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WORKING PAPER SERIES

NO 1317 / APRIL 2011

FINANCIAL IMBALANCES AND FINANCIAL FRAGILITY

by Frédéric Boissay

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In 2011 all ECB publications feature a motif taken from the €100 banknote.

NOTE: This Working Paper should not be reported as representing the views of the European Central Bank (ECB). The views expressed are those of the author and do not necessarily reflect those of the ECB.

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Macroprudential Research Network

This paper presents research conducted within the Macroprudential Research Network (MaRs). The network is composed of economists from the European System of Central Banks (ESCB), i.e. the 27 national central banks of the European Union (EU) and the European Central Bank. The objective of MaRs is to develop core conceptual frameworks, models and/or tools supporting macro-prudential supervision in the EU.

The research is carried out in three work streams:
1. Macro-financial models linking financial stability and the performance of the economy;
2. Early warning systems and systemic risk indicators;
3. Assessing contagion risks.

MaRs is chaired by Philipp Hartmann (ECB). Paolo Angelini (Banca d'Italia), Laurent Clerc (Banque de France), Carsten Detken (ECB) and Katerina Šmídková (Czech National Bank) are workstream coordinators. Xavier Freixas (Universitat Pompeu Fabra) acts as external consultant and Angela Maddaloni (ECB) as Secretary.

The refereeing process of this paper has been coordinated by a team composed of Cornelia Holthausen, Kalin Nikolov and Bernd Schwaab (all ECB).

The paper is released in order to make the research of MaRs generally available, in preliminary form, to encourage comments and suggestions prior to final publication. The views expressed in the paper are the ones of the author(s) and do not necessarily reflect those of the ECB or of the ESCB.

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CONTENTS

Abstract. This paper develops a general equilibrium model to analyze the link between financial imbalances and financial crises. The model features an interbank market subject to frictions and where two equilibria may (co-)exist. The normal times equilibrium is characterized by a deep market with highly leveraged banks. The crisis times equilibrium is characterized by bank deleveraging, a market run, and a liquidity trap. Crises occur when there is too much liquidity (savings) in the economy with respect to the number of (safe) investment opportunities. In effect, the economy is shown to have a limited liquidity absorption capacity, which depends –inter alia– on the productivity of the real sector, the ultimate borrower. I extend the model in order to analyze the effects of financial integration of an emerging and a developed country. I find results in line with the recent literature on global imbalances. Financial integration permits a more efficient allocation of savings worldwide in normal times. It also implies a current account deficit for the developed country. The current account deficit makes financial crises more likely when it exceeds the liquidity absorption capacity of the developed country. Thus, under some conditions –which this paper spells out– financial integration of emerging countries may increase the fragility of the international financial system. Implications of financial integration and global imbalances in terms of output, wealth distribution, welfare, and policy interventions are also discussed.

JEL Classification Numbers: E21, F36, G01, G21

Keywords: Financial Integration, Global Imbalances, Asymmetric Information, Moral Hazard, Financial Crisis

Non-Technical Summary

This paper develops a general equilibrium model that describes causal relationships between financial integration, current account imbalances, and financial crises. Although stylized, the model is able to account for some important features of the recent crisis, like the reversal in leverage of market-based financial institutions and the sudden collapse of the wholesale financial market. The financial market is shown to improve (in terms of efficiency) the allocation of liquidity within the banking sector, and from the banking sector to the real sector. However, frictions between lenders and borrowers impair its functioning and, in particular, it may collapse when there is too much liquidity available compared with the number of (safe) investment opportunities. In effect, the financial market is shown to have a limited liquidity absorption capacity, which depends on the productivity of the real sector, the ultimate borrower. The model boils down to a familiar supply and demand nexus on the wholesale financial market. What makes this nexus non standard is the peculiar form of the aggregate fund demand curve, which is hump shaped due to the market frictions.

Extending the model to a two country framework, I present results in line with the recent literature that shows that the financial integration of financially under-developed countries is conducive to global imbalances. But I also go one step further by showing how and when such global imbalances make the international financial system fragile. In normal times, financial liberalization is found to increase welfare at the world level, but also to benefit to emerging countries and be detrimental to developed countries. Financial integration also makes financial crises more likely when the degree of financial development in the integrated emerging countries is too low and when capital flows toward developed countries are too large. The present paper also argues that one possible cause of the recent crisis is that the US productivity slowdown as of 2004 impaired US' liquidity absorption capacity precisely when more foreign capital was flowing in. It also shows that, when it materializes, a financial crisis reduces welfare in all countries.

Finally, I use the model to discuss the effects of two types of policy intervention. The first policy is one where central banks offer a deposit facility. I show that there exists a threshold for the real deposit facility rate above which financial crises are ruled out. The second policy corresponds to Basel III's minimum liquidity coverage ratio. I show that there exists an interval for this ratio over which financial crises are ruled out while the efficiency of the wholesale financial market is preserved.

1 Introduction

The aim of this paper is to model the relationship between global imbalances and financial crises while accounting for the following features of the recent financial crisis:

Feature 1 (Leveraged market-based banking sector): The crisis followed upon the rapid development of the market-based banking sector and the surge in this sector's leverage. Leverage of broker-dealers increased about threefold during the six year expansion that preceded the crisis (figure A). As a result, broker-dealers' total assets rose dramatically, up to 90% of US quarterly GDP in mid-2007 (figure B). These developments came along with the greater importance of broker-dealers in the supply of credit to the real economy, as documented by Adrian and Shin (2008b).

Feature 2 (External imbalances): The United States has run a persistent current account deficit since the early 1990s. Figure C illustrates this evolution as a ratio of world GDP. Starting at the end of the 1990s, the counterpart of these deficits has been mainly driven by large surpluses in Asian emerging market economies.

Feature 3 (Domestic imbalances): The financial deepening process in the run-up to the recent crisis was not accompanied with comparable changes in the real sector. On the contrary: US labour productivity growth was positive over this period but started to slow down significantly already in 2004 (figure D), falling from an average year-on-year growth rate of 1.65% in 2001-2004 to a year-on-year growth rate of 0.9% in 2005-2007. Kahn (2009) and Brackfield and Oliveira-Martins (2009) attribute this productivity slowdown mainly to the construction sector.

Feature 4 (Liquidity dry-up): The crisis materialized itself as a sudden and complete freezing of liquidity in key financial markets (see, e.g., Gorton and Metrick, 2009), an abrupt deleveraging in the market-based banking sector (figures A and B) as well as falls in international trade (figure C), productivity, and aggregate output (figure D).

The sudden change from boom to collapse has been so remarkable that one representation of the crisis is a model with two possible equilibria, one close to a frictionless financial market with an efficient allocation of resources, and the other characterized by a collapse in trade (Portes, 2009, Gorton, 2010). In the present paper I formalize this idea, and model financial fragility as the coexistence of two self-fulfilling multiple equilibria on the wholesale financial market. The model is simple and ultimately boils down to the standard nexus between aggregate supply and aggregate demand of funds. The crucial, non standard feature of the model is the form of the aggregate demand curve, which is hump-shaped due to market frictions. In other terms, aggregate demand reaches a maximum for some rate of interest, reflecting the limited liquidity absorption capacity of the wholesale financial market. The financial market is shown to be fragile and subject to runs whenever the supply of funds exceeds its absorption capacity, which depends on the productivity of the real sector, the ultimate borrower of funds.

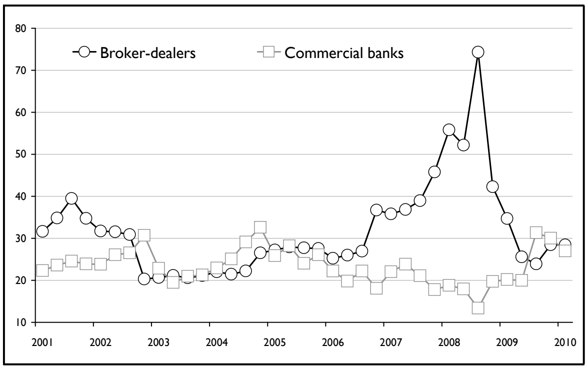


Fig. A: Leverage ratios

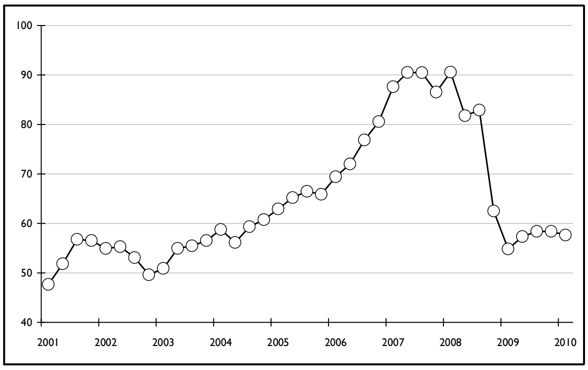


Fig. B: Broker-dealers total assets as a % of GDP

Fig. A: Total liabilities/(total assets-total liabilities). Source: US Flows of Funds (Federal Reserve).

Fig. B: Total assets in % of quarterly GDP. Source: US Flows of Funds (Federal Reserve).

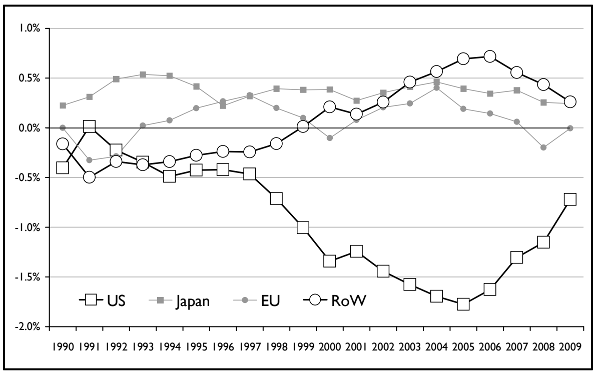


Fig. C: Current account by region

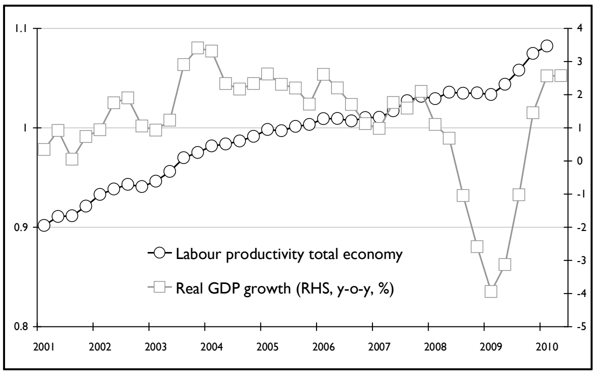


Fig. D: US labour productivity and GDP growth

Fig. C: Source: World Bank (WDI)

Fig. D: Labour productivity index normalized to one in 2005. Source: BIS and OECD (based on Bureau of Labour Statistics data).

I build the model in two steps. The first step consists in modelling capital flows among competitive heterogeneous banks through an interbank market. The model is static, has one period, and involves only one country. There is a continuum of banks born with some initial wealth. Each bank has access to a specific retail loan market and a specific and non-diversified pool of entrepreneurs. A bank may lend its resources either to entrepreneurs or to other banks on the interbank market. This market develops because banks are heterogeneous with respect to the probabilities that their respective pools of entrepreneurs default on retail loans. The expected returns on retail loans depend both on these default probabilities and on aggregate productivity in the real sector. Depending on their expected returns on retail loans, banks may choose to be either on the demand side or the supply side of the interbank market. Basically, banks with risky retail lending opportunities prefer to lend to other, more efficient banks rather than to their own pool of entrepreneurs. In contrast, efficient banks prefer to borrow on the interbank market in order to increase the size and total return of their retail loans. While the interbank market overall improves the allocation of liquidity among banks, it is also subject to frictions that prevent the economy from reaching the First Best allocation. Two types of frictions are considered jointly:

moral hazard and asymmetric information. Moral hazard arises because of limited contract enforceability. It is assumed that borrowing banks may misuse ("divert") the funds raised on the market, e.g. by investing into sub-prime mortgages. The net opportunity cost of diversion depends on the degree of contract enforceability, the expected return on retail loans, and leverage. To raise funds banks must discipline themselves by limiting their leverage (typically, banks must "have enough skin in the game"). The moral hazard problem alone is unable to generate self-fulfilling multiple equilibria, though. In effect, what makes banks' beliefs matter in the model is that information is asymmetric, in the sense that lending banks do not observe borrowing banks' expected returns on retail loans. Although lenders do not know individual borrowers' true quality and incentives to run away, they are able to infer the average borrower quality from the market return. For example, in a low market return environment even inefficient banks may prefer to borrow and operate on their retail loan market rather than lend on the interbank market, and so low interest rates arouse counterparty fear. Multiple self-fulfilling equilibria arise from lenders' beliefs about borrowers' quality. If lenders believe that borrowers are safe and do not need to be incentivized –a case of low counterparty fears, then they will tolerate high leverage and borrowers will be able to demand large loans. Aggregate demand for funds will be high, and so will the equilibrium market return. Since high market returns keep risky bankers away from the demand side, counterparty risk will indeed turn out to be limited. This equilibrium is what I will refer to as a "normal time" equilibrium. It is characterized by a deep interbank market and highly leveraged market-based banks (feature 1). In contrast, pessimistic beliefs may also be self-fulfilling, trigger a market run, a sudden liquidity dry-up, and a deleveraging process that are consistent with the observed developments in the financial sector during the recent crisis (feature 4). Such coordination failures are only possible when real sector productivity is too low, i.e. below what would be needed to maintain borrowers' incentives (feature 3).

In the second step, I analyze the effects of international capital flows on financial fragility. To do so, I extend the basic setup to a two-country framework. The two countries are identical (i.e. they have the same size, technology, distribution of banks, etc.), except with respect to the degree of development of their respective domestic financial systems. Contract enforceability is assumed to be weaker in the less financially developed ("emerging") country than in the financially "developed" country. In this context, financial integration is accompanied with positive net capital flows from the emerging to the developed country (feature 2) that improve the allocation of savings worldwide. However, under some conditions that will be discussed, current account imbalances are shown to generate financial fragility. The reason is that by exerting downward pressures on interest rates and market returns capital flows from the emerging country give inefficient and risky banks incentives to borrow funds. The mere possibility that such banks may enter the demand side of the market feeds counterparty fears and makes the financial market prone to coordination failures and freezing.

