotonically. Following common practice in the RBC literature, we compare unconditional second moments in the model and in the data. We want to find out if rational inattention improves the performance of the model in this standard exercise. Table 1 reports selected unconditional moments for the model (column "Perfect information") and for the quarterly post-war data from the United States.[37] The comparison is familiar. Let us focus on the persistence of growth rates. The first-order autocorrelations of employment, investment, and output growth are positive in the data but negative in the model. In the model these variables inherit the autocorrelation of exogenous productivity growth.[38]

In the RI equilibrium, the impulse responses of employment, investment, and output become hump-shaped (Figure 3, lines with circles). These impulse responses are hump-shaped even though there are no adjustment costs. The first-order autocorrelations of employment, investment, and output growth become positive (Table 1, column "Rational inattention"). The model matches well the first-order autocorrelation of employment growth in the data, even though rational inattention is the only source of inertia and the marginal cost of attention is small. The model underpredicts somewhat the serial correlation of output and investment growth.

Section 4 explained the effects of rational inattention one input at a time. In this section the new feature is that rational inattention induces delay in the demand for both inputs, capital and labor, at the same time. Figure 3 shows the impulse response of the firms' conditional expectation

[37]We use the data from Eusepi and Preston (2011). The sample period is 1955Q1-2007Q4. Productivity is defined as real GDP divided by hours worked, measured as in Francis and Ramey (2009). See Data Appendix in Eusepi and Preston (2011). The unconditional moments from the model are computed from the equilibrium MA representation of each variable.

[38]The model matches well the standard deviation of consumption, investment, and productivity relative to output, while underpredicting the volatility of hours worked. The model matches well the correlation of consumption, hours worked, and investment with output, while overstating the correlation of productivity with output. Finally, the model matches well the first-order autocorrelation of consumption growth. It turns out that rational inattention has little effect on these predictions of the model. See Table 1.

of productivity to a productivity shock. The impulse response is hump-shaped because the firms' beliefs are anchored on the steady state and evolve gradually. The rational inattention effect turns out to be sufficient to bring the first-order autocorrelations of employment, investment, and output growth in the model approximately into line with the data.

The amount of inattention in the model, governed by the parameter λ, can be compared to survey data on expectations. Coibion and Gorodnichenko (2015) show that models with an informational friction predict a regression relationship between the average forecast error and forecast revision in a cross-section of agents. Suppose that firms in this model report their forecasts of output. Let $\hat{y}_{t+\tau|t}$ denote the period t average forecast of output in period $t+\tau$, where τ is a positive integer. The average forecast error, $y_{t+\tau} - \hat{y}_{t+\tau|t}$, is positively related to the average forecast revision, $\hat{y}_{t+\tau|t} - \hat{y}_{t+\tau|t-1}$. The regression coefficient increases in the size of the informational friction, in this model governed by the value of λ. Coibion and Gorodnichenko (2015) and Bordalo et al. (2020) estimate this regression relationship using survey data on forecasts of a number of variables. Typically, these authors report coefficients in the range of 0.3-1.4.[39] We repeat their estimation using quarterly data on median forecasts of output (real GDP) from the U.S. Survey of Professional Forecasters for the period 1968Q4-2019Q4 obtained from the Federal Reserve Bank of Philadelphia. Focusing on $\tau = 3$, we estimate a regression coefficient of 0.76 with a standard error of 0.30, which is a finding in the typical range from the previous work.[40] Next, we simulate data from our model with the parameter values used in this section, including the value of λ. When we run the same regression on the simulated data, on average across the simulations we obtain a coefficient of 0.96. We conclude that the amount of inattention in the model is consistent with the survey data on expectations. It is remarkable that the first-order autocorrelations of employment, investment, and output growth in the model are approximately in line with the data and, at the same time, the model is consistent with the survey data on expectations.

[39]See in particular Coibion and Gorodnichenko (2015), Table 1 and Figures 1-2, and Bordalo et al. (2020), Table 3.

[40]Coibion and Gorodnichenko (2015) and Bordalo et al. (2020) also focus on $\tau = 3$. Both papers report results for forecasts of output *growth* but not output *level*. Some observations on forecasts of the level of output cannot be used due to base year changes in the dataset; furthermore, we remove as outliers the top 1 percent of forecast errors and revisions.

5.2 The effects of news about future productivity

Consider the PI equilibrium with $h \geq 1$. Figures 4 and 5 show the impulse responses with $h = 2$ and $h = 4$, respectively (lines with points). The shock is drawn in period 0 while productivity changes in period h. A news shock causes a wealth effect. Consumption and leisure are normal goods, and therefore households want to consume more (save less) and work less after a positive news shock. Firms have no incentive to increase labor demand before productivity improves, while households reduce labor supply due to the wealth effect. As a result, hours worked fall. With capital predetermined and current productivity unchanged, output contracts. On impact firms have no incentive to increase investment, while the wealth effect reduces desired saving by households. Consumption rises while investment declines.[41] The model fails to produce comovement in response to news about future productivity. It predicts an output contraction after news that productivity will improve. An increase in expected productivity creates an incentive to invest in the period before productivity improves (period $h - 1$), but this incentive is more than offset by a rise in the cost of capital. Employment, investment, and output fall in period 0, keep falling through period $h - 1$, and increase only once productivity improves in period h.

With a high elasticity of intertemporal substitution, the model predicts a fall in consumption and a rise in labor input and investment. The substitution effect due to an increase in the real interest rate dominates the wealth effect in this case, pushing consumption down and labor supply up. "However, no combination of parameters can generate a joint increase in consumption, investment, and employment." (Lorenzoni (2011), p.539.)

Consider the RI equilibrium (Figures 4-5, lines with circles).[42] In both figures, equilibrium employment is positive in period 0 and keeps rising thereafter. The firms' conditional expectation of current productivity increases on impact of a news shock, which pushes up labor demand. In general equilibrium, the desire of households to reduce labor supply is pulling employment down. It turns out that the rational inattention effect on labor demand is strong enough to *more than offset* the wealth effect on labor supply. As a result, hours worked rise in equilibrium.

[41] The wealth effect reduces desired saving, which increases the cost of capital. Firms respond by cutting investment and paying out a higher dividend; with more dividend income, households increase consumption.

[42] In the economy with $h = 4$ we set $\lambda = (22/100,000)C^{-\gamma}Y$ (with $h = 4$ the model needs a higher λ than with $h = 2$ to produce comovement after a news shock). With $h = 4$ the per period expected profit loss equals $15/100,000$ of steady-state output.

Figures 4-5 show the impulse response of investment in general equilibrium ("RI general equilibrium") and the impulse response of investment by rationally inattentive firms of measure zero when other firms have perfect information ("RI partial equilibrium," line with asterisks). In partial equilibrium, investment is positive in period 0 and keeps rising thereafter. The conditional expectation of productivity by rationally inattentive firms increases on impact, which pushes up investment demand. In general equilibrium, the desire of households to reduce saving for a given level of output is pulling investment down. We find that the rational inattention effect on investment demand approximately offsets the wealth effect on desired saving. The response of investment on impact of a news shock is close to zero (whereas it is nearly -3 percent in the perfect information equilibrium). Note also that in the RI equilibrium investment rises between period 0 and period h (this is particularly clear in Figure 5), in contrast to the PI equilibrium where it keeps falling.

