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Exchange rate forecasting with DSGE models

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Abstract

We run a real exchange rate forecasting “horse race”, which highlights that two principles hold. First, forecasts should not replicate the high volatility of exchange rates observed in sample. Second, models should exploit the mean reversion of the real exchange rate over long horizons. Abiding by these principles, an open-economy DSGE model performs well in real exchange rate forecasting. However, it fails to forecast nominal exchange rates better than the random walk. We find that the root cause is its inability to predict domestic and foreign inflation. This shortcoming leads us toward simpler ways to outperform the random walk.

Keywords: Forecasting, Exchange Rates, New Open Economy Macroeconomics, Mean Reversion.

JEL classification: C32, F31, F37.

Non-technical summary

Economic theory provides policymakers with clear guidance on how the competitiveness channel operates in the aftermath of a wide set of disturbances, such as monetary, productivity, risk premium or foreign shocks. However, there is a cloud hanging over this aspect of international economics, namely that these conjectures may have limited empirical significance, given the systematic failure of macro models to beat even the naïve random walk (RW) in exchange rate forecasting (the "exchange rate disconnect puzzle"). The question then naturally arises of whether international macro models are rich enough to be meaningful. Layers of complexity are typically added to improve their realism. For example, including in the features of the model the currency of trade invoicing may help the model to capture better the degree of exchange rate pass-through. Similarly, distinguishing the currency of denomination of asset and liabilities, may lead to a better description of the dynamics of external debt, which may be essential to better understand real exchange rate movements in emerging countries. On the other hand, imposing too many restrictions on the data generating process, either theoretically or in the estimation phase, may prove disadvantageous from a pure forecasting perspective given the higher number of estimated parameters.

Every cloud has a silver lining, however. The exchange rate disconnect puzzle has spurred economists to look for new directions of research with success. Open-economy dynamic stochastic general equilibrium (DSGE) models are clearly a major accomplishment from the theoretical perspective. The empirical literature has also shown why, by properly accounting for estimation error, exchange rate models may be better than we usually think. The consensus in the literature has also shifted back to the pre-1970s view that real exchange rates do not move randomly, but tend to revert to a slow-moving equilibrium. This particular finding raises a question, however. Why don't the mean-reverting properties of the real exchange rate, which are embedded in most new open-economy models, give them an edge in exchange rate forecasting vis-à-vis the RW?

The aim of this paper is to answer this very question. We evaluate the forecasting performance of a state-of-the-art open-economy DSGE model. Our goal is to cross-check whether this framework, albeit conceptually more appealing than the macro models of the 1970s, has the same disappointing performance out of sample. The results are encouraging.

First the good news: we find that our preferred DSGE model is able to forecast real exchange rates consistently better than the RW for three out of five countries at medium-term horizons and performs comparably for the other two. This suggests that a mean reverting real exchange rate, which is an inherent property of our preferred DSGE model, is a helpful feature rather than an obstacle from a forecasting perspective. Moreover, we indicate that there are two other forecasting tools that are more difficult to beat than the RW. We label the first one “AR fixed” since it is a simple autoregressive process of order one, where the autoregressive parameter is fixed by the modeler. The other successful competitor at medium-term horizons is a Bayesian VAR model, in which the modeler sets the prior that the real exchange rate reverts to its sample mean (MBVAR model).

Two reasons explain their success. Firstly, the “AR fixed” model, and to a lesser extent the MBVAR model, minimizes the errors at short horizons by mimicking the RW. Secondly, both models exploit the mean reversion of real exchange rates at longer horizons (in line with long-term Purchasing Power Parity). The way they do this is model specific. The “AR fixed” model foresees a constant adjustment of the real exchange rate to the recursive sample mean (“trivial dynamics”). The MBVAR model projects instead a richer adjustment process towards the recursive steady state (“no economic story”). By contrast, the DSGE model foresees a dynamic of adjustment to the steady state which depends on the type of structural shocks that have tilted the real exchange rate away from its equilibrium (“macroeconomic story”).

The key appeal of the DSGE model is that it provides a consistent macroeconomic explanation of how a wide set of variables adjust towards their equilibrium. The real exchange rate adjustment implied by the model is consistent with current account sustainability and convergence of inflation to its steady state. The concept of equilibrium exchange rate is also well defined. Empirically the model captures better the directional change of the real exchange rate. There is however a price in terms of complexity, which on the whole leads to a just minimal improvement in its forecasting performance relative to its closest competitors.

The bad news is that, if used consistently, the DSGE model encounters severe difficulties in forecasting nominal exchange rates. The reason is that it wrongly projects the relative adjustment of domestic and foreign prices. This negative result is nonetheless insightful because it helps us to reconcile the forecastability of real exchange rates with the exchange rate disconnect puzzle. The difficulty of macro models to beat the RW in exchange rate forecasting lies to a large extent in their difficulty in forecasting well inflation rates. Therefore, it is not surprising that the RW can be beaten also in nominal exchange rate forecasting, but not with a fully consistent DSGE model. This can be accomplished by employing the real exchange rate forecasts delivered by our three best models and, as a second step, assuming that all of the adjustment takes place via the nominal exchange rate. This reveals that the RW is not invincible even at horizons of one or two years.