Related literature. The core modelling of the financial market is inspired from Aghion and Bolton (1997), where agents can choose to be borrowers or lenders. This feature is crucial in the present model to the extent that endogenous switches of risky banks from the supply to the demand side of the interbank market are the cause of sudden rises in counterparty fears and liquidity dry-ups. The moral hazard problem builds upon Holmström and Tirole (1997), with the difference that the private benefit from diversion is endogenous. A number of recent papers have used the Diamond and Dybvig (1983) framework as a basis to model the interbank transactions that arise as banks face liquidity shocks. This framework indeed proves particularly useful to study market liquidity problems and the costly liquidation of long term assets (e.g. Goldstein and Pauzner, 2005, Castiglionesi et al. 2010, Malherbe, 2010). Here, in contrast, the focus is on banks' funding liquidity problems. I do not assume idiosyncratic ex post liquidity (preference) shocks to make the interbank market emerge. Instead this market develops ex ante because banks' intermediation technologies are idiosyncratic: some banks have better retail loan opportunities than others. For this reason, the interbank market can be viewed more broadly as a wholesale financial market, rather than as a short term money market. In this context, market runs will take the form of sudden increases in margin requirements, as opposed to early fund withdrawals.

This paper belongs to the literature that diagnoses reversals in market-based bank leverage (or margin requirements, or haircuts) as the core mechanism behind the recent financial crisis and the collapse of a number of segments of the wholesale financial market (e.g. repo, asset backed commercial paper, etc.). In this recent literature leverage may reverse following an exogenous, adverse aggregate shock when banks finance long term assets with short term debt instruments (i.e. when there is a maturity mismatch) and face margin requirements (Adrian and Shin, 2008a, Geanakoplos, 2009). Here, in contrast, reversals in leverage follow upon the coordination failures and switches from the normal to crisis times equilibria that may occur when there is too much liquidity available in the interbank market.[1] Bebchuk and Goldstein (2010) also explain sudden funding liquidity problems by coordination failures, but they focus on the retail loan market. Importantly, the present paper also connects the literature on leverage cycles and the collapse of the wholesale financial market with that on global imbalances (Reinhart and Reinhart, 2008 ; Caballero et al., 2008 ; Mendoza et al., 2009). Mendoza et al. (2009), for example, show that financial integration can lead to large global imbalances when countries differ in the degree of domestic financial development. However, they do not discuss the causal link between global imbalances and financial fragility. Caballero and Krishnamurthy (2009) also analyze the relationship between external imbalances and financial fragility. In their paper the definition of financial fragility is different. It refers to the developed economy's banks selling safe assets to foreign investors while keeping the equity part of their domestic retail loans, which makes them

[1] The idea to model the crisis as a sudden regime switch is also favoured by Gorton (2010, page 20), who notes that "a *lot of macroeconomists think in terms of an amplification mechanism. So you imagine that a shock hits the economy. The question is: What magnifies that shock and makes it have a bigger effect than it would otherwise have? That way of thinking would suggest that we live in an economy where shocks hit regularly and they're always amplified, but every once in a while, there's a big enough shock ... So, in this way of thinking, it's the size of the shock that's important. A "crisis" is a "big shock." I don't think that's what we observe in the world. We don't see lots and lots of shocks being amplified. We see a few really big events in history: the recent crisis, the Great Depression, the panics of the 19th century. Those are more than a shock being amplified. There's something else going on. I'd say it's a regime switch—a dramatic change in the way the financial system is operating."*

more exposed to bad exogenous domestic shocks. There is no interaction between these banks, though, and no modelling of the interbank market. Castiglionesi et al. (2010) show that financial integration makes systemic crises less likely but more extreme. Their setup is different: by allowing risk sharing and cross-country liquidity insurance, financial integration gives banks incentives to increase their lending and balance sheet's maturity mismatch, which reduces banks' resilience to aggregate shocks. However, as they consider the financial integration of identical countries, there is no current account imbalance and no discussion on the link between the financial integration of emerging market economies and global imbalances. The present paper shows both how the financial integration of such countries generates current account imbalances and how these imbalances make the financial system more fragile. Finally, Martin and Taddei (2010) recently analyzed the effects of financial integration on business cycles when credit markets are subject to both moral hazard and asymmetric information. However, they do not model market freezes and restrict their analysis to the case of a small open economy.

Outline of the paper. The paper proceeds as follows. Section 2 sets up the baseline one country model. Section 3 presents the extension to two countries and discusses the effects of financial integration on global imbalances and financial fragility. Section 4 discusses some selected policy implications, and section 5 concludes.

2 Model Setup

I consider a competitive economy populated with a mass one continuum of risk neutral agents, who live one period from date 0 to date 1. There is one good in the economy, which agents may consume at date 1; every unit of good consumed yields one unit of utility. For an expositional reason that will be explained in a moment, I interpret agents as bankers. It will be convenient to think of each banker as living on an island populated with one local, representative entrepreneur. Hence there is a continuum of bankers, entrepreneurs, and islands. I will index bankers, entrepreneurs, and islands by p, with $p \in [0, 1]$. Every entrepreneur has one project that requires some initial investment at date 0 but does not have any wealth at date 0 to self-finance this investment. In contrast, every banker is born with one unit of good as initial wealth at date 0, which he may either store or lend to his local entrepreneur. It is assumed that a given banker cannot lend directly to the entrepreneurs on other islands. Every unit of good stored at date 0 yields one unit of good at date 1. In the rest of the paper, I will interpret the good stored as "cash". In contrast, bankers are heterogeneous with respect to retail loans's expected returns, in the sense that retail loans on island p pay off R unit of goods at date 1 (per unit of good lent) with probability p, and nothing with probability $1 - p$ –as described in figure A1 in the appendix. In the paper I will interpret R as capturing real sector's productivity in the economy; it is invariant across islands. There are several ways to interpret probability p. It may reflect entrepreneur p's idiosyncratic productivity. Or it may reflect banker p's skills in monitoring and supporting the entrepreneur's project. The more skillful banker p, the higher the probability of success of entrepreneur p's project. (In this latter case, p could reflect the quality of the bank lending relationship in island p.) From a technical standpoint these two interpretations are immaterial for what will matter is that banker p's retail loan portfolio may not yield anything at date 1 –this assumption that banker p's retail loan portfolio is not diversified will play an important role in the analysis. This heterogeneity creates scope for an interbank or, more generally, a wholesale financial market to develop, where skillful bankers borrow from unskilled, inefficient bankers. For simplicity, the ps are assumed to be uniformly distributed over interval $[0, 1]$. I will denote by r the gross interest rate paid on interbank loans, and by ρ the expected gross return on financial assets. These interest rates are endogenous. To make things interesting I assume that storage is an inefficient technology, i.e.,

Assumption 1 (Productivity Parameter): $R > 1$.

Because returns on retail loans are stochastic and bankers may default on their interbank debt, ρ may be lower than r. Moreover, r cannot be above the return on retail loans R (otherwise, there would be no demand for funds) and ρ cannot be below that on storage (otherwise there would be no supply). Hence, one has (provided that the interbank market exists): $1 \leqslant \rho \leqslant r \leqslant R$.[2] By raising funds on the financial market skillful bankers are able to extend their supply of funds on their respective retail loan markets and to increase their expected returns from retail lending. I call such bankers "borrowers" and denote by ϕ the amount borrowed by banker p per unit of initial wealth. Because ϕ is the ratio of market funding to banker's equity, I will call it "leverage"; it is endogenously determined and would depend on p in a frictionless world. Leverage is perfectly and publicly observable and, therefore, contractible upon. (In other terms, each lender can observe how much other lenders have lent to a given borrower, i.e. he can observe the borrower's balance sheet.) In contrast, for unskilled bankers it may be more profitable to lend on the wholesale financial market rather than use the storage technology or lend to the domestic entrepreneur. I will call such bankers "lenders". The higher (lower) p, the higher (lower) banker p's expected return on retail loans, and the more incline is banker p to borrow (lend) from (to) other bankers. Therefore there will exist an endogenous cutoff level $\overline{p}$, above (below) which bankers borrow (lend). This endogeneity of the distribution of lenders and borrowers is a crucial feature of the present model.

Assumption 2 (Banker p's Decisions): *Bankers take the market return ρ (as well as the market rate r) as given. Given ρ (and r) banker p decides whether, and how much, he borrows or lends so as to maximize his expected profit.*

Assumption 2 that bankers are price takers is consistent with them being atomistic and competitive in the wholesale financial market. I will denote by $d \in \{l, b\}$ the decision to lend (i.e. $d = l$) or borrow (i.e. $d = b$) on the wholesale financial market, and by ϕ the amount borrowed by banker p per unit of wealth, with $\phi \geqslant 0$. I do not exclude a priori that ϕ is a function of p

[2] In particular, in a frictionless world the expected returns on interbank loans depend on the ps and non-arbitrage requires they be the same across all bankers, i.e. $pr(p) = \rho$ (see appendix 7.2).

but I omit the p for notational purpose. Banker p's objective consists in maximizing his date 1 expected profit

$$\max_{d\in\{l,b\},\phi} \pi(p) \equiv \mathbf{1}_{d=l}.\rho + \mathbf{1}_{d=b}.p\left(R + \phi\left(R - r\right)\right) \tag{1}$$

with respect to his decisions d and ϕ, where $\mathbf{1}_{d=l,b}$ is a dummy equal to one if $d = l, b$ and zero otherwise. If banker p becomes a lender, then it is optimal for him to lend all his wealth, so that his expected return is equal to ρ. If banker p becomes a borrower on the wholesale market and finances his island's entrepreneur, then his expected return is equal to $p\left(R + \phi\left(R - r\right)\right)$, where $R - r$ is the borrower's rent per unit of leverage. Because this rent is positive it is always optimal for borrowers to lever as much funds as possible. In a frictionless world the most skillful banker would be able to borrow the full amount of savings available in the economy, and the economy would reach the First Best allocation: only the safe entrepreneur ($p = 1$) would be financed. To make things interesting, I assume that the wholesale financial market is subject to frictions that prevent the economy from reaching the First Best. Two types of frictions are considered jointly: moral hazard and asymmetric information. The benchmark economies when there is no friction, when there is asymmetric information only, and when there is a moral hazard problem only are analyzed in Appendix 7.2. As shown in this appendix, the outcomes of these economies are fairly straightforward and none of them features multiple equilibria. Therefore, for the sake of space, I focus on the economy with both frictions; these two frictions are described below.

Moral Hazard The moral hazard problem resembles Holmström and Tirole (1997)'s. I assume that at date 0 borrowers have the possibility to store funds aside, run away and consume the return on storage at date 1. I will refer to this as cash diversion. Concrete examples of such private benefits would be the commissions levied by brokers on abusive mis-selling of mortgages, credit cards, and other loan products.[3] It is assumed that running away is costly and that bankers must in this case sacrifice a fraction of every diverted good. I model this by assuming that the net return of cash diversion per unit of cash diverted is equal to γ, with

Assumption 3 (Diversion Cost Parameter): $0 \leqslant \gamma \leqslant 1$.

This net return is thus lower than the return from storage in the absence of diversion, and the overall return from diversion is $\gamma\left(1 + \phi\right)$ –the key assumption here is that the gain from diversion increases with leverage, not that it is proportional. Following Mendoza et al. (2009), I interpret parameter γ as an indicator of the degree of enforcement of financial contracts and, therefore, as an indicator of financial development of the economy (this point will be discussed in more details in section 3). The lower γ, the more costly to divert funds ($\gamma = 0$ corresponds to the absence of moral hazard). The overall structure of bankers' payoffs is summarized in Figure A1 in the appendix. The moral hazard problem takes place *ex ante*, as described in Table 1. Following

[3] Gerardi et al. (2010) show empirical evidence of loan mis-selling in the US prior to the recent financial crisis. In the present setup, however, cash diversion will only act as an out-of-equilibrium threat and shall never materialize itself in equilibrium.

diversion, borrowers do not pay their debt and lenders do not get any payment at date 1. As it will become clear in a moment, diversion is a simple and useful shortcut to introduce a limited borrowing capacity, as it implies that to raise funds bankers must have enough skin in the game and limit their leverage.

Date 0:	
1.	Given ρ, banker p simultaneously decides whether he stores, lends ($d = l$) or borrows ($d = b$)
2.	Borrower p demands a quantity ϕ of funds. Given ϕ, lenders decide whether they lend to p.
3.	Loan contracts are signed once aggregate supply equals aggregate demand
4.	Borrower p decides whether or not he diverts the funds
Date 1:	
1.	If he did not divert, borrower p gets net return $(1+\phi)R - \phi r$ with probability p, and nothing otherwise. If he diverted he gets $\gamma(1+\phi)$
2.	Each lender on the wholesale market gets ρ as return

Table 1: Time line

Asymmetric Information There is an asymmetry of information between borrowers and lenders in the sense that the ps are privately known. Lenders do not observe borrowers' skills and do not know every borrower's incentive to divert cash. Since skillful bankers are unable to distinguish themselves as skillful, they are also unable to commit themselves to behave better than unskilled borrowers. Indeed, if lenders were to naïvely accept to lend on the basis of borrowers' announced skills and tolerated a higher leverage for more skillful borrowers,[4] then all borrowers would claim being skillful and obtain large loans. Obviously no lender is willing to lend so much since in this case borrowers would divert the funds. Moreover, it is easy to see that borrowers' objective function (see relation (1) for $d = b$) does not satisfy the single-crossing property, which would otherwise make it possible for borrowers to truthfully reveal their types through an appropriate menu of contracts.[5] Indeed, assume that borrowers face a menu of contracts $\{r(\widetilde{p}), \phi(\widetilde{p})\}$ that stipulates the interest rate and the maximal leverage allowed for borrowers claiming $\widetilde{p}$. Since from borrower p's perspective what matters for the choice of the contract is the rent on leverage $\phi(\widetilde{p})(R - r(\widetilde{p}))$ (see expression (1)), and since this rent is independent of p, borrower p would always pick the contract that yields the highest rent. In effect, this contract would be the unique (non-revealing) loan contract remaining on the market.

[4] It is easy to show that in the symmetric information equilibrium borrowers' leverage increases with p – see appendix 7.2.3.

[5] A menu of contracts could be revealing if, for example, for a given increase in the borrowing rate, skillful borrowers were willing to accept a relatively lower increase in leverage than less skillful borrowers. That is, if the marginal rate of substitution between ϕ and r at any given (r, ϕ) pair, $\left(\frac{d\phi}{dr}\right)_{\pi=\overline{\pi}}$, decreased with p. Since in this case skillful borrowers would value leverage relatively more (at the margin) than unskilled borrowers, it would be possible to design a menu of contracts that associates higher borrowing rates with higher leverage in such a way that skillful borrowers would reveal themselves by picking the high rate contract (in such contracts, leverage would typically be a concave function of the borrowing rate). However, from expression (1) one can see that the marginal rate of substitution between ϕ and r is independent of p: $\left(\frac{d\phi}{dr}\right)_{\pi=\overline{\pi}} = \frac{\phi}{R-r}$. Hence, borrowers' objective function does not satisfy the single-crossing property.

It follows that the only loan contract (r,ϕ) signed in equilibrium is identical for all borrowers. Given the market rate r (and return ρ), borrowers all demand the same loan ϕ. Borrower p's optimization problem therefore consists in maximizing his expected profit (see expression (1) for $d=b$) with respect to leverage ϕ under the constraint that the expected return on retail loan is above the expected return on wholesale loan (participation constraint), and under the constraint that he can credibly commit himself not to divert the funds (incentive compatibility constraint).