With capital predetermined and an increase in employment in period 0, the impulse response of output on impact of a news shock is positive. Output increases further between period 0 and period h, as employment and investment rise. The rational inattention effect on input demand induces an output expansion in response to a news shock.

Consider in more detail what affects investment in general equilibrium. Investment rises on impact of a positive news shock relative to the PI equilibrium. As a result, the cost of capital increases (the expected consumption growth rate rises) and the profit-maximizing capital input of an individual firm falls (equation (9)). Capital is a strategic substitute. An individual firm demands less capital when other firms invest more. This general equilibrium feedback effect turns out to be very strong. The coefficient on the expected consumption growth rate in the first line of equation (9) equals -504.[43] The coefficient on the expected consumption growth rate depends on the depreciation rate, δ, and the exponent on labor in the production function, ϕ, among others. In Section 4.2, with full capital depreciation and without labor input ($\delta = 1$, $\phi = 0$), this coefficient falls in absolute value by two orders of magnitude, to -1.5, implying that the strategic substitutability is much weaker. The impulse response of equilibrium investment on impact of a news shock is positive in this case.

In Figures 4-5 note also that consumption increases somewhat when firms become subject to

[43] Labor is also a strategic substitute. However, the general equilibrium dampening of labor demand due to a higher wage is weak. The coefficient on the wage in the second line of equation (9) equals -2.9.

rational inattention. Households consume more because rationally inattentive firms overestimate productivity and produce more than in the PI equilibrium. The opposite happens conditional on a standard productivity shock (Figure 3). Consumption declines somewhat when firms become subject to rational inattention. Households consume less because rationally inattentive firms underestimate productivity and produce less than in the PI equilibrium.

What is the optimal signal? In problem (10)-(16) the firm can in principle choose a multi-dimensional signal process, consisting of signals on elements of the state vector ξ_t, signals on linear combinations of the elements of ξ_t, or both. We find that a *one-dimensional* signal on *all elements* of the state vector is optimal. A one-dimensional signal on all elements of the state vector confounds current with expected future productivity. Furthermore, we find that the impulse response of the optimal signal to a news shock is positive on impact (Appendix Figure 1, upper-left panel, $h = 2$). Agents choose a low-dimensional representation of the state, and getting a news shock into the beliefs early on reduces the underreaction later on. To simplify, the message to firms from a positive signal realization is: "Hire and invest, productivity is either already up or about to rise (and it is not that important precisely when productivity rises)."

As in Section 5.1, we can compare the amount of inattention in the model to the SPF data. When we run the Coibion-Gorodnichenko regression on data simulated from the economy with $h = 2$ (with $\tau = 3$), on average we obtain a coefficient of 1.17. This amount of inattention is consistent with the survey data on expectations.[44] With $h = 4$ the model needs a higher marginal cost of attention to produce an increase in employment after a positive news shock. When we run the Coibion-Gorodnichenko regression on data simulated from the economy with $h = 4$ (with $\tau = 3$), on average we obtain a coefficient of 2.81. This amount of inattention is somewhat greater than implied by the SPF data.[45]

To summarize, in the RI-RBC model the growth rates of employment, investment, and output are about as persistent as in the data, with an amount of inattention consistent with the survey data on expectations. Moreover, rational inattention causes an increase in the demand for capital and labor on impact of a positive news shock. In general equilibrium, the rational inattention effect on labor demand more than offsets the wealth effect on labor supply and therefore employment

[44] Recall that in the SPF data the analogous regression coefficient is 0.76 with a standard error of 0.30.

[45] It seems plausible that in the real world decision-makers in small and medium-sized firms perceive the state of the aggregate economy with more noise than professional forecasters.

and output rise. The rational inattention effect on investment demand approximately offsets the wealth effect on desired saving.

In the rest of Section 5, we focus on an empirically motivated comparative static experiment. Appendix E contains additional numerical results. We analyze the behavior of the labor wedge in the model, lower the returns to scale in capital and labor (so far they are nearly constant, $\alpha + \phi = 0.33 + 0.65 \approx 1$), and compare the model to models with physical adjustment costs.

5.3 Changing macroeconomic volatility

The impulse responses in the model depend on how much attention agents choose to pay, and the optimal attention varies with the environment. So far in Section 5 we set the volatility of the productivity process based on the post-war U.S. data. Specifically, we set $\sigma = 0.008$ to match the standard deviation of the quarterly growth rate of TFP adjusted for capacity utilization in the period 1955Q1-2007Q4. The TFP growth rate was less variable in the second half of this sample than in the first half, a part of the decline in macroeconomic volatility known as the Great Moderation. The standard deviation of the TFP growth rate decreased from 0.9 percent (1955Q1-1984Q4) to 0.7 percent (1985Q1-2007Q4). Let us resolve the model with $\sigma = 0.009$ (higher volatility) and again with $\sigma = 0.007$ (lower volatility).

We find that in the lower volatility economy ($\sigma = 0.007$) the period 0 impulse response of labor input to a news shock is positive (like in the baseline with $\sigma = 0.008$). A news shock produces positive comovement of consumption and employment. In the higher volatility economy ($\sigma = 0.009$), the period 0 impulse response of labor input is negative. Here a news shock produces negative comovement of consumption and employment (Appendix Figure 1, upper-right panel, $h = 2$). The reason behind the change in the sign is intuitive. With higher volatility agents pay about 50 percent more attention to the state of the economy, and therefore the impulse response of employment is closer to the perfect information RBC model, than with lower volatility. This effect is strong enough to change the sign of the impulse response of employment to a news shock. Thus, the model suggests that empirical researchers who study different sample periods can be expected to reach conflicting conclusions regarding comovement.[46] Note also that the imperfect information

[46]Recall from the introduction that Barsky and Sims (2011) and Kurmann and Sims (2021) do not find business cycle comovement after a news shock, while Görtz, Tsoukalas, and Zanetti (2022b) who focus on data since the onset

version of the baseline RBC model in which firms take actions based on signals of the form "current productivity a_t plus noise" or "future productivity a_{t+h} plus noise" does not produce business cycle comovement, regardless of the value of σ or the amount of noise.[47]

The SPF data support the view that agents pay less attention to the macroeconomy since the onset of the Great Moderation than before. Coibion and Gorodnichenko (2015, Section III.A) make this point in detail. In Section 5.1, we reported the result from running the Coibion-Gorodnichenko regression on median forecasts of output from the SPF for the period 1968Q4-2019Q4 (with $\tau = 3$). We estimated a coefficient of 0.76 with a standard error of 0.30. Let's split the sample in half and rerun this regression in the two subsamples.[48] In the first half of the sample the coefficient falls to 0.48 (the standard error is 0.38). In the second half of the sample the coefficient rises to 1.21 (the standard error is 0.48). This finding is in line with the hypothesis of "less attention since the onset of the Great Moderation than before."[49]

6 Rational inattention by firms and households

We now add rational inattention on the side of households. We focus on how a small amount of inattention by households changes the equilibrium from Section 5. This appears to be the first time in the literature that a general equilibrium model is solved in which all agents are subject to rational inattention and prices, which the agents take as given, adjust so that markets clear (here, the wage adjusts to equate labor demand and supply and the price of a mutual fund share adjusts to equate asset demand and supply).[50]

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of the Great Moderation find comovement. Görtz, Tsoukalas, and Zanetti (2022b) also show that when they use the identification assumptions of Barsky and Sims (2011) or Kurmann and Sims (2021) and focus on the data since the onset of the Great Moderation, they do find comovement.