1 Introduction

There is hardly anything more fascinating and nerve-wracking in international finance than attempting to understand exchange rates. Little can be said about the international transmission of shocks or the cross-border impact of monetary policy without a good understanding of what drives them. But how much do we really know? We tend to typically lean on economic theory to tell us a plausible story of how exchange rates react to a set of model-based disturbances, such as monetary, productivity, risk premium or foreign shocks. Yet there is a dichotomy between our aspiration towards knowledge of how open economies operate and the view, firmly established in the empirical literature since Meese and Rogoff (1983), that macroeconomic models are outperformed even by a trivial random walk (RW) in nominal exchange rate forecasting (the "exchange rate disconnect puzzle"). Notwithstanding several attempts to overturn this result (e.g. Mark, 1995), this conclusion has been confirmed by a multiplicity of analyses, for example in the comprehensive studies by Faust et al. (2003) or Cheung et al. (2005). To this very date there is a tension between our a priori belief that exchange rates are driven by fundamentals and the desperate chase after the perfect model that would help us to forecast them systematically better than naïve benchmarks (see Rossi, 2013, for a survey).

Every cloud has a silver lining, however. International economists have responded to the exchange rate disconnect puzzle by exploring different avenues of research and making considerable headway. From an economic perspective, a clear highlight has been the development of richly specified open-economy dynamic stochastic general equilibrium (DSGE) models. Following the seminal work of Obstfeld and Rogoff (1995), several specifications were proposed to examine a variety of economic issues through the development of two-country models (Devereux and Engel, 2003) or small open-economy models (Gali and Monacelli, 2005; Justiniano and Preston, 2010). An important step forward in this strand of research was that these models can be now brought to the data via the use of advanced estimation techniques (An and Schorfheide, 2007). From an econometric perspective, recent writings have also convincingly presented evidence that models estimated with large panels of data are able to outperform the RW in exchange rate forecasting (Engel, 2013; Ince, 2014). This suggests that the dismal forecasting performance of exchange rate models can be partly attributed to estimation rather than mis-specification error.

In the context of real exchange rate forecasting, it is also important to notice that there has been a considerable reappraisal of the Purchasing Power Parity (PPP) theory as a long-term concept (Taylor and Taylor, 2004). This notion has been strengthened by means of panel unit root techniques, which indicate that real exchange rates are better described as stationary processes rather than RWs (Sarno and Taylor, 2002). The majority of the literature now takes for granted that real exchange rates are mean reverting and focuses more narrowly on how to explain their slow adjustment process (Imbs et al., 2005). Long-term PPP was also shown to be a helpful concept to forecast real exchange rates, both in the context of simple autoregressive models (Ca' Zorzi et al., 2016) and Bayesian vector autoregressive (BVAR) models centered around the Dornbusch framework (Ca' Zorzi et al., 2015).

Central to the discussion is whether these advances in the literature really represent a breakthrough toward the resolution of the exchange rate disconnect puzzle. Ex-ante there are plenty of reasons to doubt that open economy DSGE models can do better than the models of the 1970s, especially as the foreign block in these models does not contribute to better forecasts of inflation and output (Kolasa and Rubaszek, 2016).

There are however also good grounds for optimism. Long-term PPP is an intrinsic feature of open economy DSGE models and therefore should give them an edge in an exchange rate forecasting race with respect to the RW. Some indication of this was found in the studies by Adolfson et al. (2007b) and Christoffel et al. (2011), who showed that, at least for the case of the euro, the real exchange rate can be forecasted more accurately with an open-economy DSGE model than with the RW or BVAR models. To the extent that this result stays robust for other currencies, a longer sample span or tougher benchmarks and could be extended to nominal exchange rates, it would be clearly an important step forward. As we shall see, our comprehensive set of results will paint a less rosy picture.

The key aim of this paper is to provide a thorough evaluation of how well a state-of-the-art open economy DSGE model performs in real but also nominal exchange rate forecasting. To this end, we estimate the open-economy DSGE model proposed by Justiniano and Preston (2010), separately for Australia, Canada, the United Kingdom, the euro area and the United States, and carefully evaluate the quality of the forecasts it generates. The country coverage, the long evaluation sample and a set of diagnostic tools make our study arguably more comprehensive than any of the previous work that has evaluated the forecasting performance of DSGE models, especially in an open-economy context.

We also apply one of the key lessons of the recent forecasting literature and avoid easy sparring partners (Giacomini, 2015). More specifically, we bring into the forecasting race six competing models. The first is the "twin" DSGE model, which is identical from a theoretical perspective, but allows for a linear trend in the real exchange rate in the measurement equation to improve the in-sample fit. We include this specification in our forecasting race because it is common practice to detrend the real exchange rate (and other variables) before estimation, as was done inter alia by Bergin (2003, 2006) or Justiniano and Preston (2010). Next, we have three BVAR models. Two of them are standard. The third one exploits the methodology of Villani (2009) to elicit the prior that the real exchange rate reverts to its recursive mean (MBVAR). The last two models are atheoretical. One is the classical RW model, which is a popular benchmark in exchange rate forecasting competitions. The other is a simple first-order autoregressive (AR) process, which assumes that the forecasted variable gradually converges to its mean at the speed that is set by the modeler. We label this model as "AR fixed", as was done by Faust and Wright (2013) in their work on inflation forecasting.