Before I explicit borrower p's participation and incentive compatibility constraints, one comment is in order regarding the interpretation of agents as "bankers". Here agents do not perform any of the specific tasks that would justify an interpretation in terms of "traditional" commercial banking, whereby bankers are intermediaries between depositors and borrowers. For example, they provide no payment services, perform no asset transformation, there is no delegated monitoring, etc. To the extent that they borrow and lend to each other on the wholesale financial market agents should rather be viewed as "market-based" intermediaries, like broker-dealers or investment bankers. This interpretation has no material implication for the model, however, and one could also view the agents as other non-bank financial firms, large non-financial firms, or any other type of levered investors who have specific investment opportunities on the one hand, and raise funds through the financial market on the other hand.[6]

2.1 Participation and Incentive Compatibility Constraints

Since technology and profit is linear, it is easy to see that a banker p either borrows or lends, but never does both. When he lends it is optimal for him to lend his entire initial wealth, in which case he gets return ρ. It is easy to see from (1) that banker p borrows ($d^*=b$) if

$$p\,(R+\phi\,(R-r))\geqslant\rho, \tag{PC}$$

and lends ($d^*=l$) otherwise. Constraint (PC) is thus borrower p's participation constraint. Since bankers whose p satisfies

$$p\geqslant\overline{p}\equiv\frac{\rho}{R+\phi\,(R-r)} \tag{2}$$

[6] Mechanism design theory establishes that deposit based banks may arise endogenously as part of an efficient arrangement. Typically, as coalitions of agents banks are able to provide insurance against liquidity shocks (Diamond and Dybvig, 1983), or share information (Boyd and Prescott,1986). More recently, Mattesini et al. (2010) rationalized the existence of banks by the presence of commitment issues rather than informational frictions. The focus of the present paper is different. First, I do not seek to explain why banks exist, and in the present setup potential coalitions between bankers into larger and perfectly diversified financial institutions would be ruled out by the moral hazard problem. Second, I am primarily interested in *market-based* banks (as opposed to deposit-based banks) because of their increasing importance in the economy and central role in the recent crisis, as Adrian and Shin (2008b) have documented: "*broker-dealers have traditionally played market-making and underwriting roles in securities market. However, their importance in the supply of credit has increased dramatically in recent years with the (...) changing nature of the financial system toward one based on capital market, rather than one based on the traditional role of the bank as intermediating between depositors and borrowers.*" (p. 1).

borrow funds, the expected return from the loans to the pool of borrowers is by definition equal to

$$\rho \equiv r \int_{\overline{p}}^{1} p \frac{dp}{1-\overline{p}} = \frac{1+\overline{p}}{2} r. \tag{3}$$

This relation means that the risk premium r/ρ is the inverse of the average repayment probability of the pool of borrowers, $(1+\overline{p})/2$. Although lenders do not observe the type of each borrower, they are able to infer the type $\overline{p}$ of the marginal borrower based on the market return ρ and the market rate r. No banker will be willing to lend if there exist borrowers $p \geqslant \overline{p}$ whose return from cash diversion $\gamma(1+\phi)$ is above both the expected return on retail lending and above the return on financial assets. Borrower p knows of this in advance, and so takes care never demand too high a loan ϕ, such that

$$\gamma(1+\phi) \leqslant \max\{\overline{p}(R+\phi(R-r)), \rho\}. \tag{IC}$$

The above constraint can be interpreted in two different ways, reflecting bankers' trade-offs between cash diversion and retail lending on the one hand, and between cash diversion and wholesale lending on the other hand (see also figure A1). It can be seen as borrowers' incentive compatibility constraint $(\gamma(1+\phi) \leqslant \overline{p}(R+\phi(R-r)))$, but also as lenders' incentive compatibility constraint $(\gamma(1+\phi) \leqslant \rho)$, to the extent that lenders too could potentially borrow and divert funds. By construction, the two terms inside the max operator are identical (see relation (2)). Constraint (IC) is at the core of the present model. It requires that leverage be incentive compatible for *all* borrowers, including the marginal one, and therefore guarantees that all borrowers have the incentive to use the funds borrowed for retail lending. This may seem extreme at first sight (indeed, this rules out cross-subsidization between borrowers, whereby virtuous borrowers would pay a higher cost of funding to compensate for the losses on peccant borrowers) but it is a necessary condition for the market to clear. To understand this point, consider for a moment a situation where lenders would accept to lend to over-levered borrowers (so that constraint (IC) does not hold). Then one would have $\gamma(1+\phi) > \rho$, which means that bankers would be better off borrowing and diverting the cash rather than lending to other bankers. In this case, however, there would be no supply of funds on the wholesale market, which contradicts the fact that some bankers lend in the first place. Such situation cannot be an equilibrium. Now assume that at the market rate r all lenders accept to lend only to borrowers whose leverage satisfies constraint (IC), except one deviating lender, who tolerates a higher leverage. In this case, *all* bankers would demand a loan to this deviating lender, who would then be exposed to cash diversion, face a lower repayment probability and, ultimately, obtain a lower expected return. In particular, because some borrowers would divert cash, the average repayment probability of the pool of borrowers would be less than 1/2 and the lender's expected return less than $r/2$, which is below the equilibrium market return (see relation (3)). It follows that in equilibrium no lender has interest in granting a loan that violates constraint (IC).[7] The program of banker p consists in maximizing

[7] This result reflects the existence of strategic complementarities between lenders (see Cooper and John, 1988). Indeed, by raising aggregate demand, an increase in leverage tolerance by all lenders except one works to increase

his expected profit with respect to d and ϕ –see (1)– under the incentive compatibility constraint (IC). I am now ready to define an equilibrium:

Definition 1 (Equilibrium): *An equilibrium of the wholesale financial market is a couple (ρ^*, ϕ^*) for the expected market return ρ^* and leverage ϕ^* such that (i) ϕ^* is optimal given ρ^* and (ii) the wholesale financial market clears.*

I solve the equilibrium in three steps. First, I derive the optimal leverage that maximizes borrower p's expected profit under constraint (IC) and determine the type of the marginal borrower $\overline{p}$. This permits me to derive the aggregate supply and demand curves (second step), and eventually to solve for the equilibrium (third step).

2.2 Optimal Leverage and Marginal Borrower

Since it is optimal for the borrowers to demand as big a loan as possible, the incentive compatibility constraint binds and one has

$$\phi^* (\rho) = \frac{\rho - \gamma}{\gamma}. \tag{4}$$

The positive relationship between ϕ and ρ is an important feature of the present model but at odds with standard asymmetric information models, like Stiglitz and Weiss (1981)'s, which predicts on the contrary that borrowing constraints should be more stringent when the market return goes up. Two important differences with this type of models are worth mentioning. First, in these models lenders usually have market power, whereas here the wholesale market is competitive and bankers are price takers. Second, in Stiglitz and Weiss credit rationing is due to the adverse selection of borrowers and the fact that the identity of the lenders and borrowers is fixed exogenously. In the present model there is no adverse selection, and bankers choose on which side of the market they operate. This choice depends on the return on financial assets. When ρ decreases, for example, the net present value of retail loans increases and turns positive for the lenders with the highest ps. As a result, these bankers shift from the supply to the demand side of the wholesale financial market and become borrowers: $\overline{p}$ goes down. The drop in $\overline{p}$ as two distinct implications. First, new borrowers are less efficient than the borrowers already present in the market and therefore the average repayment probability diminishes. This contrasts with adverse selection models where a decrease in ρ would on the contrary work to improve the average quality of borrowers. (These models assume in general a mean preserving spread distribution of returns –or a similar mechanism, which makes the best borrowers leave the market when the interest rate increases.) Second, the drop in $\overline{p}$ also implies that the marginal borrower's incentive to divert cash increases, which arouses lenders' fear of diversion. Understanding lenders' increasing fear borrowers reduce their leverage in order to access the market: each of them demands a loan such that even the least efficient, marginal, borrower can commit himself not to run away. Hence the positive relationship between ϕ and ρ. Overall, this positive relationship results from the joint

the equilibrium market return ρ^*, and therefore gives this one lender incentive to raise his leverage tolerance as well.

effects of moral hazard and asymmetric information. Because of the moral hazard problem lenders put a cap on borrowers' leverage. Because of the asymmetry of information the marginal borrower exerts a negative externality on the whole pool of borrowers. Indeed, not only does his incentive compatibility constraint determine his own leverage, but it also determines the leverage of all the other borrowers. (To see this point, compare constraint (IC) with the incentive compatibility constraint (A3) –in appendix 7.2.3– that would prevail in a symmetric information world.) Since the marginal borrower's tolerated leverage increases with the marginal borrower's skills, leverage goes down whenever new, less efficient bankers enter the demand side of the market or, in other words, when ρ decreases ($\partial\phi/\partial\rho > 0$). This "leverage effect" works to decrease the aggregate demand for funds when market return goes down and is responsible for the hump shaped form of the aggregate demand curve (see figure 1). It is useful for the determination of aggregate demand and supply to express $\overline{p}$ as a function of ρ. Substituting ϕ and r out of relations (2), (3), and (4), one can characterize the marginal borrower $\overline{p}$ as follows (see appendix 7.3):

$$R\overline{p}^2 + (R + \gamma - 2\rho)\,\overline{p} - \gamma = 0 \underset{\overline{p}\in[0,1]}{\Rightarrow} \overline{p} = \varphi(\rho) \equiv \frac{2\rho - R - \gamma + \sqrt{(2\rho - R - \gamma)^2 + 4\gamma R}}{2R}. \qquad (5)$$

It is easy to check that $\varphi'(\rho) > 0$, which means that the number of lenders increases as market return goes up (for $\rho \geqslant \theta$). Given ρ, an increase in R increases the opportunity cost of investing into financial assets, and so reduces the number of lenders $\partial\overline{p}/\partial R < 0$. A rise in γ has the opposite effect. By raising the incentive to divert cash, it triggers deleveraging, lowers the overall return on retail loans, and raises the opportunity cost of borrowing; hence the rise in the number of lenders ($\partial\overline{p}/\partial\gamma > 0$).

2.3 Aggregate Funds Supply and Demand

I am now in the position to derive the aggregate supply and demand curves. When $\rho < 1$, bankers prefer storage over wholesale lending and there no supply of funds in this case. When $\rho \in (1, R]$ bankers $p \leqslant \overline{p}$ become lenders and aggregate supply is then equal to $\overline{p}$. When $\rho = 1$ bankers $p \leqslant \overline{p}$ are indifferent between storage and wholesale lending, so that aggregate supply is undetermined (but below $\overline{p}$). Finally, when $\rho > R$ all bankers supply funds, meaning that aggregate supply is equal to 1. Hence, aggregate supply $S(\rho)$ takes the following form:

$$S(\rho) = \left\{ \begin{array}{cl} 0 & \text{if } \rho < 1 \\ \in [0, \varphi(1)] & \text{if } \rho = 1 \\ \varphi(\rho) & \text{if } \rho \in (1, R] \\ 1 & \text{if } \rho > R \end{array} \right\}. \qquad (6)$$

On the demand side, when $\rho \in [1, R]$ bankers $p \geqslant \overline{p}$ become borrowers and borrow ϕ, so that aggregate demand equals $(1 - \overline{p})\,\phi$. When $\rho < 1$, the opportunity cost of borrowing is the return on storage: aggregate demand is constant and the same as when $\rho = 1$. Finally, when $\rho > R$ no banker wants to be a borrower, and aggregate demand is null. Aggregate demand $D(\rho)$ can therefore be expressed as

$$D(\rho) = \left\{ \begin{array}{cc} (1-\varphi(1))\left(\frac{1-\gamma}{\gamma}\right) & \text{if } \rho < 1 \\ (1-\varphi(\rho))\left(\frac{\rho-\gamma}{\gamma}\right) & \text{if } \rho \in [1, R] \\ 0 & \text{if } \rho > R \end{array} \right\}. \tag{7}$$

In equilibrium, the aggregate demand for funds corresponds to the total amount of funds that flow from bankers with $p < \overline{p}$ to bankers with $p \geqslant \overline{p}$. It is driven by two opposite forces. On the one hand, all things being equal, a rise in ρ works to decrease the number of borrowers and, therefore, aggregate demand for funds ($\varphi'(\rho) > 0$). On the other hand, it works to increase leverage per head ($\partial\phi/\partial\rho > 0$). These forces result in the aggregate demand curve being strictly concave over interval $(1, R)$, $D'(R^-) = -\varphi'(R)\left(\frac{R}{\gamma} - 1\right) < 0$, and (for γ large enough) $D'(1^+) > 0$ and hump-shaped.[8]

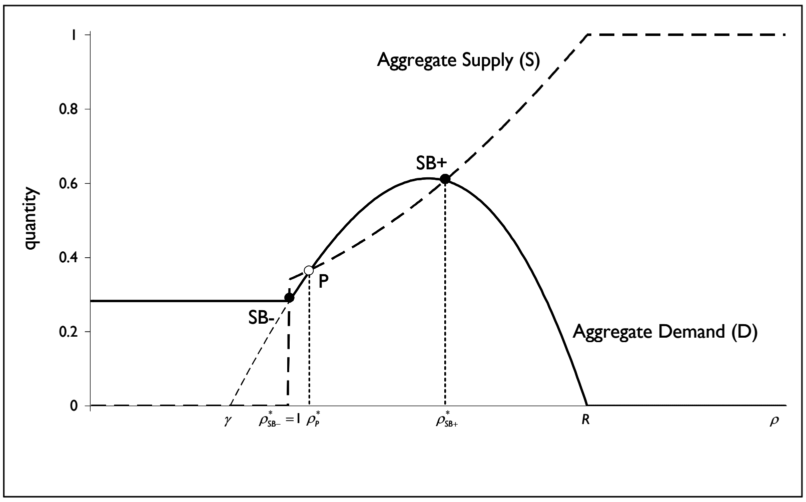


Figure 1: Multiple self-fulfilling equilibria
with $R = 2.5$; $\gamma = 0.7$

This hump shape reflects the limited liquidity absorption of the economy. The fact that aggregate demand reaches a maximum for some market return $\rho > 1$ (see figure 1) means that borrowers may be unable to absorb the whole supply of funds whenever it is too high. It also reflects the negative externality that the marginal borrower imposes on the other, more efficient borrowers when it enters the demand side of the market. Which of the two forces prevails depends on the prominence of this externality, which is more severe when it affects many borrowers (i.e. when $\overline{p}$ and ρ are low). It follows that aggregate demand increases (decreases) with ρ for low (high) values of ρ. In addition, since $\partial\overline{p}/\partial\gamma > 0$, it is easy to see that $\partial S(\rho)/\partial\gamma \geqslant 0$ and $\partial D(\rho)/\partial\gamma \leqslant 0$. As the incentive to divert cash increases (higher γ) bankers' borrowing capacity