[47] Recall the discussion of the optimal confounding signal in Section 4.1.

[48] The SPF sample starts only in 1968 and therefore it seems reasonable to split the sample in half, rather than divide it into unequal "before" and "after" the onset of the Great Moderation subsamples. That alternative approach, however, happens to yield regression results very similar to the ones reported here.

[49] In Section 5.2 we also ran the same Coibion-Gorodnichenko regression on data simulated from the baseline rational inattention economy, obtaining a coefficient of 1.17 ($h = 2$). Repeating this regression in the model with $\sigma = 0.009$ and $\sigma = 0.007$ yields coefficients of 0.78 and 1.42, respectively.

[50] In Maćkowiak and Wiederholt (2015), all firms and households are also subject to rational inattention but in each market one side of the market sets the price while the other side chooses the quantity. Moreover, there is no

Loss in utility from suboptimal actions. Each household j chooses a signal about the state of the economy to maximize the expected discounted sum of utility. The household recognizes that a more informative signal requires more attention, which is costly. Proceeding analogously to Section 3, we derive an expression for the expected discounted sum of losses in utility when actions of household j deviate from the utility-maximizing actions – the actions the household would take if it had perfect information in every period. To obtain this expression, we compute the log-quadratic approximation to the expected discounted sum of utility at the non-stochastic steady state. After a change of variables similar to Section 3, we arrive at

$$\sum_{t=0}^{\infty} \beta^t E_{j,-1} \left[\frac{1}{2} \left(\tilde{x}_t - \tilde{x}_t^* \right)' \tilde{\Theta} \left(\tilde{x}_t - \tilde{x}_t^* \right) \right], \tag{21}$$

where

$$\tilde{x}_t = \begin{pmatrix} \omega_V \left(q_{jt} - q_{jt-1} \right) \\ \gamma \left[\omega_V \left(\frac{1}{\beta} q_{jt-1} - q_{jt} \right) + \omega_W l_{jt} \right] + \eta l_{jt} \end{pmatrix} \tag{22}$$

$$\tilde{\Theta} = -C^{1-\gamma} \gamma \begin{bmatrix} \left(1 - \frac{1}{1+\frac{\eta}{\omega_W \gamma}} \right) \frac{1}{\beta} & 0 \\ 0 & \frac{1}{1+\frac{\eta}{\omega_W \gamma}} \frac{1}{\gamma^2} \end{bmatrix}$$

$$\tilde{x}_t^* = \begin{pmatrix} z_t - (1-\beta) \sum_{s=t}^{\infty} \beta^{s-t} E_t \left[z_s \right] + \left(1 + \omega_W \frac{\gamma}{\eta} \right) \frac{1}{\gamma} \beta \sum_{s=t}^{\infty} \beta^{s-t} E_t \left[r_{s+1} \right] \\ w_t - \gamma \left(\omega_W w_t + \omega_D d_t \right) \end{pmatrix}, \tag{23}$$

$z_s \equiv \omega_W \left(w_s + \frac{1}{\eta} w_s \right) + \omega_D d_s$, and $r_{s+1} \equiv \beta v_{s+1} - v_s + (1-\beta) d_{s+1}$. See Appendix F.[51]

This objective has a simple interpretation. The first element of $\tilde{x}_t$ is the change in asset holdings. The second element of $\tilde{x}_t$ is the component of the marginal rate of substitution between consumption and leisure that is directly controlled by the household through the choice of asset holdings, q_{jt}, and hours worked, l_{jt}. The vector $\tilde{x}_t^*$ is the utility-maximizing action in period t. It is optimal to increase asset holdings when income is high relative to permanent income or when the return on saving is high. It is optimal to equate the marginal rate of substitution between consumption and leisure to the wage. When the household deviates from these optimal choices, the household loses an amount of utility determined by the matrix $\tilde{\Theta}$. This matrix is diagonal, because a *sub*optimal marginal rate of substitution between consumption and leisure does not affect the optimal change

capital in that model.

[51] The coefficients ω_V, ω_W and ω_D denote the steady-state ratios V/C, WL/C and $D/$C, respectively.

in asset holdings, and a *sub*optimal change in asset holdings does not affect the optimal marginal rate of substitution between consumption and leisure.[52] The tildes in $\tilde{x}_t$, $\tilde{x}_t^*$, and $\tilde{\Theta}$ indicate that these are the action, the optimal action, and the penalty matrix for a household rather than a firm.

The attention problem of a household. We assume that the household chooses asset holdings, q_{jt}, and hours worked, l_{jt}, in every period t. One can also think of the household as choosing the vector $\tilde{x}_t$ in equation (22). These two formulations are equivalent so long as the household knows its own past action q_{jt-1}, which is the case if $\mathcal{I}_{jt-1} \subset \mathcal{I}_{jt}$, where $\mathcal{I}_{jt}$ denotes the period t information set of household j. Since the matrix $\tilde{\Theta}$ is diagonal, the best response in period t given any information set $\mathcal{I}_{jt}$ is the conditional expectation of $\tilde{x}_t^*$, $\tilde{x}_t = E\left(\tilde{x}_t^* | \mathcal{I}_{jt}\right)$. The attention problem of household j then has the exact same form as the attention problem of firm i, (10)-(16). We posit again that the optimal action $\tilde{x}_t^*$ can be written as a linear function of a state vector, $\tilde{x}_t^* = \tilde{G}'\tilde{\xi}_t$, where the state vector has the property that it follows a first-order VAR process $\tilde{\xi}_{t+1} = \tilde{F}\tilde{\xi}_t + \tilde{\mu}_{t+1}$ with an innovation that follows a Gaussian vector white noise process with covariance matrix $\Sigma_{\tilde{\mu}}$. In the RBC model, $\tilde{x}_t^*$ is given by equation (23). We explain in the next paragraph how we find the state vector $\tilde{\xi}_t$.