The key contribution of this paper is to show that, in order to deliver real exchange rate forecasts of high quality, models must abide by two principles. As a first priority, they must produce "conservative" forecasts, in the sense that they should not attempt to explain a large fraction of the exchange rate volatility out of sample. Although this implies a tendency to underpredict the scale of exchange rate movements, at least large forecast errors from assigning an excessive weight to the in-sample short-term dynamics are avoided. The second principle to which models must conform is that they should exploit any mean reverting tendency of the real exchange rate. All our core results become entirely intuitive if we keep these principles in the back of our mind.

The first finding of our study is that the (baseline) DSGE model performs almost as well as the RW in the short-run, while it is clearly better for three currencies and comparable for two in the medium-run. This is perfectly understandable in light of the principles mentioned above if we consider that this DSGE model produces short-term forecasts that are a bit less "conservative" (and hence less successful) than the RW but are consistent with the mean reverting properties of real exchange rate data (and hence perform better over the medium run).

The second finding is that the twin DSGE model (with trend) is much less accurate than the baseline DSGE (without trend). The reason is that the twin model delivers forecasts that are neither sufficiently conservative nor mean reverting. The lesson that we draw from this result is that attempts to improve the in-sample fit of DSGE models, e.g. by detrending the real exchange rate or other variables, can be counterproductive out of sample. This is true even for those currencies where the mean reversion property is relatively weak.

The third finding is the most important of the paper. We show that there is not an appreciable difference in the performance of the DSGE, "AR fixed" and MBVAR models. The "AR fixed" model is particularly hard to beat since it provides at the same time very conservative forecasts in the short-run and mean-reverting forecasts in the medium run. The MBVAR model is instead more competitive at longer horizons. This highlights that the conclusions of Adolfson et al. (2007b) and Christoffel et al. (2011) no longer hold when confronted with more competitive benchmarks.

The fourth finding is even more humbling, as we show that the exchange rate disconnect puzzle has not yet been solved. Like less sophisticated models in the past, the baseline DSGE model encounters considerable difficulties in beating the RW in nominal exchange rate forecasting. We show how it is possible to reconcile the forecastability of real exchange rates with the exchange rate disconnect puzzle. The key difficulty of the DSGE model in nominal exchange rate forecasting vis-à-vis the RW is its failure to capture adequately both domestic and foreign inflation.

The fifth and more positive finding is that, learning from this failure, one can in the majority of cases defeat the RW in nominal exchange rate forecasting at horizons longer than one or two years. To accomplish this, it is enough to take the real exchange rate forecasts delivered by the DSGE, "AR fixed" or MBVAR models and simply use them as nominal exchange rate forecasts. This means that it is preferable to assume that relative prices follow a RW rather than to rely on the volatile forecasts for this ratio that are derived with the DSGE model.

In this paper we also discuss how the choice of the best economic model clearly goes beyond a narrow forecasting evaluation criterion. The strength of the DSGE model is that it foresees a path of real exchange rate adjustment that has a structural interpretation. The elusive concept of equilibrium exchange rate is also meaningfully defined. Moreover, over longer horizons the DSGE model projects the direction of real exchange rate changes better than both "AR fixed" and MBVAR models. These findings are encouraging considering that DSGE model-based estimates of the steady-state exchange rate can be quite volatile and sensitive to the addition of new observations. This feature puts it at a disadvantage relative to the "AR fixed" model, which is resilient to estimation error and spurious in-sample dynamics. The weakness of the DSGE model is that its complexity has a limited pay-off in pure forecasting terms, while its inability to forecast nominal exchange rates calls into question its full reliability for public policy.

The remainder of the paper is structured as follows. Section 2 presents the models at the start of the forecast race. Section 3 describes the data and the design of the forecasting competition. Section 4 presents and explains the main results, emphasizing the main takeaways of our analysis. This section also discusses the different concepts of equilibrium exchange rate and the real exchange rate dynamics that stem from each modeling strategy. Section 5 concludes.

2 Round-up of forecasting methodologies

We consider the following competitors in our forecasting horse race.

DSGE model

Our key theoretical reference is the DSGE model developed by Justiniano and Preston (2010), which is a generalization of the simple small open-economy framework of Gali and Monacelli (2005). In this model households maximize their lifetime utility, which depends on consumption and labor, the latter being the only input to production. The consumption good is a composite of domestic and foreign goods. Both domestic producers and importers operate in a monopolistically competitive environment and face nominal rigidities á la Calvo. Monetary policy is conducted according to a Taylor-type rule. The foreign economy is exogenous to the domestic economy.