[8] Indeed, one has $\varphi'(\rho) > 0$, $\varphi''(\rho) > 0$, and $D''(\rho) = -\varphi''(\rho)\left(\frac{\rho}{\gamma} - 1\right) - \frac{2}{\gamma}\varphi'(\rho)$. Hence, $D''(\rho) < 0$. Moreover, from (7) one gets: $D'\left(1^+\right) > 0 \Leftrightarrow 1 - \gamma < \frac{1-\varphi\left(1^+\right)}{\varphi'\left(1^+\right)}$. When $\gamma \nearrow 1$ the left-hand side of the inequality goes to zero, while the right-hand side is strictly positive (from (5)). In other terms the aggregate demand curve is hump shaped when the moral hazard problem is not too benign.

diminishes and the retail lending activity becomes less attractive with respect to financial assets. In this case the supply curve shifts upward and the demand curve shifts downward. An increase in R has the opposite effect by making retail lending more attractive relative to financial assets: $\partial S(\rho)/\partial R \leqslant 0$ and $\partial D(\rho)/\partial R \geqslant 0$. Given the above aggregate demand and supply, the market clearing condition, which determines ρ^*, reads

$$S(\rho^*) = D(\rho^*). \tag{8}$$

2.4 Equilibrium

The aggregate supply and demand curves are represented in figure 1, for a case where the moral hazard problem on the financial market is neither too severe nor too benign (i.e. when productivity R is neither too high nor too low and γ is above a certain threshold $\overline{\gamma}$ –see proposition 1 and relation (A10) in appendix 7.4) and multiple equilibria coexist. Figure 2 illustrates two other possible and interesting configurations. Those are represented by points $SB-$, P, and $SB+$. It is easy to see that only equilibria $SB-$ and $SB+$ are locally *tatonnement* stable, in the sense that any small perturbation to the equilibrium price would bring the price back to equilibrium as a result of a standard Walrasian tatonnement process.[9] In contrast, equilibrium P associated with expected return ρ_P is unstable and, as such, is of limited relevance; I will not discuss this equilibrium further in the paper. Which equilibrium is ultimately reached depends on bankers' beliefs about the odds that borrowers run away. Since in this paper I am only interested in the conditions of coexistence of multiple equilibria and not in which equilibrium is ultimately selected when $SB-$ and $SB+$ coexist, I will not address the issue of equilibrium selection here. Proposition 1 below describes the conditions of existence and uniqueness of equilibria $SB-$ and $SB+$.

Proposition 1 (Equilibrium): *There exist a threshold $\overline{\gamma}$ (with $0 < \overline{\gamma} < 1$) and functions $\widehat{R}(\gamma)$ and $\overline{R}(\gamma)$, with $\overline{R}(\gamma) \geqslant \widehat{R}(\gamma) \geqslant 1$, $\widehat{R}'(\gamma) > 0$, $\overline{R}'(\gamma) > 0$ $\forall\gamma \in (0,1]$, and $\lim_{\gamma\searrow 0}\overline{R}(\gamma) = \lim_{\gamma\searrow 0}\widehat{R}(\gamma) = 1$, such that:*[10]

i. If $\gamma \in (0,\overline{\gamma}]$ then $\widehat{R}(\gamma) = \overline{R}(\gamma)$; If $\gamma \in (\overline{\gamma},1]$ then $\widehat{R}(\gamma) < \overline{R}(\gamma)$;
ii. Equilibrium $SB+$ exists if and only if $R > \widehat{R}(\gamma)$ and equilibrium $SB-$ exists if and only if $R \leqslant \overline{R}(\gamma)$;
iii. If it exists $SB+$ is characterized by $\left(\rho^*_{SB+}, \phi^*_{SB+}\right)$ where ρ^*_{SB+} is the largest solution to (8) and $\phi^*_{SB+} = (\rho^*_{SB+} - \gamma)/\gamma$.[11] *If it exists $SB-$ is characterized by $\left(\rho^*_{SB-}, \phi^*_{SB-}\right)$ with $\rho^*_{SB-} = 1$ and $\phi^*_{SB-} = (1-\gamma)/\gamma$.*

Proof: See appendix 7.4.

[9] See Mas-Colell et al., 1995, section 17H, and the discussion in appendix 7.5.

[10] For the value of $\overline{\gamma}$ and the explicit forms of functions $\overline{R}(\gamma)$ and $\widehat{R}(\gamma)$ see expressions (A10), (A8), and (A9) in appendix 7.4.

[11] Replacing (5) into (6), (7), and (8), one gets (for $\rho \in (1, R]$): $D(\rho) - S(\rho) \geqslant 0 \Leftrightarrow \frac{R}{\gamma} \geqslant \psi(\overline{p}) \equiv \frac{\overline{p}^2+1}{(1-\overline{p}^2)\overline{p}}$, with $\overline{p} = \varphi(\rho)$ (where $\varphi(.)$ is defined in (5)).

The threshold $\widehat{R}(\gamma)$ in proposition 1 corresponds to the minimum level of productivity that is necessary to reach $SB+$. Condition $R > \widehat{R}(\gamma)$ alone is not sufficient to guarantee that the economy will reach this equilibrium because $SB-$ may also exist when $R \in \left(\widehat{R}(\gamma), \overline{R}(\gamma)\right]$, in which case the wholesale market is subject to coordination failures. To see this point, suppose for a moment that all bankers are pessimistic, in the sense that everyone believes that even unskilled borrowers (who are prone to diversion) demand funds. To protect themselves against cash diversion lenders will tolerate only low leverage, implying –all things being equal– a low aggregate demand and a low equilibrium market return, so that even unskilled borrowers will indeed be willing to demand funds (thereby validating bankers' initial beliefs). The economy will reach $SB-$ (see figure 1), which I will refer to as the "crisis time" equilibrium. This situation is akin to a market run, where every lender who believes that other lenders tolerate low leverage will also tolerate low leverage.

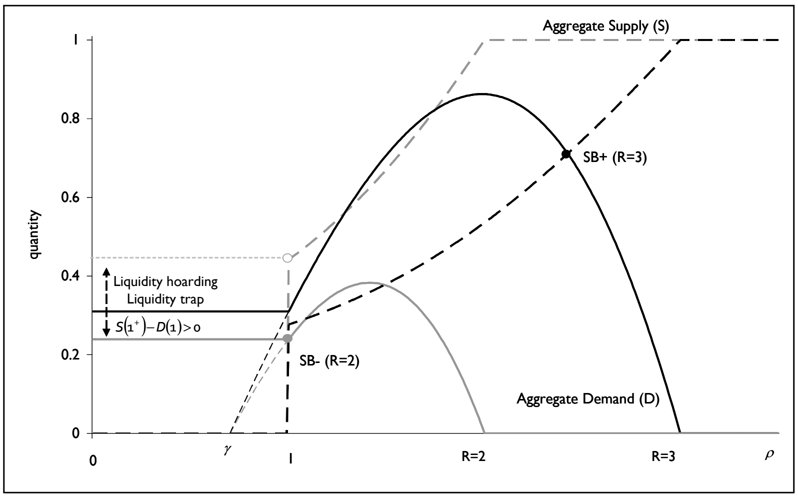


Figure 2: Productivity and financial market equilibrium
with $R = 2$ (grey), $R = 3$ (black) ; $\gamma = 0.7$

In contrast, if all bankers are optimistic about counterparty risk, then the market return will be high ($\rho^*_{SB+} > 1$) and justify, ex post, bankers' optimism. The economy will then reach $SB+$, which I will refer to as the "normal time" equilibrium. Both equilibria $SB-$ and $SB+$ are consistent and compatible with a given productivity level $R \in \left(\widehat{R}(\gamma), \overline{R}(\gamma)\right]$. Note however that $SB+$ and $SB-$ never co-exist when the moral hazard problem is benign, i.e. when $\gamma < \overline{\gamma}$, since $\widehat{R}(\gamma) = \overline{R}(\gamma)$ in this case, and that $SB+$ always exists and is unique when $\gamma = 0$ (see the discussion and figure A4 in appendix 7.4.1). Figure 2 completes the description of the possible equilibria. Here productivity R varies from $R = 2$ to $R = 3$. High productivity $R = 3$ generates a relatively high demand for funds with respect to the total amount of liquidity available in the economy (here normalized to one) and results in a relatively high equilibrium market return, which crowds inefficient borrowers out of the demand side of the market. Since the remaining borrowers have little incentive to divert cash, they do not need to be disciplined through stringent limits on leverage. In this case the equilibrium is characterized by high market return ρ^*_{SB+}, high leverage ϕ^*_{SB+}, and an efficient financial market. There is no crisis. This contrasts with the case $R = 2$, where in equilibrium $SB-$ hardly any funds are channeled to skillful bankers.

Liquidity Hoarding and Liquidity Trap Equilibrium $SB-$ features a high risk premium, deleveraging, and liquidity hoarding. At the market return $\rho^*_{SB-} = 1$ the total quantity of funds available is $S(1^+)$ but lenders are indifferent between financial assets and storage. Moreover, at this market return the incentive to divert cash is so high that lenders do not tolerate a leverage above $\phi^*_{SB-} = \frac{1}{\gamma} - 1$, for a higher leverage would not be incentive compatible. Because at this market return the constrained aggregate demand is lower than aggregate supply, not all funds are channelled to the wholesale financial market (and ultimately to entrepreneurs). The quantity of funds that is not invested, $S(1^+) - D(1) = \overline{p}^*_{SB-} - \left(1 - \overline{p}^*_{SB-}\right)\phi^*_{SB-} \geqslant 0$, is stored until date 1.[12] In other terms, bankers hoard liquidity.[13] In this equilibrium an exogenous increase in the aggregate supply of liquidity would have no effect on the equilibrium market return and on the real economy. The crisis equilibrium thus presents features akin to the traditional Keynesian liquidity trap. Moreover, the financial system is subject to two types of inefficiencies. First, a fraction of the total liquidity available in the economy is kept idle within the banking sector. Second, retail loans also reach low p entrepreneurs, whose expected productivity is lower than that of the entrepreneurs who would otherwise be financed during normal times (see proposition 2iv below). The crisis equilibrium thus exhibits "zombie lending" similar to what Caballero et al. (2008), for example, have documented in the case of Japan in the 90s. In the rest of the paper I will refer to the existence of the crisis equilibrium as "financial fragility" (definition 3). Section 4 will discuss how policy interventions may help avoid such undesirable outcome.

Definition 3 (Financial Fragility): *The wholesale financial market is fragile whenever the crisis time equilibrium $SB-$ exists, i.e. whenever $R \leqslant \overline{R}(\gamma)$.*[14]

As proposition 1 suggests, the threshold $\overline{R}(\gamma)$ is an increasing function of γ. This means that when the retail loan market is less developed then real productivity must be higher to rule out crises. This productivity threshold corresponds to the very productivity level for which $SB-$ exists but there is no liquidity hoarding, that is $S(1^+) = D(1)$, and can be obtained by solving this latter equation numerically (see also relation (A8) in appendix 7.4.2). Proposition 2 compares the crisis time and the normal time equilibria.

Proposition 2 (Comparison Between $SB-$ and $SB+$):

(i) $\phi^*_{SB-} \leqslant \phi^*_{SB+}$ *(leverage is higher in $SB+$)*

(ii) $1 < \frac{r^*_{SB+}}{\rho^*_{SB+}} \leqslant \frac{r^*_{SB-}}{\rho^*_{SB-}}$ *(the credit risk premium is strictly positive and lower in $SB+$)*

[12] By definition of $\overline{R}(\gamma)$, $R \leqslant \overline{R}(\gamma) \Leftrightarrow S(1^+) \geqslant D(1)$, which means that $\rho^*_{SB-} = 1$ clears the wholesale market.

[13] Storage is crucial in the present model. The assumption that there exists a storage technology is a simple way to introduce "liquidity hoarding" into the model. Another, equally important, implication is that without storage bankers would not be able to divert liquidity in the first place and the economy would always reach the First Best equilibrium -see the analyzes of the benchmark economies in appendices 7.2.1 and 7.2.2.

[14] To my knowledge there is no universal definition of the notion of "financial fragility". This definition somewhat differs from –but is not inconsistent with– Allen and Gale (2005)'s, who define financial fragility as a situation where a "*small aggregate shock in the demand for liquidity leads to disproportionately large effects in terms of default or asset-price volatility*" (p. 543).

(iii) $1 < \frac{R}{r^*_{SB+}} \leqslant \frac{R}{r^*_{SB-}}$ *(the funding liquidity risk premium is strictly positive and lower in SB+)*

(iv) $\overline{p}^*_{SB-} \leqslant \overline{p}^*_{SB+}$ *(the banking sector is more efficient in SB+)*

Proof: See appendix 7.6.

In normal times the expected market return on financial assets is relatively high, unskilled bankers prefer to lend rather than borrow so that the pool of borrowers is composed of efficient bankers only. This has two consequences in terms of funding liquidity and credit risks. First, borrowers are able to raise a large amount of funds (proposition 2i), which results in high aggregate demand and a high equilibrium market rate r^*. The implied narrowing of the spread R/r^* reflects bankers' ability to borrow against the present value of retail loans or, in Brunnermeier (2009)'s language, the degree of funding liquidity.[15] Put differently, the spread R/r^* can be viewed as a funding liquidity risk premium. Driven by lenders' fear that borrowers divert the cash, this premium is lower in normal times (proposition 2iii) when only skillful bankers raise funds (i.e. when $\overline{p}$ is high –see relation (A11) in appendix for a formal proof). Second, a high expected market return on financial assets has also a negative effect on credit risk since, conditional on not diverting, borrowers are less likely to default. Hence the credit risk premium r^*/ρ^* too is lower in $SB+$ than in $SB-$ (proposition 2ii). Overall, the interest rate spread on retail loans R/ρ^* is the (geometric) sum of a credit risk premium r^*/ρ^*, due to the risk that entrepreneurs default, and a funding liquidity risk premium R/r^*, due to the risk that borrowers divert the funds.

Financial Market Depth and Funding Liquidity In the literature more is known about the relationship between financial market depth and market liquidity -i.e., the ease with which assets are traded, than about the relationship between market depth and *funding* liquidity. Pagano (1986), for example, develops a model where market liquidity is positively related to the size of the financial market and "trading volume and absorptive capacity of the market tend to feed positively on each other: more traders make more active trade, and vice versa" (p. 256). In contrast, as in Bebchuck and Goldstein (2010), proposition 2 suggests there is a positive correlation between the volume of trade (point i) and funding liquidity (point iii) since both increase from crisis to normal times. I complete the description of the equilibrium with the comparative statics analysis of variations in the two parameters of the model R and γ (see proposition 3).