Definition and computation of equilibrium. The definition of the RI equilibrium remains the same as stated in Section 3.3, except that in period -1 each firm *and* each household solve its respective attention problem.[53] The aggregate dynamics now depend on the signal process chosen by firms and on the signal process chosen by households. We compute the rational expectations equilibrium, where firms hold the correct belief about F and Σ_μ and households hold the correct belief about $\tilde{F}$ and $\Sigma_{\tilde{\mu}}$, using a guess-and-verify method. In the first step, we guess a finite-order ARMA process for the wage, w_t, and a finite-order ARMA process for consumption, c_t, which implies a process for the cost of capital, $\gamma E_t\left(c_{t+1} - c_t\right)$. We obtain G, F and Σ_μ in equations (11)-(12) from equation (9). In the second step, we solve the attention problem of firm i, and compute the process followed by k_{it}, l_{it}, y_t, k_t, i_t, and d_t. We express the solution for dividends, d_t, as a finite-order ARMA process. In the third step, we turn to the attention problem of household j.

[52] A given change in asset holdings can be financed with different combinations of consumption and hours worked. One of these combinations equates the marginal rate of substitution between consumption and leisure to the wage.

[53] The firm's attention problem is essentially unchanged. Households no longer have the same consumption level in this version of the model. We assume that each firm values profits according to the marginal utility of consumption of the average household.

From equation (23), the household's optimal action $\tilde{x}_t^*$ depends on w_t, d_t, and v_t (the first element $\tilde{x}_{1,t}^*$ depends on w_t, d_t, and v_t, and the second element $\tilde{x}_{2,t}^*$ depends on w_t and d_t). The price of a mutual fund share v_t adjusts so that in equilibrium asset demand equals asset supply, $\int_0^1 q_{jt}dj = 0$. To impose this market-clearing condition, we compute the process for v_t such that $\tilde{x}_{1,t}^* = 0$ given the guess for w_t and the solution for d_t. Since $\tilde{x}_{1,t}^* = 0$, the perfect tracking of $\tilde{x}_{1,t}^*$ requires no attention and the solution to the attention problem has the feature that $\tilde{x}_{1,t} = 0$, which implies that $q_{jt} = 0$ and $\int_0^1 q_{jt}dj = 0$. The guess for w_t, the solution for d_t, and the second line in equation (23) yield $\tilde{F}$, $\Sigma_{\tilde{\mu}}$, and $\tilde{G}$. We solve the attention problem of household j and use the solution, together with the second line in equation (22) and $q_{jt} = 0$, to compute the process followed by the household's hours worked, l_{jt}. We compute the aggregate consumption process from the equation $c_t = \omega_W \left(w_t + \int_0^1 l_{jt}dj \right) + \omega_D d_t$.[54] In the fourth step, we update the guess for the wage process and the guess for the cost of capital process in step 1 until the labor demand process obtained in step 2 equals the labor supply process obtained in step 3 and the cost of capital process guessed in step 1 is consistent with the aggregate consumption process obtained in step 3.

We assume the same parameter values as in Section 5, except that the marginal cost of attention to a household, which we call $\tilde{\lambda}$, no longer equals 0 as is implicit there.[55] We focus on a small $\tilde{\lambda}$ to learn how a small amount of inattention by households changes the equilibrium. We set $\tilde{\lambda} = (1/100,000)\, C^{1-\gamma}$ when $h = 0$ and $\tilde{\lambda} = (3/100,000)\, C^{1-\gamma}$ when $h = 2$.[56]

Results. Figure 6 shows the equilibrium with firms and households subject to rational inattention (lines with asterisks). The top row is the case of $h = 0$. The bottom row is the case of $h = 2$. The PI equilibrium (lines with points) and the equilibrium from Section 5 with rationally

[54] This equation follows from aggregating the log-linearized flow budget constraint of household j and imposing asset market clearing.

[55] The derivation of the household's objective assumes that η is a strictly positive number, whereas $\eta = 0$ in Section 5. Therefore we now set η equal to a very small, strictly positive number (so that utility is approximately linear in hours worked). The equilibria studied in Section 5 are essentially identical whether $\eta = 0$ or η equals a very small, strictly positive number.

[56] This means that the household's marginal cost of attention is equal to $1/100,000$ of steady-state consumption ($3/100,000$, respectively) per period. In equilibrium, the per period expected utility loss from inattention is equal to $5/1,000,000$ of steady-state consumption with $h = 0$ and $8/1,000,000$ with $h = 2$. In principle, one could choose the values of λ and $\tilde{\lambda}$ jointly to maximize the model's fit to, e.g., the unconditional moments in the data from Table 1. However, one would need to solve the model many times and each solution is time-consuming.

inattentive firms and perfectly informed households (lines with circles) are displayed for comparison.

Begin with a standard productivity shock, $h = 0$. On impact rational inattention by households reduces labor supply for a given wage, because it takes time for households to recognize that working conditions have improved. To restore equilibrium in the labor market the wage rises (the impulse response of the wage is stronger on impact when households are rationally inattentive than when they have perfect information). A higher wage depresses investment demand (the profit-maximizing capital stock is decreasing in the expected wage, see equation (9)). In equilibrium employment, investment, output, and consumption fall compared with the equilibrium from Section 5.1. Rational inattention by households adds further dampening and delay to the impulse responses of these variables to a productivity shock.

Next, consider a news shock, $h = 2$. Rational inattention by households has two effects in the model. It weakens the wealth effect because a news shock is an instantaneous change in the present value of income and rational inattention creates a dampened and delayed reaction of consumption and leisure to this change in permanent income. In addition, under rational inattention the labor supply decision becomes forward-looking, which makes households even more willing to supply labor at a given wage on impact of a positive news shock. The payoffs from future work rise, and the optimal signal of households confounds the payoff from current work with the payoffs from future work. Both effects of households' rational inattention strengthen comovement. To restore equilibrium in the labor market the wage falls (the impulse response of the wage is weaker on impact when households are rationally inattentive than when they have perfect information). A lower wage stimulates investment demand. In equilibrium employment, investment, and output rise on impact of a news shock, compared with the equilibrium from Section 5.2, while consumption falls. Once productivity rises in period 2, the wage increases while employment, investment, and output fall compared with the equilibrium from Section 5.2 (this is the effect already seen in the impulse responses to a productivity shock in top row of Figure 6).

We conclude that rational inattention by households *adds* further persistence to the growth rates of employment, investment, and output and *strengthens* comovement after a news shock. With rationally inattentive households employment, investment, and output are *even higher* on impact of a positive news shock.