The model features a number of rigidities that have been emphasized in the applied DSGE literature (Christiano et al., 2005; Smets and Wouters, 2007), also in the open economy context (Adolfson et al., 2007a). Due to the local currency pricing assumption, the law of one price does not hold in the short-run. International financial markets are assumed to be incomplete. Consumption choice is subject to habit formation and prices of non-optimizing firms are partially indexed to past inflation. Finally, the model includes a rich set of disturbances that affect firms' productivity, importers' markups, households' preferences, risk in international financial markets, monetary policy, as well as the dynamics of three foreign variables: output, inflation and the interest rate. As documented by Justiniano and Preston (2010), this model provides a reasonable characterization of the data for Australia, Canada and New Zealand. Importantly, it is consistent with the empirical finding of a disconnect between exchange rate movements and domestic variables, as cost-push and risk premium shocks explain most of the variation in the exchange rate but little of that in inflation and output.

For all countries considered in this paper, the model is estimated using eight macroeconomic times series. These are the following three pairs for the domestic and foreign economy: the log change in output ($\Delta\tilde{y}$ and $\Delta\tilde{y}^*$), inflation ($\Delta\tilde{p}$ and $\Delta\tilde{p}^*$) and the short-term interest rate ($\tilde{i}$ and $\tilde{i}^*$), and additionally the domestic country's current account to GDP ratio ($\tilde{ca}$) and the log change in the real exchange rate ($\Delta\tilde{q}$). In this respect, we make two important departures from Justiniano and Preston (2010). First, our set of observable variables includes the current account balance rather than the change in the terms of trade. This is motivated by our focus on the real exchange rate dynamics and the well-established connection between this variable and the current account in the equilibrium exchange rate literature (Williamson, 1994) or the external balance assessment methodology of the IMF (Phillips et al., 2013). Second, and unlike some of the previous studies, in our baseline specification we do not demean the log-difference in the real exchange rate prior to estimation. A model variant in which we allow for a linear trend in this variable is also considered, but turns out to be far less successful.

As is standard in the literature, we use Bayesian methods to take the DSGE models to the data (An and Schorfheide, 2007), making the same prior assumptions for the estimated parameters as Justiniano and Preston (2010).[1] The openness

[1] Justiniano and Preston (2010) estimate their model for two countries considered in this paper (Australia and Canada) and we use the same prior assumptions for the remaining three. Since our main conclusions do not hinge on the results obtained for Australia and Canada, it is unlikely that the DSGE model receives an unfair advantage in our forecasting race due to a choice of priors that aims to improve the model fit, a concern recently raised by Gurkaynak et al. (2013). Note also

parameters are calibrated based on each country's average share of imports and exports in GDP. We correct these shares for the import content of exports calculated by the OECD to compensate for the lack of this feature in the model.

More details on the model's assumptions and derivations, as well as prior distributions used in the estimation, can be found in Justiniano and Preston (2010). In the Appendix, we list all equations making up the log-linearized version of the model, explain the link between its variables and the empirical data described in the next section, and present some details on the calibration and estimation of the model parameters.

BVAR models

It is well known that DSGE models have a restricted infinite-order VAR representation (Fernandez-Villaverde et al., 2007), which explains why VARs have been widely used in the forecasting literature evaluating DSGE models. However, because of the large number of parameters and short time series, classical estimates of unrestricted VAR coefficients are often imprecise and forecasts are of low quality due to large estimation error. A common method to tackle this problem is to apply Bayesian VAR techniques. We follow this route by considering three BVAR models that are estimated using the same times series as in the case of the DSGE model. These three specifications differ in the choice of whether the real exchange rate and other regressors are differenced prior to estimation, and on whether we impose the prior that the real exchange rate is mean reverting. In particular, we consider a BVAR in "levels" (LBVAR, for $\tilde{y}$, $\tilde{y}^*$, $\tilde{p}$, $\tilde{p}^*$, $\tilde{i}$, $\tilde{i}^*$, $\tilde{ca}$ and $\tilde{q}$), one expressed as a mixed model of variables expressed in "levels" and "differences" (DBVAR, for $\Delta\tilde{y}$, $\Delta\tilde{y}^*$, $\Delta\tilde{p}$, $\Delta\tilde{p}^*$, $\tilde{i}$, $\tilde{i}^*$, $\tilde{ca}$ and $\Delta\tilde{q}$) and one where we exploit the methodology of Villani (MBVAR, for $\Delta\tilde{y}$, $\Delta\tilde{y}^*$, $\Delta\tilde{p}$, $\Delta\tilde{p}^*$, $\tilde{i}$, $\tilde{i}^*$, $\tilde{ca}$ and $\tilde{q}$) to elicit the prior that the real exchange rate is mean reverting. In all cases we use the specification with four lags as the models are fitted to the data of quarterly frequency.

As regards the details of the estimation process, we use the standard Normal-Wishart prior proposed by Kadiyala and Karlsson (1997) for LBVAR and DBVAR models and assume a normal-diffuse prior for the MBVAR as in Villani (2009).

that we use a flat prior for trend inflation and hence our findings are immune to the criticism of Faust and Wright (2013).