Proposition 3 (Effects of Productivity and Diversion Cost):

(i) In SB+: $\frac{d\phi^*_{SB+}}{dR} \geqslant 0, \frac{d\rho^*_{SB+}}{dR} \geqslant 0, \frac{d\overline{p}^*_{SB+}}{dR} \geqslant 0$; $\frac{d\phi^*_{SB+}}{d\gamma} \leqslant 0, \frac{d\rho^*_{SB+}}{d\gamma} \leqslant 0, \frac{d\overline{p}^*_{SB+}}{d\gamma} \leqslant 0$

(ii) In SB−: $\frac{d\phi^*_{SB-}}{dR} = 0, \frac{d\rho^*_{SB-}}{dR} = 0, \frac{d\overline{p}^*_{SB-}}{dR} \leqslant 0$; $\frac{d\phi^*_{SB-}}{d\gamma} \leqslant 0, \frac{d\rho^*_{SB-}}{d\gamma} = 0, \frac{d\overline{p}^*_{SB-}}{d\gamma} \geqslant 0$

Proof: See appendix 7.7.

[15] Brunnermeier (2009, p. 91): "*Funding liquidity, the ease with which expert investors and arbitrageurs can obtain funding, is distinct from market liquidity. Funding liquidity is high —and markets are said to be "awash with liquidity"— when it is easy to borrow money, either uncollateralized or with assets as collateral*".

As the partial derivatives of (5) with respect to R and ρ suggest, an increase in R has opposite effects on $\overline{p}^*_{SB+}$. On the one hand, given ρ higher returns on retail loans attract bankers to the demand side of the interbank market ($\partial\overline{p}/\partial R < 0$). On the other hand, the rise in R also relaxes the borrowing constraint (see (IC)), which generates a higher demand for fund per borrower (ϕ^*_{SB+} increases), a higher aggregate demand, a higher equilibrium market return (ρ^*_{SB+} goes up), and works to reduce the number of borrowers ($\partial\overline{p}/\partial\rho > 0$). Proposition 3i shows that this second effect prevails (i.e. $\overline{p}^*_{SB+}$ ultimately goes up) and that in normal times a rise in R implies fewer but more leveraged borrowers on the interbank market (proposition 3i). (I.e. it improves the efficiency of the banking sector). The intuition is that the least efficient borrowers ultimately prefer to become lenders in order to reap benefits from the most efficient bankers' productivity gains. The effect of γ on $\overline{p}^*_{SB+}$ is subject to a similar, albeit opposite, trade-off. Given ρ, a higher γ increases borrowers' incentive to divert funds and diminishes lenders' leverage tolerance (ϕ^*_{SB+} goes down), which by reducing the return on retail loans also reduces the number of borrowers ($\partial\overline{p}/\partial\gamma > 0$). As the aggregate demand for funds diminishes, however, so does ρ^*_{SB+}, which works to reduce the number of lenders. Again, this latter effect dominates and ultimately $\overline{p}^*_{SB+}$ goes down. In crisis times, in contrast, the market only adjusts through bankers' liquidity hoarding behavior, not through market return, which is constant (proposition 3ii). Hence the above trade-off does not exist and only the direct effects of R and γ on borrowers' incentives are at work. It follows that increases in R and γ have negative and positive effects on $\overline{p}^*_{SB-}$, respectively (proposition 3ii).

3 Financial Integration, Imbalances, and Fragility

As sections 2.3 and 2.4 suggest the model ultimately boils down to a familiar supply and demand nexus. What makes this nexus non standard is the peculiar form of the fund demand curve, which financial frictions constrain and distort into a hump shape. (Figure A2 in the appendix shows that the demand curve would be monotonically decreasing in the market return in the absence of asymmetric information.) From a graphical perspective, the robustness of the financial system depends on where the supply curve stands with respect to the demand curve (see figure 2). Any exogenous upward shift in the supply curve makes the financial system closer to fragility. As many policy makers and academics (e.g. Bernanke, 2005, 2007, Caballero et al., 2008) have pointed out, one important cause of excess liquidity in developed countries (notably the US) in the run up to the recent crisis was the global saving glut and incapacity of the domestic banking sectors in emerging market economies (notably Asia excluding Japan) to generate financial assets from domestic real investments, as well as their propensity to instead invest domestic savings into developed countries' financial assets. The purpose of this section is to model this phenomenon as an upward shift in the fund supply curve on the interbank market. To do so I now consider a two-country (or -region) model. The two countries have the same features as the economy described so far: each of them is populated with a continuum of bankers, entrepreneurs, and islands, has the same productivity and storage technology, etc. However, following Mendoza et al. (2009), one country is assumed financially "developed", with $\gamma = \gamma_d$, while the other is "emerging", with $\gamma = \gamma_e$ and $\gamma_e \geqslant \gamma_d$. This latter inequality means that bankers have more incentives to divert funds in the emerging country than in the developed country. This is the only feature that differentiates the two countries, which I index by $i = e$ (for "emerging"), d (for "developed"). The country of origin of a given banker is perfectly observable. I first described in section 3.1 the current account imbalances that arise following the financial integration of the two countries. In section 3.2 I discuss the conditions under which these imbalances generate financial fragility, and in section 3.3 I discuss the implications of financial integration in terms of output and welfare.

3.1 Current Account Imbalances

In autarky, bankers located in country i trade with each other through country i's domestic wholesale financial market. There are two distinct interbank markets, and for each of them the equilibrium is characterized as in proposition 1, for $\gamma = \gamma_i$. The interbank market of country d is characterized by a higher expected return ($\rho_d^* > \rho_e^*$) as well as by fewer but more leveraged borrowers ($\overline{p}_d^* > \overline{p}_e^*$ and $\phi_d^* > \phi_e^*$) –from propositions 2 and 3. After financial integration bankers from the two countries trade with each other through the international wholesale financial market. Since the international fund demand (supply) is equal to the sum of the domestic demands (supplies), the fund demand (supply) curve on the integrated market is also hump shaped (monotonically increasing). In equilibrium the international expected market return ρ^* clears the integrated financial market

$$S_e(\rho^*) + S_d(\rho^*) = D_e(\rho^*) + D_d(\rho^*), \tag{9}$$

where the aggregate supply and demand of funds by bankers in country i, $S_i(\rho)$ and $D_i(\rho)$, are defined in (6) and (7) for $\gamma = \gamma_i$. Capital is assumed to flow freely across countries. In effect, it goes from the emerging (where expected returns on financial assets are originally lower) to the developed country until market returns are equalized. Figure 3 depicts the current account balance of country d as the difference between domestic savings and domestic investment, i.e. $S_d(\rho^*) - D_d(\rho^*)$, in percentage of country d's domestic output for various values of γ_e (see section 3.3 below for the definition of output). It appears as a function of γ_e that decreases with γ_e in $SB+$ (plain line curve), is constant in $SB-$ (dashed horizontal line), and is surjective when both $SB-$ and $SB+$ coexist. In normal times, the current account deficit widens as γ_e increases because the incentives to divert funds are stronger and counterparty risks higher in this case, which leads lenders from country e to further invest into country d. Borrowers in country d are also able to raise more funds than those in country e ($\phi_d^* > \phi_e^*$ –see relation (4)). As a result, the rent on leverage is higher in country d and, all things being equal (in particular given that the productivity level and market return are the same in both countries), there are more borrowers in this country than in country e ($\overline{p}_e^* > \overline{p}_d^*$ –see relation (5)). As γ_e goes up, the flow of funds from e to d intensifies and leverage in country d's banking sector as a whole increases. More precisely, leverage decreases at the bank level (i.e. goes ϕ_d down) but increases at the banking sector's level (i.e. $(1 - \overline{p}_d)\,\phi_d$ goes up), which is consistent with feature 1 in the Introduction.

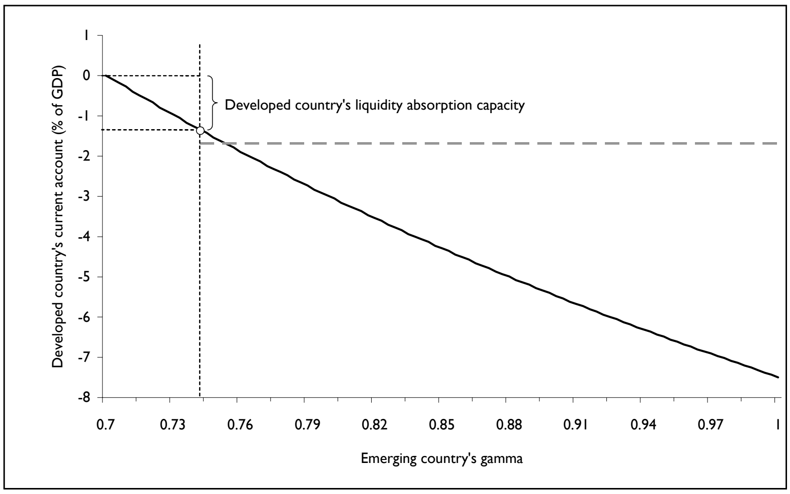


Figure 3: Developed country's current account balance $S_d(\rho^*) - D_d(\rho^*)$ in $SB+$ (plain line) and $SB-$ (dashed line) with $R = 3$, $\gamma_d = 0.7$, and $\gamma_e \geqslant 0.7$

Given country d's limited liquidity absorption capacity, however, it may be that financial integration makes the international financial system more fragile. As figure 3 shows, there is indeed a threshold for γ_e (represented by the dotted vertical line) above which capital flows from the emerging country are so large that even inefficient bankers may enter the demand side of the financial market. If lenders believe that risky bankers are on the other side of the market, then they may run off the market and trigger a crisis. During a crisis international trade collapse and country d's deficit is smaller than in normal times due to bankers hoarding liquidity. (In percentage of country d's output, the current account deficit may be larger in the crisis time equilibrium for some low γ_e since output is also lower in this case –figure 5a illustrates this latter point.) In effect, the developed economy is only able to cope with a limited current account deficit without any threat to the financial sector. For example, for $R = 3$ this limit is at about 1.4% of country d's GDP, which corresponds to the intersection of the plain line curve with the vertical dotted line in figure 3. Any increase in the current account deficit above this capacity gives rise to multiple equilibria with the possibility of coordination failures onto the crisis time equilibrium. The relationship between financial integration and financial fragility is discussed in more details next section.

3.2 Financial Fragility

Figure 4 reports the minimum level of productivity $\overline{R}(\gamma)$ that is required to rule out the financial crisis equilibrium (see definition 3), for various degrees of financial development γ_e in the emerging country, with $\gamma_e \geqslant \gamma_d$, in autarky and under financial integration. Under financial integration, $\overline{R}(\gamma_e, \gamma_d)$ is computed in a similar way as in the previous section, i.e. it is obtained by solving equation $S_e(1^+) + S_d(1^+) = D_e(1) + D_d(1)$ numerically with respect to R. The financial system is fragile in country i when $R \leqslant \overline{R}_i$. The lower dotted line corresponds to the productivity threshold $\overline{R}_d$ for the developed country in autarky when $\gamma_d = 0.7$. It is constant because in autarky the wholesale financial market of country d is independent of γ_e.

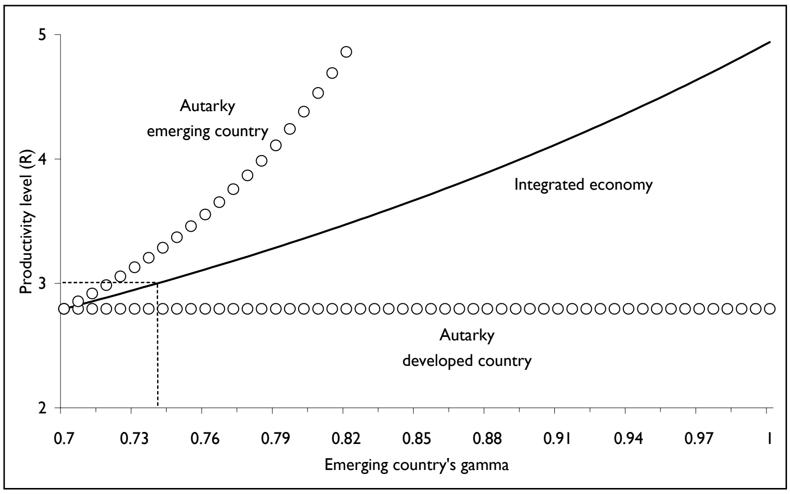


Figure 4: Minimum productivity $\overline{R}_i$ for various values of γ_e before (dotted lines) and after (plain line) financial integration with $\gamma_d = 0.7$ and $\gamma_e \geqslant 0.7$

The upper dotted line corresponds to the productivity threshold $\overline{R}_e(\gamma_e)$ for the emerging country in autarky for various values of γ_e above 0.7. It increases monotonically with γ_e (from proposition 1, $\overline{R}_e'(\gamma_e) > 0$). It is easy to see that the emerging (developed) country in autarky has the most (least) fragile financial system. Finally, the plain line curve in the middle corresponds to the case of full financial integration $\overline{R}(\gamma_e, \gamma_d)$. Productivity thresholds under autarky and financial integration are identical when $\gamma_e = \gamma_d = 0.7$, which basically means that –all other things being equal– financial integration of equally developed countries has no effect on financial fragility. This is intuitive, since in this case financial integration does not imply any net capital flow or imbalance across countries. In fact, in the present model, financial integration only matters when it involves countries with different degrees of development.[16] As they flow to country d, country e's savings exert downward pressures on the wholesale market expected return, which is conducive to market runs. Financial integration raises the minimum productivity level required in country d to rule out such runs and, given R, makes country d's financial sector more fragile. This result is illustrated by the fact that the plain line curve increases with respect to γ_e (i.e. $\overline{R}_1'(\gamma_e, \gamma_d) > 0$). For $R = 3$ (see the horizontal dotted line in figure 4), for example, the crisis time equilibrium exists whenever γ_e is above the threshold represented by the vertical dashed line: whenever $\gamma_e > 0.74$ the wholesale market is fragile. (Note that, by construction, this threshold is the same as in figure 3.) One important conclusion of this section is therefore that financial integration of emerging countries jeopardizes the financial system if the degree of financial development of these countries is too low given the level of productivity in the real sector. Implications in terms of output and welfare are discussed below.

[16] Things would be different if I assumed ex post idiosyncratic shocks and risk aversion, in which case financial integration of ex ante identical countries would have risk sharing and liquidity insurance effects, as in Castiglionesi et al. (2010). Instead, the focus here is on wealth re-allocation and efficiency effects of financial integration.