Remark. We have assumed that households choose how much to save in the mutual fund and how much to work, (q_{jt}, l_{jt}). One could assume instead that households choose how much to consume and how much to work, (c_{jt}, l_{jt}). When households have perfect information, it makes no difference whether households choose q_{jt} or c_{jt}. When households are subject to rational inattention, it does make a difference whether households choose q_{jt} or c_{jt}, but we conjecture that the difference in equilibrium dynamics would be small. The reason is the following. The household sector as a whole cannot save by adjusting holdings of the mutual fund. The price of a share in the mutual fund always has to adjust so that in equilibrium asset demand equals asset supply, $\int_0^1 q_{jt} dj = 0$. The household sector as a whole *can* save by having firms invest, which implies lower dividends in the current period and higher dividends in future periods. Firms always aim to make the investment decision in the best interest of the households; hence, the cost of capital in the profit-maximizing action (9) equals $\gamma E_t (c_{t+1} - c_t)$. Furthermore, due to asset market clearing, aggregate consumption always equals $c_t = \omega_W \left(w_t + \int_0^1 l_{jt} dj \right) + \omega_D d_t$. This key mechanism is present in the model independent of whether households are choosing holdings of the mutual fund or consumption. However, there are two differences between these two versions of the RI-RBC model. First, when households choose consumption instead of the mutual fund holdings, the labor supply decision of households will be somewhat different, because it will become somewhat easier to equate the marginal rate of substitution between consumption and leisure to the wage (the second entry on the right-hand side of equation (22) will become $\gamma c_{jt} + \eta l_{jt}$ and the second entry on the right-hand side of equation (23) will become w_t). Second, when households choose consumption instead of mutual fund holdings, the consumption decision of some households will imply a positive $q_{jt} - q_{jt-1}$, while the consumption decision of other households will imply a negative $q_{jt} - q_{jt-1}$. The price of a mutual fund share will still always have to adjust so that in equilibrium $\int_0^1 q_{jt} dj = 0$.

7 Conclusions

This paper has two main messages:

- The RI-RBC model features positive autocorrelations in growth rates after a productivity shock and comovement after a news shock. The positive autocorrelations in growth rates are due to the fact that agents put a positive weight on their priors. The comovement after news shocks is caused by the fact that agents choose a low-dimensional representation of the state and try to get news into beliefs early.

- Solving standard DSGE models with rational inattention is feasible. The RI-RBC model has two variable inputs (capital and labor), two endogenous costs of inputs (the cost of capital and the wage), an endogenous mutual fund share price, and rationally inattentive firms and households. Solving this model is feasible. We hope that researchers will build on this methodological contribution and study other DSGE models with rational inattention.

When we introduced rational inattention on the side of households, we assumed that they choose how much to save in the mutual fund and how much to work. In future work, it would be interesting to solve the model under the assumption that households choose how much to consume and how much to work.

Insights from the model may help future empirical research. The rational inattention explanation for comovement after news shocks can potentially rationalize why researchers who study different sample periods can reach conflicting conclusions regarding comovement in the data. Furthermore, Miyamoto and Nguyen (2020) and Hirose and Kurozumi (2021) show that including survey data on expectations, in addition to macroeconomic data, in estimation of a DSGE model leads to more precise estimates of news shocks. In the RI-RBC model, the average forecast of productivity underreacts to a productivity shock and initially overreacts to a news shock. This difference may help identify news shocks in the data even more precisely. As another example, the literature finds that the average forecasts of macro variables in the survey data display a combination of underreaction and some overreaction (Angeletos, Huo, and Sastry, 2020, Kohlhas and Walther, 2021). The model suggests that rational inattention may help explain this pattern.

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Table 1: Business cycle statistics

	Data	Model, $h = 0$	
		Perfect information	Rational inattention
Relative standard deviation			
σ_c/σ_y	0.55	0.56	0.59
σ_l/σ_y	0.92	0.66	0.58
σ_i/σ_y	2.89	3.05	2.94
σ_a/σ_y	0.52	0.46	0.51
Correlation			
$\rho_{c,y}$	0.79	0.78	0.81
$\rho_{l,y}$	0.86	0.85	0.83
$\rho_{i,y}$	0.90	0.93	0.92
$\rho_{a,y}$	0.40	1.00	0.99
First-order serial correlation			
Δc	0.27	0.23	0.28
Δl	0.41	-0.06	0.44
Δi	0.35	-0.06	0.14
Δy	0.30	-0.05	0.13
Δa	-0.06	-0.05	-0.05

Data: United States, 1955Q1-2007Q4, from Eusepi and Preston (2011).
Model: Unconditional moments computed from the equilibrium MA representation of each variable.