For the model in levels (LBVAR), we use the standard RW prior. For the mixed models (MBVAR and DBVAR), we follow Adolfson et al. (2007b) and Villani (2009), centering the prior for the first own lag at zero for the differenced variables and at 0.9 for the variables in levels. All other VAR coefficients are centered at zero. As regards the dispersion of the prior distributions, we assume that they are tighter for higher lags (*decay* hyperparameter is set to 1) and choose the conventional value of 0.2 for the *overall tightness* hyperparameter. In the case of the MBVAR model, we additionally set the prior variance for cross-variable coefficients to lower values than for their own lags (*weight* hyperparameter equal to 0.5). The steady-state prior for the real exchange rate is centered at its recursive mean, with tightness such that the 95% interval coincides with the ±2.5% range around this mean. As we will see below, this choice ensures that the equilibrium exchange rate in "AR fixed" and MBVAR models is almost the same. As regards the remaining economic variables, we take standard values suggested by the literature. The 95% interval is defined as 0.5% ±0.25% for steady-state (quarterly) inflation and output growth, 1.0% ±0.25% for the (quarterly) interest rate, and 0% ±1.5% for the current account to GDP ratio.

Atheoretical benchmarks

We also let two atheoretical models into the race. The first one is the most widely used benchmark in the exchange rate forecasting literature, i.e. the naïve RW model proposed by Meese and Rogoff (1983). From the perspective of the forecasting practitioner, there is nothing more conservative than assuming that no changes occur over the forecast horizon. We also propose another atheoretical model, which practitioners all know very well and consists in simply assuming that the variable of interest gradually returns to its average past value. Since in this method the parameter that determines the speed of convergence to the mean is set by the modeler, we label this method as "AR fixed". This method was recently shown by Faust and Wright (2013) to be very competitive relative to several other methods for inflation. More generally, they demonstrate that a reasonable gliding path between two good boundary forecasts, one for the starting point and one for the long-term value, tends to produce very competitive forecasts. The "AR fixed" model shares with the RW the convenient feature that it is not subject to estimation error. At the same time, it is more appealing than the RW as, consistently with the macro literature, it foresees that the real exchange rate is mean reverting.

In the empirical application we set the autoregressive parameter of the "AR fixed" model to 0.95, which is consistent with the half-life adjustment of just over three years. This is within the range between three and five years suggested by Rogoff (1996) in his influential survey on the persistence of real exchange rates. However, the analysis that we present here is robust to any value in this range. This high duration was described by Rogoff himself to be among one of the six major puzzles in international economics and has sparked a large body of literature. A number of studies have also shown that aggregation bias in both the time and product dimensions may help to reconcile this high duration of the adjustment process with faster convergence at the product and sectoral levels (Imbs et al., 2005; Crucini and Shintani, 2008; Bergin et al., 2013). The writings of Kilian and Zha (2002), Murray and Papell (2002), and Rossi (2005) have however questioned the existence of the puzzle given the large estimation uncertainty that emerges using both Bayesian and classical techniques. Irrespective of these controversies, Ca' Zorzi et al. (2016) have shown that imposing a half-life in Rogoff's range is good for exchange rate forecasting.

3 Data

We use quarterly data over the period 1975:1 to 2013:4 for Australia, Canada, the United Kingdom, the euro area and the United States to construct the following eight time series for each of the five economies:

$\tilde{y}$, $\tilde{y}^*$	GDP per capita, calculated as a ratio of real GDP to the size of the population (log, seasonally adjusted)
$\tilde{p}$, $\tilde{p}^*$	CPI index (log, seasonally adjusted)
$\tilde{i}$, $\tilde{i}^*$	short-term nominal money market rate
$\tilde{ca}$	current account balance-to-GDP ratio (seasonally adjusted)
$\tilde{q}$	CPI-based real effective exchange rate (log)

We employ final and not real-time data. Not only does this allow us to compile a larger dataset, it also ensures consistency in the way we calculate aggregate foreign variables. The extension to real-time data is clearly of great interest but goes well beyond the scope of this paper.[2]

To compile such a large dataset we have extracted the data from various databases: the OECD Main Economic Indicators, IMF International Financial Statistics, European Commission AMECO and ECB Area Wide Model databases (Table 1 provides the relevant tickers). For each of the five countries, the foreign sector (variables denoted with stars) is represented by the other four economies plus Japan. This aggregation is carried out on the basis of the narrow effective exchange rate weights published by the Bank for International Settlements (Klau and Fung, 2006). More specifically, we compute the average values of these weights over the period 1993-2010 for the relevant countries and subsequently adjust them so that they sum to unity. The obtained weights are:

	US	EA	UK	CAD	AUS	JAP	coverage
United States		34.4	7.7	31.5	1.6	24.7	67.3
Euro area	40.5		34.8	3.7	1.8	19.1	85.8
United Kingdom	18.5	70.9		2.0	1.0	7.5	91.9
Canada	81.5	9.6	2.5		0.3	6.1	90.8
Australia	32.5	30.2	8.8	2.4		26.1	74.3

The last column shows that the coverage ratio for the foreign sector ranges from 67% for the US to almost 92% for the UK.

4 Results

We assess the out-of-sample forecast performance of the baseline DSGE model and its competitors for horizons ranging from one quarter to six years. The models are estimated using recursive samples.[3]

The point forecasts discussed below were calculated as the means of draws from each model's predictive density. Note that generating the forecasts for DSGE models

[2] It is important to note that real-time vintages would be strictly necessary if we were to compare our results with expert forecasts. In our forecasting race, none of the models employs additional information that would give them an unfair advantage over other competitors.