3.3 Aggregate Output and Welfare

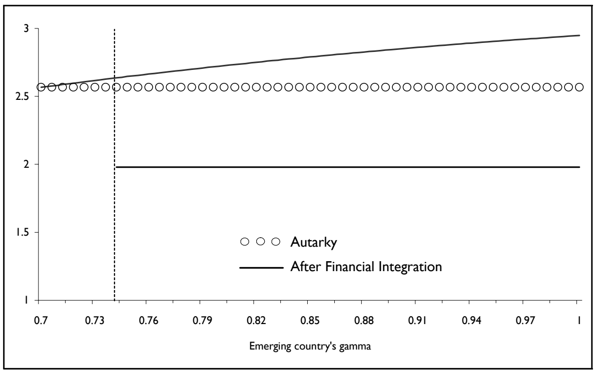
Since entrepreneurs do not make any profit welfare in country i (W_i) is defined as the sum of bankers' net gains (as defined in (1))

$$W_i \equiv \underbrace{\overline{p}_i^* \rho^*}_{\text{lenders' returns}} + \underbrace{\int_{\overline{p}_i^*}^1 \pi_i(p)\, dp}_{\text{borrowers' returns}} . \tag{10}$$

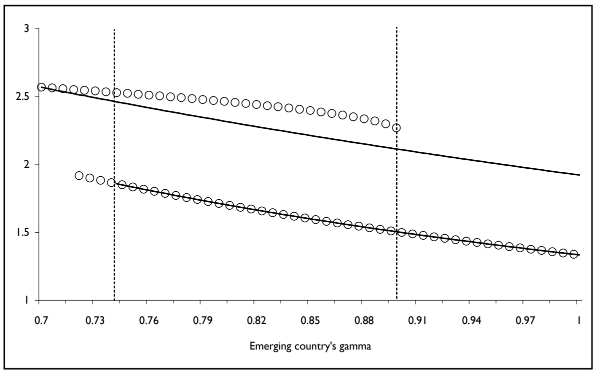
Aggregate output (Y_i) is defined as the sum of entrepreneurs' output

$$Y_i \equiv \int_{\overline{p}_i^*}^1 p\,(1+\phi_i^*)\, R dp = \frac{\left(1-\overline{p}_i^{*2}\right)(1+\phi_i^*)\, R}{2}. \tag{11}$$

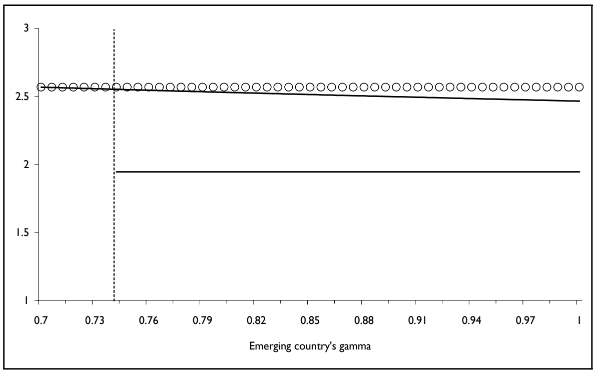
Welfare and output for various degrees of financial development γ_e are reported in figure 5. The plain lines correspond to full financial integration and the dotted lines correspond to autarky. To fix ideas, the model is parameterized so that the financial system is not fragile in country d in autarky, i.e. $SB-$ does not exist. (I use $R = 3$, $\gamma_d = 0.7$ and $\gamma_e \geqslant 0.7$.) In this case, country d has unique and constant levels of output and welfare. In country e, in contrast, equilibrium $SB+$ is unique only for low values of γ_e, does not exist for values of γ_e close to one, and coexists with $SB-$ for intermediate values of γ_e. In all cases, welfare and output are lower, and the financial system is more fragile in country e than in country d. Financial integration has two main effects. First, as discussed in the previous section, since the emerging country may export financial fragility to the developed country, financial integration reduces world aggregate welfare and output in the case a financial crisis materializes. Second, in normal times financial integration raises both output and welfare at the world level because resources are overall more efficiently used. However, it also induces a redistribution of wealth and welfare both within and across countries. Typically, output augments in the developed country thanks to higher investments but diminishes in the emerging country (figures 5a and 5b). Perhaps more surprising is the result that welfare is redistributed the other way around (figures 5c and 5d). The change in the equilibrium interest rate is detrimental to borrowers and beneficial to lenders in the emerging country (where the interest rate increases), whereas it is detrimental to lenders and beneficial to borrowers in the developed country (where the interest rate decreases). In the emerging country, welfare tends to diminish because of borrowers' welfare losses, but these losses are more than offset (i) by the welfare gains induced by the relaxation of borrowers' borrowing constraint (i.e. the rise in ϕ_e^*), (ii) by lenders' welfare gains (rise in ρ_e^*), and (iii) by the fact that the proportion of lenders ($\overline{p}_e^*$) increases. Thus, overall domestic welfare increases (figure 5d). Symmetrically, in the developed country lenders are worse off and the tightening of the borrowing constraint (decrease in ϕ_d^*) partially offsets borrowers' net gains from the lower borrowing rate. Moreover, a disproportionate share of the output gains generated in country d ultimately accrue to the lenders from country e, so that overall welfare diminishes (figure 5c). Financial liberalization thus benefits to the emerging country and is detrimental to the developed country.



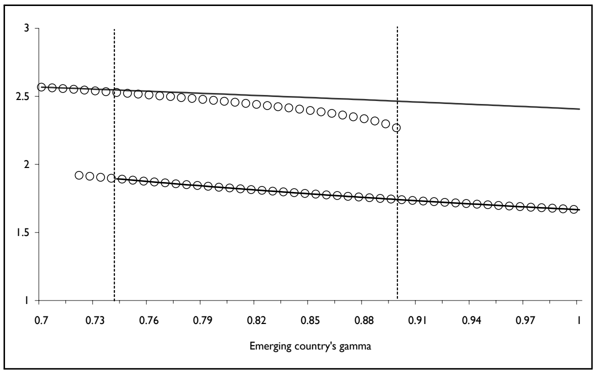
a. Output in country d (Y_d)



b. Output in country e (Y_e)



c. Welfare in country d (W_d)



d. Welfare in country e (W_e)

Figure 5: Domestic output and welfare before (dotted) and after (plain) financial integration with $R = 3$, $\gamma_d = 0.7$ and $\gamma_e \geqslant 0.7$

These predictions are opposite to Ju and Wei (2010) and Mendoza et al. (2007). The latter present a model where welfare redistribution effects induced by the change in the interest rate are much stronger than the wealth effects. The reason is that in their setup agents have concave utility and the borrowers are the poorest agents (i.e. with the highest marginal utility of consumption), who want to borrow in order to smooth their consumption. In this context, in the emerging country the redistribution from borrowers to lenders reduces domestic welfare, while in the developed country the redistribution from lenders to borrowers increases it.

4 Policy Implications

This section discusses the effects of two types of policy intervention capable of preserving financial stability when liquidity is abundant. The first policy consists of central banks providing a deposit facility and committing to remunerate deposits at a (real) rate δ. As will become clear in an instant, this policy has a fairly straightforward implication. The second policy is a micro-prudential policy that consists in a financial regulator requiring bankers to hold a minimum amount of cash, i.e. to comply with a minimum liquidity coverage ratio (LCR).

4.1 Central Bank Deposit Facility

Assume that the central banks of the two countries provide deposit facilities at the same deposit rate $\delta \geqslant 1$. With this latter condition central bank deposits henceforth replace bankers' storage technology as outside option. It is easy to see from figure 1 that there exists a certain threshold for δ above which the crisis equilibrium is ruled out. Indeed, by remunerating deposits at a high enough rate, in fact for $\delta > \rho_P$, (where ρ_P is the smallest solution to (9)) the central bank makes sure that inefficient bankers are never willing raise funds on the interbank market. Put differently, it is able to coordinate bankers' expectations onto the normal time equilibrium. Note that in equilibrium the central bank will not take any deposit, though, as all lenders will prefer to go to the wholesale market. It is therefore enough for the central bank to commit to remunerate deposits at rate $\delta > \rho_P$ in order to coordinate expectations and avoid financial crises. Such policy, however, can only help when the moral hazard problem is not too severe and the normal time equilibrium exists, i.e. when $R > \widehat{R}(\gamma)$. Interestingly, the above predictions of the model contradict Bebchuk and Goldstein (2010)'s. In their model a rise in the central bank deposit rate raises the opportunity cost of lending to entrepreneurs, which leads banks to exit the retail loan market and ultimately works to increase entrepreneurs' funding liquidity risks. Here, an increase in the central bank deposit rate also increases the opportunity cost of lending to entrepreneurs. However, as long as the deposit rate is not too high (i.e. for $\delta \leqslant \rho^*_{SB+}$), this has the virtuous effect to limit zombie lending, to keep the least efficient bankers away from the demand side of the interbank market, and to mitigate counterparty fears. As a result, interest rate spikes reduce bankers' (and entrepreneurs') funding liquidity risks. (It is easy to see that for a deposit rate $\delta \in \left(\rho^*_{SB+}, R\right)$ the interbank market would shrink and some bankers would deposit funds with the central banks, while for $\delta \geqslant R$ the interbank market would vanish.)

4.2 Liquidity Coverage Ratio

The minimum LCR designed in Basel III identifies the amount of liquid assets a bank must hold that can be used to offset the net cash outflows it may encounter under stress. The most liquid assets include cash and marketable securities from sovereigns or central banks (see Basel Committee on Banking Supervision, 2009). Retail loans and other banking book assets are considered as the most illiquid assets. The aim of the LCR is to ensure that the bank holds enough liquid assets to repay its short term wholesale debts and, more generally, short term financing subject to funding liquidity risk. The main benefit of such a policy is to reduce funding liquidity risk and ensure that the bank is able to roll over its short term debt. The cost is that it diverts the bank from financing the real sector and hinders entrepreneurs' investments. The present model lends itself particularly well to the analysis of this trade-off because it features both the assets and liabilities items Basel III's LCR is based on, namely: cash (i.e. storage) and wholesale debt that is subject to funding liquidity risk. Let $\ell \in [0, 1]$ be the minimum (liquidity coverage) ratio of cash to wholesale market debt bankers are required to hold, so that borrowers must store at least $\phi\ell$ goods as cash if they want to raise ϕ on the wholesale market. This cash is assumed non-divertible, for example because it is escrowed with the central bank (e.g. reserves) or other third party. Hence a borrower's balance sheet at date 0 is structured as follows:

A	Borrower balance sheet		L
(Retail loans)	$1+\phi-\ell\phi$	ϕ	(Interbank loan)
(Cash holding)	$\ell\phi$	1	(Endowment)

To facilitate the discussion, parameter ℓ is assumed to be the same in the emerging as in the developed country. While the LCR reduces the quantity of funds that can be diverted, it is inefficient from the banker's viewpoint for it reduces the quantity of funds channelled to the real sector. It is therefore optimal to keep cash holding to the minimum and lend as much as possible, i.e. $1+\phi-\ell\phi$, to entrepreneurs. Banker p thus maximizes his expected profit

$$\max_{d\in\{l,b\},\phi} \pi(p) \equiv \mathbf{1}_{d=l}.\rho + \mathbf{1}_{d=b}.p\left((1+\phi)R-(R-1)\ell\phi-\phi r\right), \tag{12}$$

where $R-1$ is the opportunity cost of holding cash. With probability p retail loans are paid back and borrower p's net gain is the sum of returns on both retail loans and storage minus debt repayments. With probability $1-p$ retail loans are not paid back and borrower p's gross return is that on storage, $\ell\phi$. Since $\ell<1<r$, the return on storage is insufficient to pay the entire debt and borrower p gets nothing in net terms in this case. Note that because the banker must hold cash proportional to his wholesale market debt and cash holding is inefficient, the rent on leverage may not always cover the cost of wholesale debt. In particular, when ℓ is above $(R-r)/(R-1)$ the expected profit decreases in ϕ, which implies that the interbank market does not exists. Too high a liquidity coverage ratio kills the wholesale financial market. For simplicity, in the analysis below I will derive the solution of maximization problem (12) under the conjecture that $\ell \leqslant (R-r^*)/(R-1)$ –bearing in mind that the solution shall hold only when this conjecture is true in equilibrium (this corresponds to situations (A) and (B) in figure 6 below). The model is solved in a similar way as the basic model. To keep notation simple, in what follows I drop the country indice $i=d,e$ for country-specific variables ϕ_i, $\overline{p}_i$, r_i, domestic aggregate demand and supply $D_i(\rho)$ and $S_i(\rho)$, and parameter γ_i. Bankers' incentive compatibility constraint (IC) now writes

$$\gamma(1+\phi-\ell\phi) \leqslant \max\left\{\overline{p}\left((1+\phi-\ell\phi)R+\ell\phi-\phi r\right),\rho\right\}, \tag{IC$_\ell$}$$

where, as before, the term on the left-hand side corresponds to the private benefit of the borrower when he misuses the funds (e.g. makes sub-prime retail loans). Banker p is willing to borrow on the wholesale market if and only if

$$p\left((1+\phi)R-(R-1)\ell\phi-\phi r\right) \geqslant \rho, \tag{PC$_\ell$}$$

which implies the marginal borrower is

$$\overline{p} \equiv \frac{\rho}{(1+\phi)R-(R-1)\ell\phi-\phi r}. \tag{2$_\ell$}$$

Since only bankers with $p \geqslant \overline{p}$ borrow funds the expected return from wholesale loans equals

$$\rho \equiv \int_{\overline{p}}^{1} (pr + (1-p)\,\ell)\,\frac{dp}{1-\overline{p}} = \frac{1+\overline{p}}{2}r + \frac{1-\overline{p}}{2}\ell. \tag{3_ℓ}$$

From constraint (IC_ℓ) it is easy to check that –all things being equal– leverage is a positive function of ℓ:

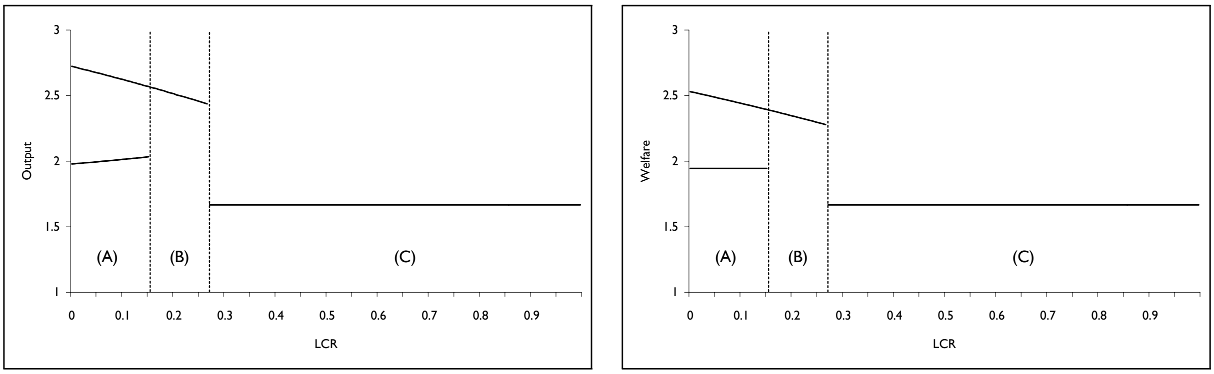
$$\phi^*(\rho) = \frac{\rho-\gamma}{\gamma(1-\ell)} \tag{4_ℓ}$$