Figure 1: Impulse responses with $\alpha = 0$

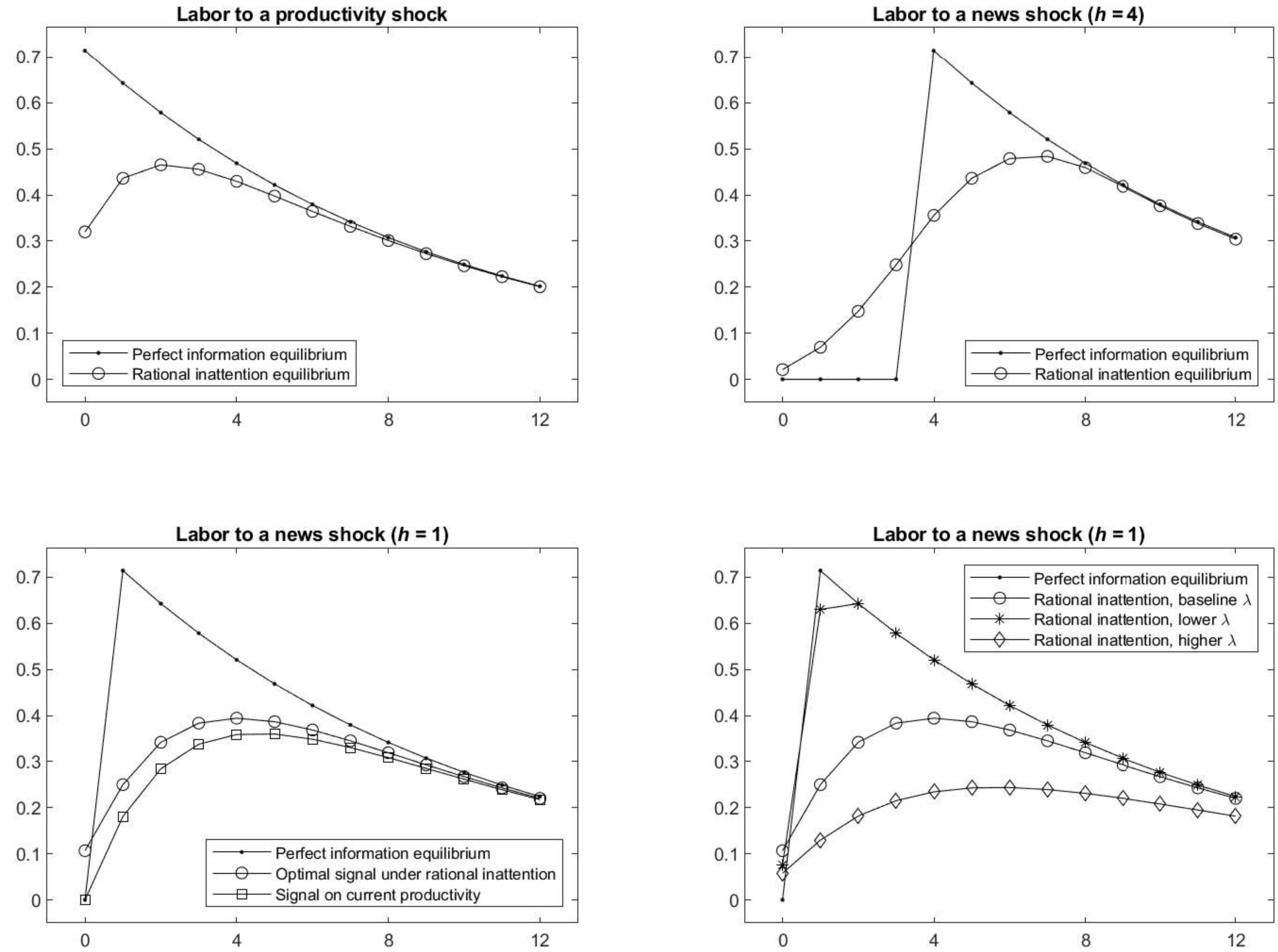


Figure 2: Impulse responses with $\phi = 0$

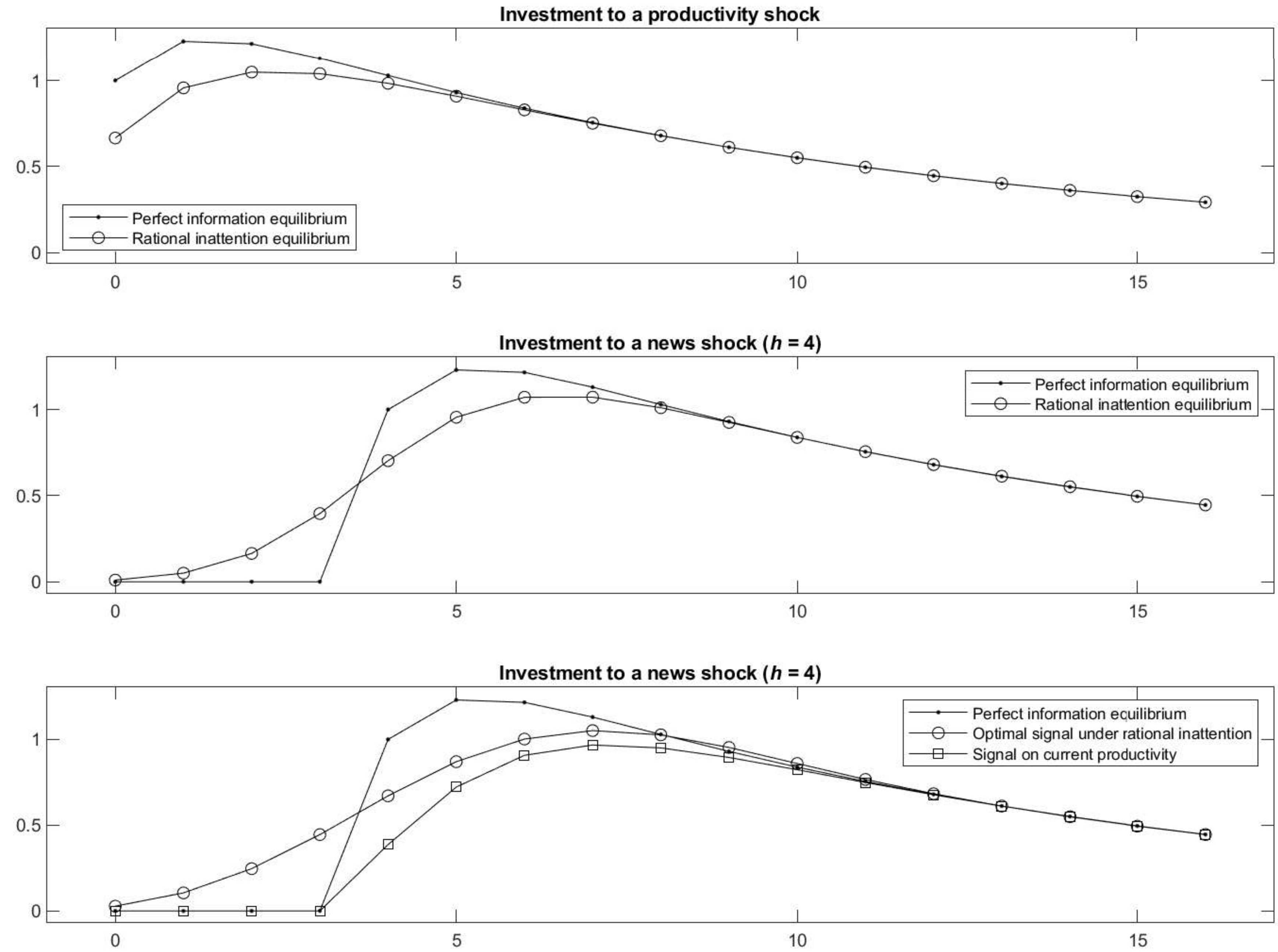


Figure 3: Impulse responses to a productivity shock