[3] The one-quarter-ahead forecasts are evaluated on the basis of 76 observations, two-quarter-ahead forecasts on the basis of 75 observations, and so forth with the 24-quarter-ahead forecasts comprising 53 observations. The first set of forecasts is elaborated with models estimated over the sample 1975:1-1994:4 for the period 1995:1-2000:4. This procedure is repeated with samples ending in each quarter from the period 1995:2-2013:3.

only required running estimation, performing convergence checks and drawing from the predictive density 760 times (since we have 76 different estimation windows for each country and two DSGE variants). The total computer time needed to execute all these steps amounted to almost half a year.[4]

Forecast accuracy

We begin our analysis by measuring the forecasting performance of the seven competing methods with the root mean squared forecast errors (RMSFEs) for the real exchange rates (Table 2). We report the RMSFE values as ratios in comparison to the RW, so that values below unity indicate that a given model outperforms the RW benchmark. We also test the null of equal forecast accuracy with the two-sided Diebold-Mariano test.

A number of key features of the results are immediately evident. The "AR fixed", MBVAR and baseline DSGE models have generally the lowest RMSFEs at medium-term horizons. They overwhelmingly beat the RW for the United States, euro area and United Kingdom, while the results are broadly similar for Australia and Canada. Of the two DSGE models, the baseline version (without trend) is consistently better than the alternative (with trend). This indicates that the attempt to explain in-sample the real exchange rates is counterproductive out of sample. Of the three BVAR models, the MBVAR is the most accurate, followed by LBVAR and, at a considerable distance, DBVAR. This suggests that differencing the real exchange rate before estimation attributes too much weight to short-term dynamics. It also reveals that having a set of priors for the long-run level of the real exchange rate enhances the out-of-sample performance of BVAR models.

To shed some light on the absolute performance of the competing models, we run the so-called Mincer-Zarnowitz regressions. This consists in regressing the realized real exchange rates q_τ on a constant and their forecast q_τ^F:

$$q_\tau = \alpha + \beta q_\tau^F + \eta_\tau. \quad (1)$$

where the number of observations τ depends on the forecast horizon. For an efficient forecasting model, the constant term should be close to zero, the slope coefficient

[4]We used the Intel(R) Xeon(R) 3.40GHz Processor.

close to unity and the fit of the regression measured by the R^2 coefficient high. Table 3 reports the outcomes for the shortest (one-quarter-ahead) and longest (six-years-ahead) horizons considered in this paper. It presents the parameter estimates for α and β, the R^2 coefficient and the p-value of the joint test that $\alpha = 0$ and $\beta = 1$. At the one-quarter horizon, the null of forecast efficiency cannot be rejected at the 5% significance level in almost all cases for the "AR fixed", baseline DSGE and MBVAR models. The third criterion required to establish efficiency is however not fulfilled as the R^2 never exceeds 5%. At the six-year horizon, where the fit of the regressions is much higher, the null of efficiency is rejected almost always for all models. In terms of efficiency, all models disappoint, even if to a different degree.

The results of the Mincer-Zarnowitz regressions are also illustrated graphically in Figures 1 and 2, which present scatter plots of the real exchange rate realizations (y-axis) versus the model-based forecasts (x-axis). Points along the 45 degree line correspond to perfect predictions, i.e. the maximum degree of efficiency possible. Observations that fall in the top-right and bottom-left quadrants are forecasts that anticipate correctly the directional change of the real exchange rate. Based on their position relative to the 45 degree line, the predictions that have the correct sign can be further split between those where the forecasted absolute change in the real exchange rate is larger (overprediction) or smaller (underprediction) than the realization. Tables 4 and 5 provide a set of indicators that summarizes the information in the scatter plots. In the upper panel we present the percentage of forecasts that have the correct sign. This is complemented by the goodness-of-fit χ^2 test to evaluate if this number is significantly different from 50% (Pesaran and Timmermann, 1992). In the second panel we show the percentage of forecasts that underpredict the realized values. The third panel reports the correlation coefficients between forecasts and actual data. The fourth and final panel contains an indicator that we label as "relative volatility" as it measures the ratio of the average absolute forecasted change in the real exchange rate to the average absolute actual change in this variable. The value of this indicator is by definition zero for the RW (very conservative) and 100% for the perfect model.

The findings of both Figure 1 and Table 4 confirm that at a short horizon none of the models perform particularly well. Most observations are distant from the 45 degree line, correlation is low and the share of forecasts that have the correct sign is almost never significantly different from 50%. At longer horizons the "AR fixed", MBVAR and baseline DSGE models prove to be much better than the competition. Albeit not perfectly aligned along the 45 degree line (Figure 2), most observations can be found in the "correct" quadrants, i.e. the top-right and bottom-left ones. For example, the euro forecasts are of the correct sign in about 70% of cases with the "AR fixed" and MBVAR models and in almost 80% with the baseline DSGE model (Table 5). In all three cases the null of a random draw equal to 50% is strongly rejected at the 1% significance level.[5] Our three best-performing models also generate forecasts that are highly and positively correlated with actual data for all currencies (except for the Australian dollar). Table 5 also shows that these three models have a strong tendency to underpredict the absolute size of exchange rate movements, which is reflected in the "relative volatility" indicator having values that are much lower than 100%. The only model that is able to describe well the scale of future real exchange rate movements is DBVAR, but typically its forecast goes in the wrong direction. To sum up, the scatter plots and statistics discussed above confirm that the "AR fixed", MBVAR and baseline DSGE models are most accurate, especially at longer horizons.