This result simply restates that borrowers have less funds to misuse when they hold cash and that by mitigating the moral hazard problem LCR works to relax bankers' borrowing constraint. Notice also that the above relations are identical to relations (IC), (PC), (2), (3), and (4) when $\ell = 0$, respectively. It is easy to re-arrange these equations in order to express $\overline{p}$ as a function $\varphi_\ell(\rho)$,[17] as well as the aggregate country supply and demand:

$$S(\rho) = \left\{ \begin{array}{cl} 0 & \text{if } \rho < 1 \\ \in [0, \varphi_\ell(1)] & \text{if } \rho = 1 \\ \varphi_\ell(\rho) & \text{if } \rho \in (1, R] \\ 1 & \text{if } \rho > R \end{array} \right\},\ D(\rho) = \left\{ \begin{array}{cl} (1-\varphi_\ell(1))\left(\frac{1-\gamma}{\gamma(1-\ell)}\right) & \text{if } \rho < 1 \\ (1-\varphi_\ell(\rho))\left(\frac{\rho-\gamma}{\gamma(1-\ell)}\right) & \text{if } \rho \in [1, R] \\ 0 & \text{if } \rho > R \end{array} \right\}. \tag{13}$$

Again, the equilibrium market return ρ^* is the highest return that solves the market clearing condition (9), under the condition that $\ell < (R-r^*)/(R-1)$ in equilibrium. When this condition is not satisfied, then $\phi^* = 0$ and there is no wholesale financial market: bankers with $p < 1/R$ store their wealth, and all bankers with $p \geqslant 1/R$ lend to their pool of entrepreneurs. I am now in the position to solve the model for various values of $\ell \in (0,1)$. The purpose of this exercise is to discuss the effects of LCR on welfare and output in the developed country, as well as on the fragility of the international financial system. To do so I start from a situation where in the absence of LCR ($\ell = 0$) both $SB-$ and $SB+$ coexist and gradually raise ℓ up to one. I set parameters to $R = 3$, $\gamma_d = 0.7$ and $\gamma_e = 0.8$. (It is easy to see from figures 4 and 5 that in this case $SB-$ and $SB+$ coexist when $\ell = 0$.) For the results see figure 6, which describes three different situations. Situation (A) is one where ℓ is too low to rule out financial crises. In normal times, any marginal increase in the LCR reduces aggregate output as less funds are channelled to productive projects, without making the financial system more robust. In crises times, the rise in LCR gradually reduces funding liquidity risk, which allows for an increase in leverage and the reduction of the excess supply of funds, hence the rise in output in this case. Figure 6 suggests that there exists a threshold for ℓ above which the moral hazard problem recedes and equilibrium $SB-$ is ruled out. This corresponds to situation (B). In this case the most efficient borrowers are able to leverage and demand enough funds so as to keep aggregate demand and market return high enough to maintain inefficient bankers on the supply side of the interbank market.

[17] More precisely, $\varphi_\ell(\rho)$ is the root (in the unit circle) of polynomial $\left[R - \ell\left(R - 2 + \frac{2\gamma}{\rho}\right)\right]\overline{p}^2 + \left[R + \gamma - 2\rho - \ell\left(R - 2 + \frac{2\gamma}{\rho} - \gamma\right)\right]\overline{p} - \gamma(1-\ell) = 0.$



a. Output in country d (Y_d) b. Welfare in country d (W_d)

Figure 6: Domestic output and welfare for various values of $\ell \in (0,1)$ with $R = 3$, $\gamma_d = 0.7$ and $\gamma_e = 0.8$

Finally, situation (C) is one where borrowers are able to raise even more funds, but are unwilling to do so because they would then have to hold too much cash against wholesale debt, which is not profitable. That is, under the initial conjecture that borrowers demand as much funds as possible one would obtain $\ell > (R - r^*) / (R - 1)$ in equilibrium, which contradicts the initial conjecture. Hence, there is no wholesale financial market in such case. Overall, this exercise suggests that there is an interval for the liquidity coverage ratio over which financial crises are ruled out while market efficiency is preserved.

5 Conclusion

This paper develops a general equilibrium model that describes causal relationships between financial integration, current account imbalances, and financial crises. Although stylized, the model is able to account for some important features of the recent crisis, like the reversal in leverage of market-based financial institutions and the sudden collapse of the wholesale financial market. The financial market is shown to improve (in terms of efficiency) the allocation of liquidity within the banking sector, and from the banking sector to the real sector. However, frictions between lenders and borrowers impair its functioning and, in particular, it may collapse when there is too much liquidity available compared with the number of (safe) investment opportunities. In effect, the financial market is shown to have a limited liquidity absorption capacity, which depends on the productivity of the real sector, the ultimate borrower. The model boils down to a familiar supply and demand nexus on the wholesale financial market. What makes this nexus non standard is the peculiar form of the aggregate fund demand curve, which is hump shaped due to the market frictions.

Extending the model to a two country framework, I present results in line with the recent literature that shows that the financial integration of financially under-developed countries is conducive to global imbalances. But I also go one step further by showing how and when such global imbalances make the international financial system fragile. In normal times, financial liberalization is found to increase welfare at the world level, but also to benefit to emerging countries and be detrimental to developed countries. Financial integration also makes financial crises more likely when the degree of financial development in the integrated emerging countries is too low and when capital flows toward developed countries are too large. The present paper also argues that one possible cause of the recent crisis is that the US productivity slowdown as of 2004 impaired US' liquidity absorption capacity precisely when more foreign capital was flowing in. It also shows that, when it materializes, a financial crisis reduces welfare in all countries.

Finally, I use the model to discuss the effects of two types of policy intervention. The first policy is one where central banks offer a deposit facility. I show that there exists a threshold for the real deposit facility rate above which financial crises are ruled out. The second policy corresponds to Basel III's minimum liquidity coverage ratio. I show that there exists an interval for this ratio over which financial crises are ruled out while the efficiency of the wholesale financial market is preserved.

6 References

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7 Appendix

7.1 Bankers' Payoff Structure

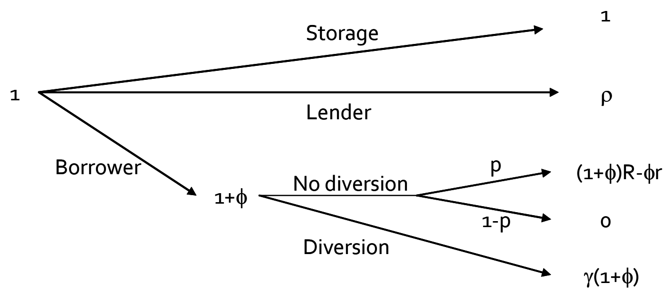


Figure A1: Payoffs

7.2 Benchmark Models

In this section I describe the First Best equilibrium of the wholesale financial market in the absence of friction (i.e. perfect information and no cash diversion), as well as the equilibria when the ps are private information and cash diversion is not possible, and when cash diversion is possible but information is symmetric. Here I show that none of these benchmark cases exhibits multiple equilibria and, therefore, that both moral hazard and asymmetric information are necessary to obtain financial fragility (as defined in definition 3).

7.2.1 Frictionless Economy

Let $\overline{p}$ be the marginal banker, who is indifferent between borrowing and lending with, by definition: $\overline{p} \equiv \rho / \left(R + \phi\left(R - r\right)\right)$. Given r, borrower p ($\forall p \geqslant \overline{p}$) maximizes his expected profit (1) with respect to ϕ, and it is optimal for him to demand an infinite quantity of funds ($\phi \to +\infty$) when $r < R$. As a result, the aggregate demand for funds is finite if and only if $r = R$, which is therefore the condition for the financial market to clear. At this rate, the most efficient banker $p = 1$ is the only banker willing to borrow. Since this banker never defaults, $\rho^* = R$, and since $\rho^* > 1$ liquidity hoarding is never a viable option for bankers, who prefer to lend on the market. In equilibrium there is no spread (i) between the interest rates on the retail and the bond markets ($r^* = R$) or (ii) between the interest rate and the return on financial assets ($r^* = \rho^*$). This absence of spread reflects the full efficiency of the financial market. The economy reaches the first best equilibrium $\left(\rho^*, D\left(\rho^*\right)\right) = (R, 1)$, which I represented by point FB in figure A2. Note that this first best corresponds to the equilibrium that would prevail in an economy where wealth would be located in the most efficient island only. (Put differently, the distribution of liquidity at date 0 does not matter.) Moreover, competition among bankers implies that the (only) borrower on the market does not draw any benefit from leverage, and profits in the financial sector are uniformly distributed across bankers (they all have the same profit R).

7.2.2 Economy with Asymmetric Information Only

Under asymmetric information all bankers have access to the same borrowing rate r and banker $p = 1$ demands an infinite quantity of funds so long $R > r$. In this case the equilibrium is unique and requires that $r^* = R$, which implies that only the most skillful banker borrows funds on the wholesale financial market. In other terms, this equilibrium is perfectly revealing. The economy reaches the First Best equilibrium even though the ps are private information.

7.2.3 Economy with Moral Hazard Only

As in the text, I solve the equilibrium in three steps. After having explicited bankers' participation and incentive compatibility constraints, I derive the optimal leverage that maximizes bankers' expected profit under these constraints and determine the type of the marginal borrower $\overline{p}$. This permits me to derive the aggregate supply and demand curves (second step), and eventually solve for the equilibrium (third step).

Participation and Incentive Compatibility Constraints In equilibrium the expected return ρ must be the same across borrowers; otherwise, all lenders would be willing to lend to the borrower with the highest expected return, and the market would not clear. Hence, the interest rate faced by borrower p is equal to

$$r(p) = \frac{\rho}{p}. \tag{A1}$$

Using the participation constraint (PC) and relation (A1), one can determine the type of the marginal borrower

$$\overline{p} \equiv \frac{\rho}{R} \equiv \varphi(\rho). \tag{A2}$$

Banker p will become borrower only if the participation constraint (PC) is satisfied, that is if $p \geqslant \overline{p}$. Because lenders observe the ϕs they have the ability to deter cash diversion by denying loans to over-levered borrowers. Therefore, leverage ϕ depends on the ps and is specific to each banker: no banker will be willing to grant a loan to borrower p if p's return from cash diversion $\gamma(1+\phi)$ is above both his expected return from retail lending, $p(R + \phi(R - r))$, and the return on wholesale lending, ρ. Banker p knows of this in advance, and so takes care never demand too high a loan ϕ, such that $(\forall p \in [0, 1])$[18]

$$\gamma(1+\phi) \leqslant \max\{p(R + \phi(R - r)), \rho\}. \tag{A3}$$

The above constraint is borrower p's incentive compatibility constraint and determines p's borrowing capacity.

[18] The max term on the right hand side emphazises that even low p bankers have the option to demand a loan and then divert the cash. To keep notation light, I do not explicitly write ϕ as a function of p but, of course, $\phi = \phi(p)$ is borrower-specific.

Optimal Leverage and Marginal Borrower The program of borrower $p \geqslant \overline{p}$ consists in maximizing his expected profit (1) for $d = b$) under constraint (A3). Using relations (A1) and (A2) one can re-write p's incentive compatibility constraint as $(\rho + \gamma - pR)\,\phi \leqslant pR - \gamma$. Because this constraint would always be satisfied for borrower $p \nearrow 1$ and aggregate demand would be infinite when $\rho + \gamma < R$, in equilibrium one necessarily has $\rho + \gamma > R$. For borrower p, it is therefore optimal to demand funds as long as the loan is incentive compatible. Thus, constraint (A3) binds and p's loan demand is equal to

$$\phi^* (p, \rho) = \frac{pR - \gamma}{\gamma + \rho - pR}. \tag{A4}$$

Relation (A4) shows that leverage increases with banker's efficiency and productivity: $d\phi/dp > 0$ and $\partial\phi/\partial R > 0$. It also decreases with the market return: $\partial\phi/\partial\rho < 0$ because, all things being equal, a rise in the return required by the market increases the cost of debt and, therefore, the incentive to divert cash. Following a rise in ρ, ϕ must diminish to maintain incentives.

Aggregate Funds Supply and Demand I am now in the position to derive the aggregate demand and supply curves. When $\rho < 1$ bankers prefer to consume early than to lend, and so the aggregate supply of funds is equal to zero. When $\rho > 1$, bankers $p \in [0, \overline{p})$ prefer to lend their unit of wealth rather consume early, while the rest of the bankers prefer to borrow. When $\rho = 1$ bankers $p \leqslant \overline{p}$ are indifferent between early and late consumption, and so the aggregate supply is undetermined (but below $\overline{p}$). Finally, when $\rho > R$ all bankers supply funds (i.e. $\overline{p} = 1$) and therefore aggregate supply equals 1. It follows that the aggregate supply of funds takes the following form[19]

$$S(\rho) = \left\{ \begin{array}{cl} 0 & \text{if } \rho < 1 \\ \in [0, \varphi(1)] & \text{if } \rho = 1 \\ \varphi(\rho) & \text{if } \rho \in (1, R] \\ 1 & \text{if } \rho > R \end{array} \right\}. \tag{A5}$$

On the demand side, when $\rho \in [R - \gamma, R]$ bankers $p \geqslant \overline{p}$ become borrowers and borrow $\phi(p, \rho)$, so that aggregate demand equals $\int_{\varphi(\rho)}^{1} \phi(p, \rho)\, dp$. As discussed above, when $\rho < R - \gamma$ the most skillful banker ($p = 1$) is not financially constrained and aggregate demand is infinite. Finally, when $\rho > R$ no banker wants to be a borrower, and aggregate demand is null. The aggregate demand $D(\rho)$ can therefore be expressed as[20]

$$D(\rho) = \left\{ \begin{array}{cl} +\infty & \text{if } \rho < R - \gamma \\ \displaystyle\int_{\varphi(\rho)}^{1} \phi(p, \rho)\, dp & \text{if } \rho \in [R - \gamma, R] \\ 0 & \text{if } \rho > R \end{array} \right\}, \tag{A6}$$

where $\phi(p, \rho)$ and $\varphi(\rho)$ are defined in (A4) and (A2), respectively. Noticing that $\rho^* = 1$ when $R \searrow 1$ and that ρ^* increases with R, it is easy to see that the equilibrium market return ρ^* is always above 1. Hence, there will be no liquidity hoarding in this economy.

[19] One has $S(\rho) = \frac{\rho}{R}$ for $\rho \in [1 - \gamma, R]$.

[20] One can show that $D(\rho) = \frac{\rho}{R}\left(1 - \ln\left(\frac{\gamma + \rho - R}{\gamma}\right)\right) - 1$ for $\rho \in [R - \gamma, R]$.

Proposition A1 (Equilibrium): *There exists one unique equilibrium characterized by the market return* ρ^* *and leverage schedule* $\{\phi^*(p,\rho^*)\}_{p=\rho^*/R}^{p=1}$, *where* ρ^* *solves relation (8) given the aggregate loan supply (A5) and demand (A6),* $\rho^* \in (R-\gamma, R)$, *and* $\rho^* > 1$.