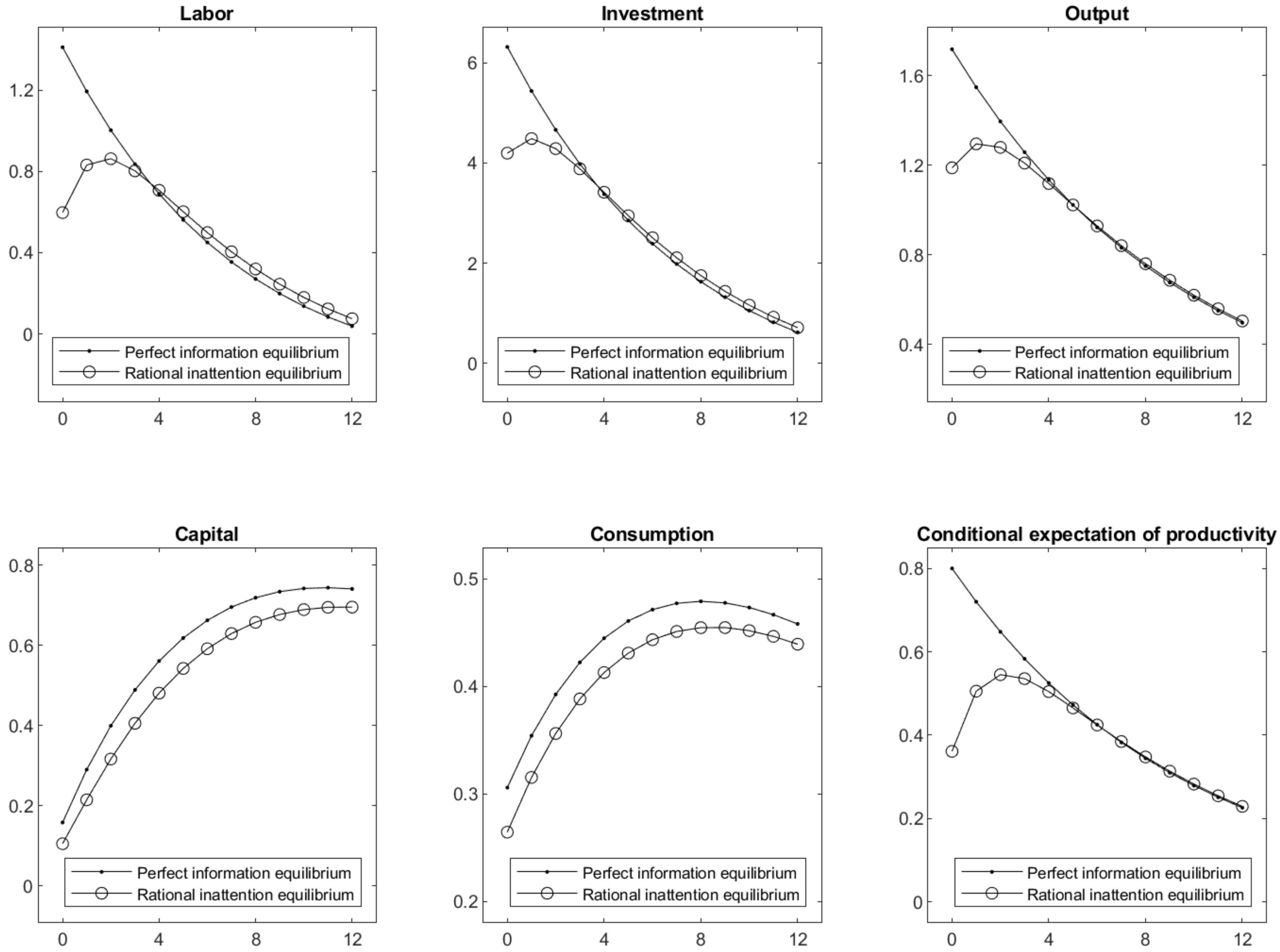


Figure 4: Impulse responses to a news shock (h = 2)

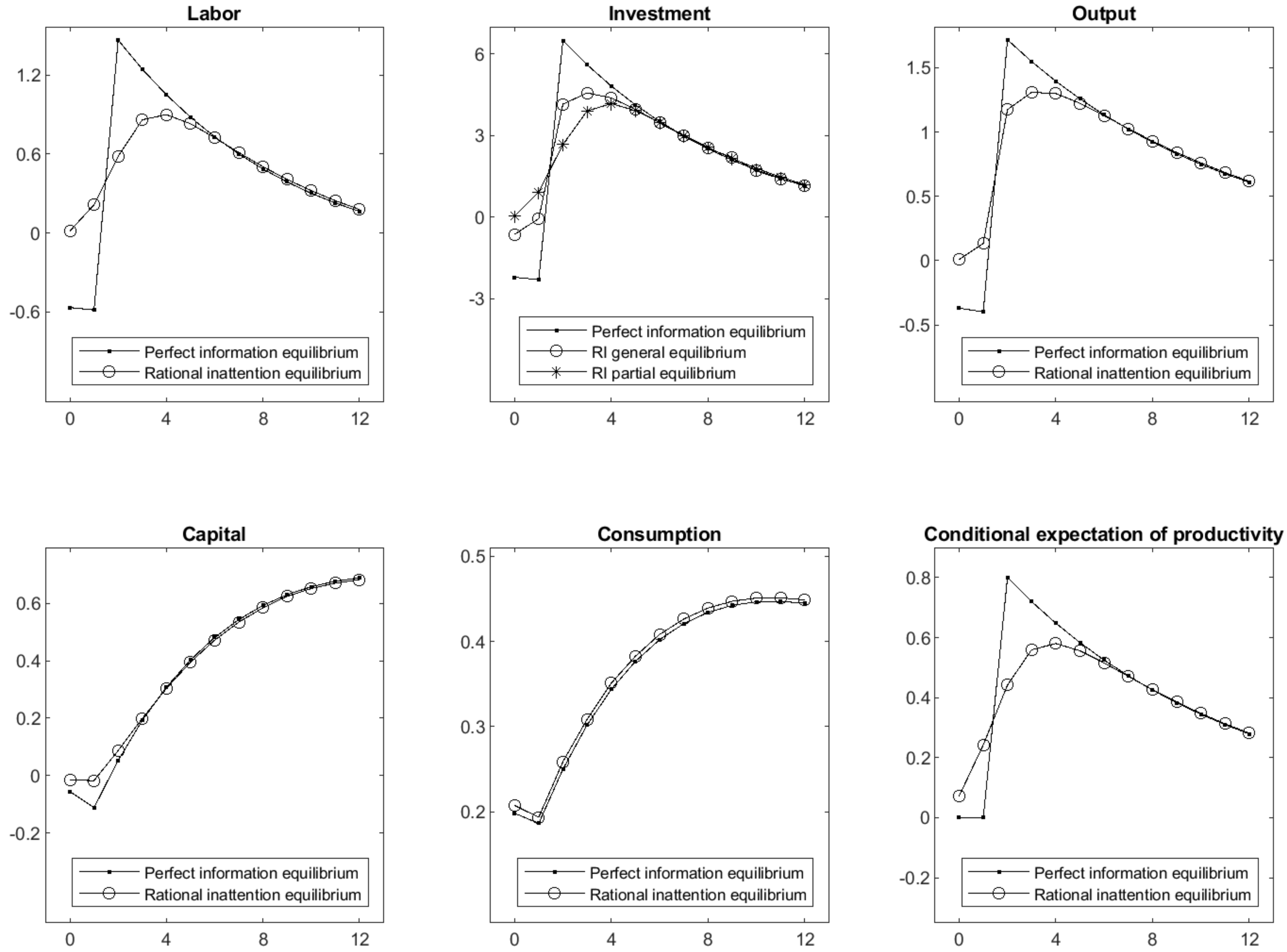


Figure 5: Impulse responses to a news shock ($h = 4$)