The anatomy of the results

In what follows we investigate what is driving the results reported above. A good way forward is to get a visual impression of the performance of the six models by plotting their forecasts for the real exchange rate at different points in time (Figure 3). A first inspection suggests that the baseline DSGE and "AR fixed" models are characterized by conservative forecasts, i.e. forecasts that do not attempt to explain a large fraction of the data variation or to anticipate the turning points. The charts also show that for the baseline DSGE, MBVAR and "AR fixed" models there is a visible mean-reversion mechanism. Conversely, the worst performing model in terms of the RMSFEs, i.e. the DBVAR model, delivers volatile forecasts, which usually are strongly influenced by recent trends. The LBVAR and DSGE (with trend) models extrapolate long-term trends rather than project their correction, hence their forecast accuracy for longer horizons is relatively low. Among the three

[5]In the case of Australia the null of a random draw is rejected at the 1% significance level in the case of the DSGE model (with trend) and at the 10% significance level in the case of LBVAR. This suggests that these models would be beaten by a set of predictions based on the toss of a coin.

mean-reverting methods, the MBVAR model has the richest dynamics, but this harms rather than enhances its short-term performance. It is clear that in our forecasting race the conservatism of the DSGE and "AR fixed" models, which also manifests itself in the low "relative volatility" indicators reported in Tables 4 and 5, shields them from incurring large forecast errors. Among the BVARs, the MBVAR is the least volatile and best performing. This is intuitive since the steady-state priors anchor its short-term forecasts within a reasonable range.

Some simple statistics help us to gauge better what role mean reversion plays in the data and how well it is captured by our forecasting tools. We start by counting how many times the real exchange rate moves toward its mean. These numbers can be found in the last row of Table 6 for each country panel. They reveal that at short horizons, e.g. one year ahead, there is no evidence of mean reversion or diversion. At the lowest end of the range we find the euro area, where the real exchange rate reverts towards its mean only 45% of the times. At the highest end we find the United Kingdom, where mean reversion takes place in 71% of the cases. The number of episodes where the real exchange rate moves towards its mean tends to increase monotonically with the length of the forecast horizon. After 24 quarters, this frequency is in the range between 64% for Canada and 89% for the United Kingdom. The only notable exception is Australia, where the indicator cannot pick up any evidence of mean reversion either in the short or long term.

It may be nonetheless misleading to just count the episodes of mean reversion without evaluating the strength of the correction. To explore this issue in greater depth, for each model, currency and forecast horizon we calculate the following statistic to measure the pace of mean reversion (PMR):

$$PMR_h = -100 \sum_{t=1}^{T} w_t \frac{q^f_{t+h} - q_t}{q_t - \overline{q}_t}, \tag{2}$$

where q^f_{t+h} is an h-step-ahead forecast for the real exchange rate elaborated at period t, $\overline{q}_t$ is the recursive sample mean and w_t indicates a weight that is proportional to the deviation of the real exchange rate from the recursive mean, i.e. $w_t = |q_t - \overline{q}_t| / \sum_{t=1}^{T} |q_t - \overline{q}_t|$.[6] The way we have designed this measure leads to very intuitive results. Positive values of the PMR statistic point to mean-reversion and negative

[6]For small deviations from the steady state, the mean-reverting forces are likely to be obscured by other short-term factors.

ones to mean divergence. If a model predicts the full return to the sample mean within a given horizon, then $PMR = 100$. Given the above definition, this statistic will be equal to zero for the RW benchmark at all horizons. For the "AR fixed" model, the formula can be derived analytically, namely $PMR_h = 100 \times (1 - 0.95^h)$. For the remaining models the PMR statistic is calculated using out of sample forecast data.

The PMR values for the six models in our forecasting race are presented in the first six rows of Table 6 for each country. The seventh row shows the corresponding statistics for realized data. It is insightful to look at the realized data first. The PMR indicator reveals that, contrary to the common presumption, mean reversion already starts at short horizons, even if very feebly. For example, the pace of mean convergence of the real exchange rate of the euro increases from less than 10% at the end of the first year to 30% after three years; it then accelerates, reaching 120% by the end of our forecast horizon. This suggests that at long horizons there are a few important episodes where the adjustment even overshot what was predicted by relative PPP. The table also shows that an entirely analogous adjustment characterizes the US dollar and the pound sterling. A steady rise in the forces of mean reversion is also detectable for the Canadian dollar, but the correction is incomplete even after six years. Finally, the evidence of mean reversion is again almost non-existent for the Australian dollar, except at very long horizons.