The equilibrium is represented in figure A2 by point SB as the intersection of the aggregate demand and supply curves. Since for the equilibrium market return some bankers $p < 1$ are willing to borrow, the financial intermediation process is less efficient than in the frictionless economy (point FB). The intuition goes as follows. For the frictionless equilibrium rate (i.e. for $r = R$) the leverage of banker $p = 1$ is now constrained due to the fact that even this banker is unable to commit not to run away. Since at this interest rate banker $p = 1$ is the only borrower on the market, and by virtue of atomicity, aggregate demand is infinitesimal. Consequently, the equilibrium requires a lower borrowing rate (i.e. that for some $p \leqslant 1$: $r(p) < R$). At such a lower rate, however, some less skillful bankers are willing to borrow, which reduces the overall efficiency of the pool of borrowers. The resulting equilibrium SB is a second best equilibrium in which (i) the return on leverage $R - r(p)$ and (ii) the risk premium $r(p)/\rho - 1$ are both strictly positive, $\forall p > \overline{p}$.

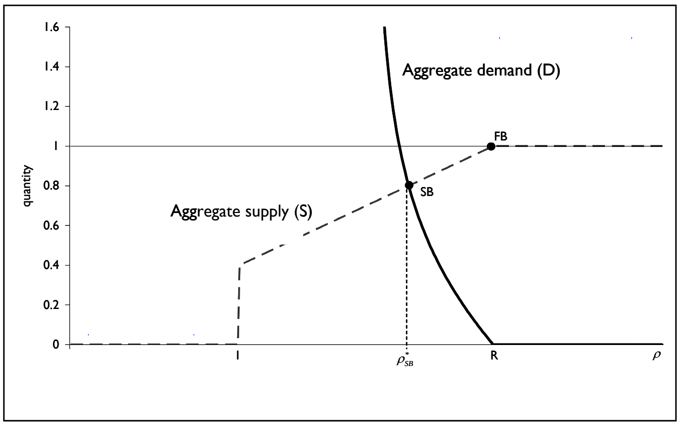


Figure A2: Financial market supply and demand - Perfect information with $R = 2.5$; $\gamma = 0.7$

The impact of productivity on the equilibrium is straightforward. Following a rise in R the opportunity cost of cash diversion increases and the moral hazard problem softens: not only are more bankers willing to become borrowers in this case $(\partial\varphi_{MH}(\rho)/\partial R < 0)$, but also borrowers are able to leverage more $(\partial\phi/\partial R > 0)$. Both the equilibrium market return ρ^* and (for a given borrower p) market rate $r^*(p)$ increase so that the market clears.

7.3 Derivation of Relation (5)

Using relation (3) I substitute r into relation (2), which yields

$$\overline{p} = \frac{\rho}{(1+\phi)R - \frac{2\rho\phi}{1+\overline{p}}}$$

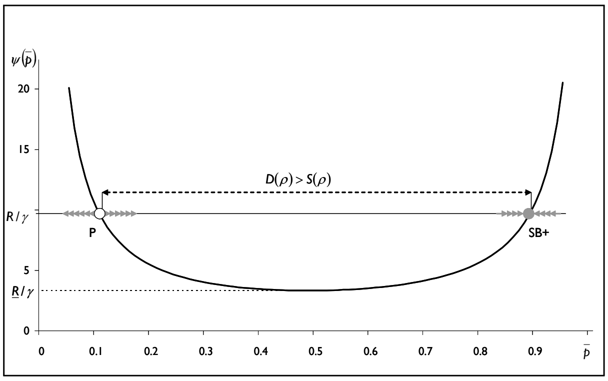
and then, using (4), I substitute ϕ into the above relation, which yields

$$\overline{p} = \frac{\gamma}{R - \frac{2(\rho-\gamma)}{1+\overline{p}}} \Leftrightarrow R\overline{p}^2 + (R + \gamma - 2\rho)\,\overline{p} - \gamma = 0.$$

7.4 Proof of Proposition 1

7.4.1 Condition of Existence of $SB+$

The wholesale market clears when $S(\rho) = D(\rho) \Leftrightarrow \overline{p} = (1-\overline{p})\,\phi \Leftrightarrow \overline{p} = (1-\overline{p})\left(\frac{\rho}{\gamma} - 1\right)$ which is equivalent to (using (5)): $\frac{R}{\gamma} = \frac{\overline{p}^2+1}{(1-\overline{p}^2)\overline{p}}$. I define $\psi(\overline{p}) \equiv \frac{\overline{p}^2+1}{(1-\overline{p}^2)\overline{p}}$ so that $D(\rho) \geqslant S(\rho) \Leftrightarrow \psi(\overline{p}) \leqslant \frac{R}{\gamma}$. The function $\psi(\overline{p})$ is convex and reaches a minimum in $\underline{\overline{p}} \equiv \sqrt{\sqrt{5}-2} \simeq 0.49$, as depicted in figure A3.



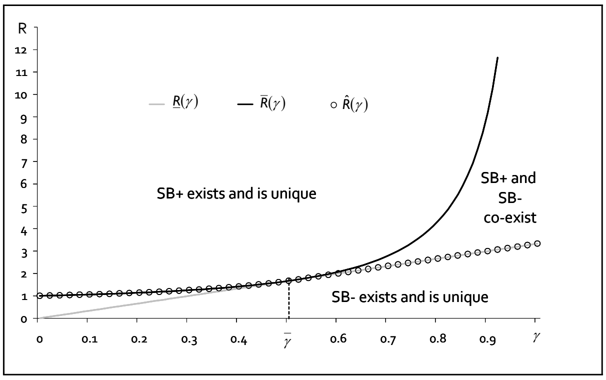


Figure A3 — Wholesale financial market equilibrium

Figure A4 — Existence and uniqueness of $SB+$ and $SB-$

If it exists, equilibrium $SB+$ is characterized by the market return $\rho^*_{SB+} = \varphi^{-1}\left(\overline{p}^*_{SB+}\right)$ (see relation (5)) where $\overline{p}^*_{SB+}$ is the root to equation $\overline{p}^*_{SB+} = \psi^{-1}\left(\frac{R}{\gamma}\right)$ that is above $\underline{\overline{p}}$ (the other root, below $\underline{\overline{p}}$, being associated with an unstable equilibrium). Therefore, $SB+$ exists if:

$$R \geqslant \underline{R}\left(\underset{(+)}{\gamma}\right) \equiv \gamma\psi\left(\underline{\overline{p}}\right) = \frac{\sqrt{5}-1}{\left(3-\sqrt{5}\right)\sqrt{\sqrt{5}-2}}\gamma \simeq 3.33\gamma, \tag{A7}$$

with $\underline{R}(\gamma)' > 0$. It remains to check that $\rho^*_{SB+} > 1$ in this case, i.e. that the storage technology is not used. By definition: $\overline{p}^*_{SB+} = \left(1-\overline{p}^*_{SB+}\right)\left(\frac{\rho^*_{SB+}}{\gamma} - 1\right) \Leftrightarrow \rho^*_{SB+} = \frac{\gamma}{1-\overline{p}^*_{SB+}}$. Hence, $\rho^*_{SB+} > 1 \Leftrightarrow \overline{p}^*_{SB+} > 1-\gamma$. Since by construction $\overline{p}^*_{SB+} \geqslant \underline{\overline{p}}$, two cases must be discussed. First, if $\underline{\overline{p}} \geqslant 1-\gamma \Leftrightarrow \gamma \geqslant \overline{\gamma} \equiv 1 - \underline{\overline{p}}$, then $\overline{p}^*_{SB+} > 1-\gamma$ and $SB+$ exists $\forall R \geqslant \underline{R}(\gamma)$. Second, if $\underline{\overline{p}} < 1-\gamma$ then

$$\overline{p}^*_{SB+} > 1-\gamma \Leftrightarrow \psi^{-1}\left(\frac{R}{\gamma}\right) > 1-\gamma \Leftrightarrow R > \frac{2-2\gamma+\gamma^2}{2-3\gamma+\gamma^2} \equiv \overline{R}\left(\underset{(+)}{\gamma}\right), \tag{A8}$$

By construction $\overline{R}(\gamma)$ is the very productivity level for which $\overline{p}^*_{SB+} = 1-\gamma \Leftrightarrow \overline{p}^*_{SB+} = \left(1-\overline{p}^*_{SB+}\right)\left(\frac{1}{\gamma}-1\right) \Leftrightarrow D(1) = S(1^+) \Leftrightarrow \rho^*_{SB+} = 1$, and it is easy to check from (A7) and (A8) that $\overline{R}'(\gamma) > 0$, $\lim_{\gamma \searrow 0} \overline{R}(\gamma) = 1$, $\overline{R}(\gamma) \geqslant 1$, $\overline{R}(\gamma) > \underline{R}(\gamma)\ \forall \gamma \in (0,1] \diagup \{\overline{\gamma}\}$, and $\overline{R}(\overline{\gamma}) = \underline{R}(\overline{\gamma})$. It follows that $SB+$ exists if and only if

$$R > \widehat{R}(\gamma) \equiv \begin{cases} \overline{R}(\gamma) & \text{for } \gamma \leqslant \overline{\gamma} \\ \underline{R}(\gamma) & \text{for } \gamma \geqslant \overline{\gamma} \end{cases}, \tag{A9}$$

where $\underline{R}(\gamma)$ and $\overline{R}(\gamma)$ are defined in (A7) and (A8), $\widehat{R}'(\gamma) > 0$, $\lim_{\gamma \searrow 0} \widehat{R}(\gamma) = \lim_{\gamma \searrow 0} \overline{R}(\gamma) = 1$, and

$$\overline{\gamma} \equiv 1 - \sqrt{\sqrt{5} - 2} \simeq 0.51. \tag{A10}$$

Note that $\lim_{\gamma \searrow 0} \widehat{R}(\gamma) = 1$ implies that $SB+$ always exists and is unique when there is no moral hazard (given assumption 1). In this case, $SB+$ converges toward the first best allocation FB described in appendix 7.2.1: $\phi^*_{SB+} \to +\infty$ and the market clears only when $r^*_{SB+} = \rho^*_{SB+} = R$.

7.4.2 Condition of Existence of $SB-$

From (6), (7) and (8), I know that $SB-$ exists if and only if $D(1) \leqslant S(1^+)$. Since $\partial S(1^+)/\partial R \leqslant 0$ and $\partial D(1)/\partial R \geqslant 0$, and $\overline{R}(\gamma)$ is (by construction) the productivity level for which $D(1) = S(1^+)$, $SB-$ exists if and only if $R \leqslant \overline{R}(\gamma)$.

7.5 Equilibrium Stability

The Walrasian tatonnement process described in definition A1 below can be thought of as a tentative trial-and-error process taking place in fictional time (t), starting with an initial fictive price that is not an equilibrium price, and run by an abstract agent (the Walrasian auctioneer) bent on finding this equilibrium price, or bent on restoring equilibrium after a random disturbance. Figure A5 below depicts the excess demand on the wholesale financial market for the same parameters as in figure 1. It shows that point P can be eliminated as unstable equilibrium.

Definition A1 (Tatonnement Stability): *An equilibrium (ρ^*, ϕ^*) is (locally) stable if, whenever the initial market return $\rho(0)$ is sufficiently close to ρ^*, the dynamic adjustment driven by the tatonnement process: $d\rho(t)/dt = D(\rho(t)) - S(\rho(t))$ causes the market return to converge to ρ^*, where $d\rho(t)/dt$ is the rate of change of the market return.*

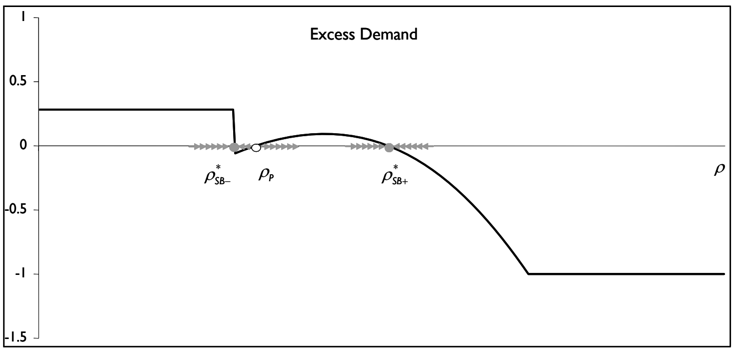


Figure A5: Tatonnement trajectories
with $R = 2.5$; $\gamma = 0.7$

7.6 Proof of Proposition 2

Point (i): $\phi^*_{SB-} \leqslant \phi^*_{SB+}$ comes from the fact $\rho^*_{_{SB+}} \geqslant \rho^*_{_{SB-}}$ (see proposition 1) and relation (4). Point (ii): $\frac{r^*_{SB+}}{\rho^*_{SB+}} \leqslant \frac{r^*_{SB-}}{\rho^*_{SB-}}$ comes directly from relation (3), the result $\varphi'(\rho) \geqslant 0$, and $\rho^*_{_{SB+}} \geqslant \rho^*_{_{SB-}}$. To show point (iii) I first replace (4) and (3) into (2) and get

$$\frac{R}{r} = 1 + \frac{\gamma(1-\overline{p})}{(1+\overline{p})\,\overline{p}}, \tag{A11}$$

where the right hand side of the relation decreases with $\overline{p}$. Since $\overline{p}^*_{SB+} = \varphi\left(\rho^*_{SB+}\right) \geqslant \varphi\left(\rho^*_{SB-}\right) = \overline{p}^*_{SB-}$, one gets $R/r^*_{SB+} \leqslant R/r^*_{SB-}$.

7.7 Proof of Proposition 3

Point (i): One has $\frac{d\overline{p}^*_{SB+}}{dR} \geqslant 0$ (see figure A1) and therefore (using (5)) $\frac{d\rho^*_{SB+}}{dR} \geqslant 0$ and (using (4)) $\frac{d\phi^*_{SB+}}{dR} \geqslant 0$. Similarly, one has $\frac{d\overline{p}^*_{SB+}}{d\gamma} \leqslant 0$ (see figure A1) and (from (5)) $\frac{\partial\rho}{\partial\gamma} \leqslant 0$ and $\frac{\partial\rho}{\partial\overline{p}} > 0$, which imply that $\frac{d\rho^*_{SB+}}{d\gamma} \leqslant 0$ and (using (4)) $\frac{d\phi^*_{SB+}}{d\gamma} \leqslant 0$. Point (ii): $\frac{d\phi^*_{SB-}}{dR} = 0$ comes from relation (4) and $\rho^*_{_{SB-}} = 1$, while $\frac{d\overline{p}^*_{SB-}}{dR} \leqslant 0$ comes from $d\varphi(1)/dR \leqslant 0$ (from (5)). Similarly, one has: $\frac{d\phi^*_{SB-}}{d\gamma} \leqslant 0$ comes from relation (4) and $\rho^*_{_{SB-}} = 1$, while $\frac{d\overline{p}^*_{SB-}}{d\gamma} \geqslant 0$ comes from $d\varphi(1)/d\gamma \leqslant 0$ (from (5)).