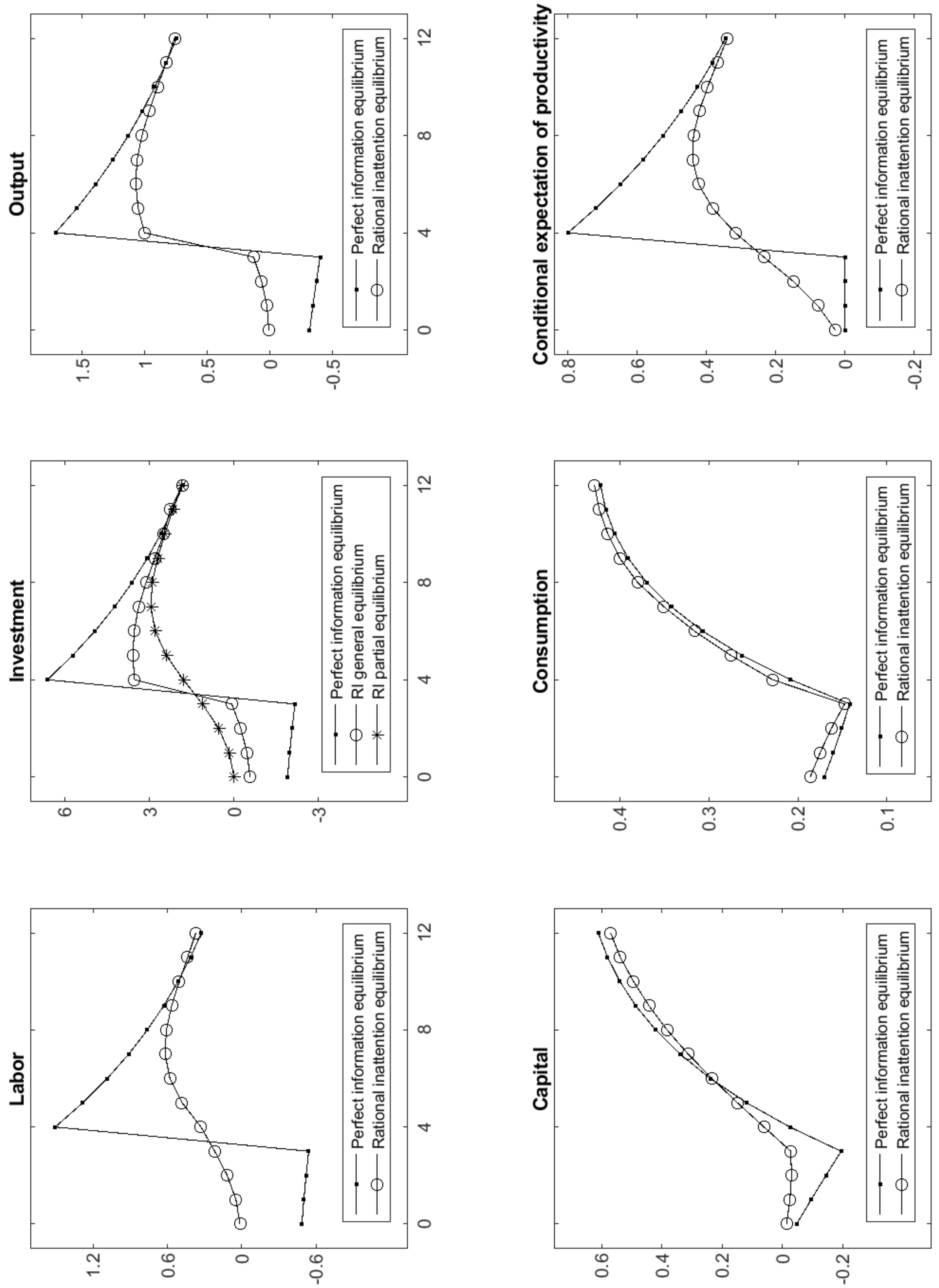
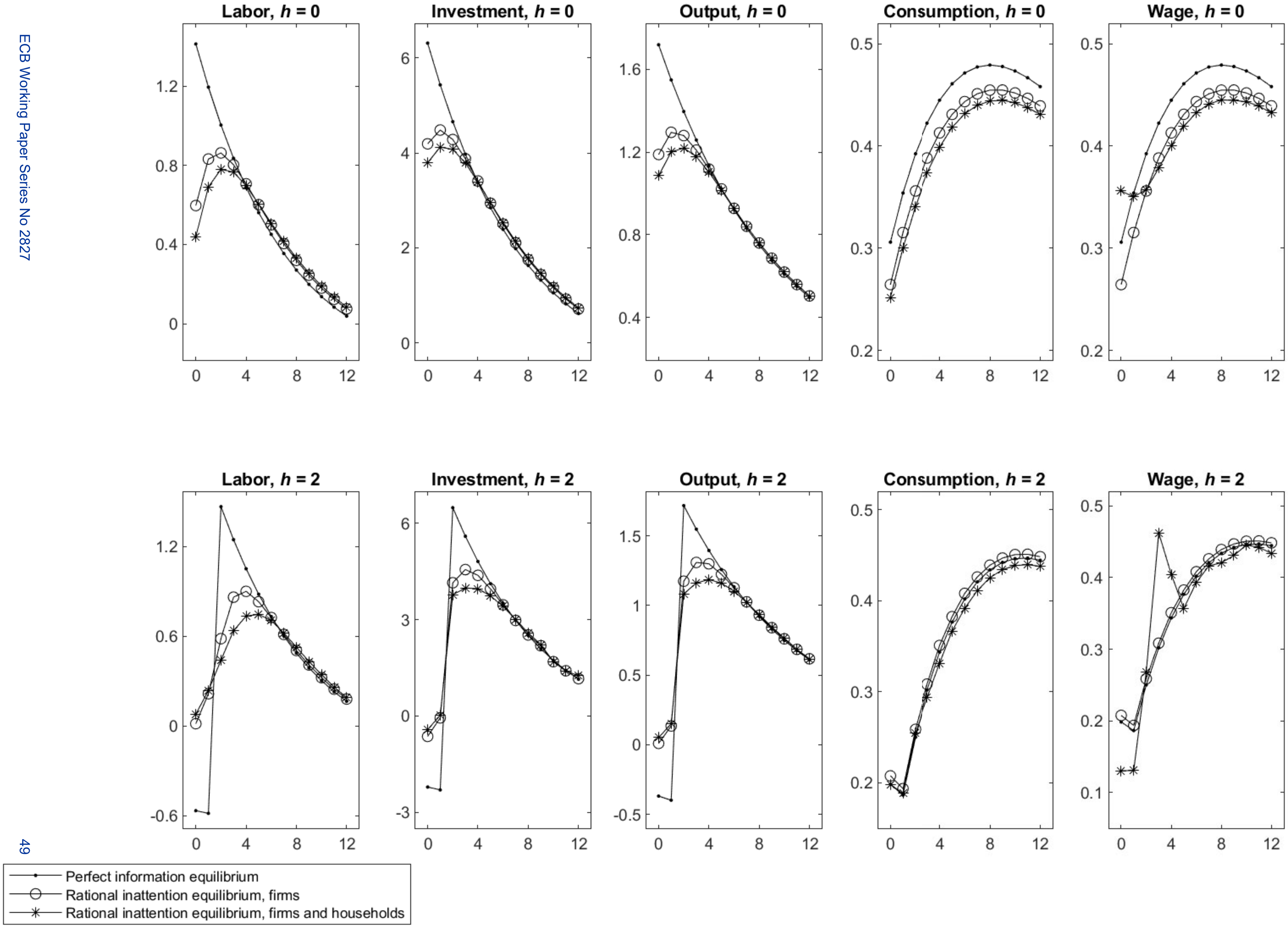


Figure 6: Impulse responses with rational inattention by firms and households



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