The PMR indicator for our best three models, i.e. the "AR fixed", MBVAR and baseline DSGE, matches the data rather well. They correctly anticipate that mean reversion plays initially a minor role but eventually becomes key. This helps us to get a grasp of why these models, in particular "AR fixed", are already competitive at short-horizons and become increasingly harder to beat at longer forecasting horizons. The other models instead miss this opportunity. The DBVAR model generally predicts mean divergence both for short-term and long-term horizons. A similar story can be told for the LBVAR model for Canada and Australia. To sum up, the failure of the standard BVARs to account for the mean-reverting property in the data is consistent with their poor forecasting performance.[7]

[7]It is also worth noting that even for Australia, where the evidence of mean reversion is weaker, our benchmark DSGE clearly beats its "twin" variant (with trend) in terms of RMSFEs.

Equilibrium exchange rates

In the previous subsection we have shown that the best forecasting models are capable of replicating the mean reversion in the real exchange rate data. It should be noted, however, that the "end-point" is not the same across models. In the case of the "AR fixed" model it is equal to the recursive mean, while for the MBVAR and DSGE models it is the model-based steady-state of the real exchange rate. It is natural to interpret these "end-points" as proxies for the long-run equilibrium exchange rate (expressed in real terms). There is indeed a bewildering plethora of different equilibrium exchange rate concepts in the literature (Driver and Westaway, 2004). The concepts employed here relate directly to two of the most popular methods used to assess imbalances (Phillips et al., 2013; Bussière et al., 2010).[8]

To see how much these theoretical differences matter empirically, we plot in Figure 4 the recursive estimates of the equilibrium real exchange rate for all three models. It can be seen that the estimates from the "AR fixed" and MBVAR models are almost the same. Although in the latter case relative PPP is only set as a long-run prior, the outcome is almost identical to the recursive mean, which suggests that the "prior" set by the modeler is informative. The differences between the recursive real exchange rate mean and the steady-state real exchange rate implied by the DSGE model are instead typically larger. The average distance between them varies between 0.7% for Australia and 2.8% for the euro area. There are however specific cases where the gap is much larger. For example, in the years after the euro was launched, the US dollar rose significantly against the euro. A retrospective look at that episode tells us that in the period 1999-2003 the real effective exchange rate of the dollar was (on average) overvalued by 11% according to the "AR fixed" and MBVAR models but by only 3% according to the DSGE model.

The DSGE model-based concept of equilibrium exchange rate is particularly appealing thanks to the strong theoretical foundations of the model. It guarantees

[8]The concept of equilibrium exchange rate implicit in the "AR fixed" model (and used to elicit the steady-state prior in the MBVAR model) is based on Equilibrium Real Exchange Rate method. This method consists in estimating a reduced-form relationship between the real exchange rate and a set of relevant fundamentals. Under PPP, the only relevant fundamental is the constant and hence the estimated equilibrium exchange rate is the recursive mean of the real exchange rate. The concept of equilibrium used in the DSGE model could be viewed instead as an elegant generalization of the Fundamental Equilibrium Exchange Rate method, where the (long-run) equilibrium exchange rate is consistent with all the variables (and not just the current account) being at the steady state.

both a mean-reversion mechanism for the real exchange rate and long-term current account sustainability. However, the estimated equilibrium exchange rate is more volatile than that implied by the other two models. There are various cases where adding just one observation to the estimation sample leads to a re-assessment of the real equilibrium exchange rate by more than 10%. We interpret this result as a sign of the sizable role of estimation error. This feature clearly puts the DSGE model at a disadvantage relative to the other two models, which instead reassess the arrival of new information only marginally and hence avoid picking up spurious in-sample dynamics (Faust and Wright, 2013). The equilibrium exchange rate calculated with the "AR fixed" or MBVAR model therefore has weaker theoretical foundations, but is more stable.

Real exchange rate dynamics

Our three preferred models also feature different dynamic adjustments to their respective steady states. In particular, Figure 5 illustrates how equilibrium is restored in the "AR fixed" and DSGE models. In the former case, this adjustment is just a simple log-linear gliding path towards the historical mean. In the latter case, the path of adjustment is more complex as it depends on the dynamic reactions of the exchange rate to eight different disturbances and the historical realizations of the shocks. For example, if the real exchange rate is tilted from its steady state by a monetary shock, the return to the equilibrium is rapid. By contrast, it takes many years to eliminate the impact of an import cost-push-shock. Moreover, the shapes of the impulse responses do not always point to a gradual return to the steady state, as they sometimes have an oscillating pattern (as for example in the aftermath of shocks affecting foreign variables). The impulse response functions presented in Figure 5 also help us explain the cross-country differences in mean reversion implied by the DSGE model. It is clear that the effects of a cost-push shock, which accounts for the bulk of real exchange rate fluctuations, especially at medium and long horizons, are more persistent for the US and Australia than for the UK and Canada, which is consistent with the statistics reported in Table 6.

In comparison to the "AR fixed" model, the DSGE framework is naturally richer since it provides forecasts that can be both time variant and country specific. In particular, there are episodes where the real exchange rate initially diverges further from the (real) equilibrium exchange rate in order to bring the current account back on a converging path towards its steady state.[9]

The fundamental question is whether this kind of structural macroeconomic argument gives the DSGE model a forecasting edge over atheoretical benchmarks such as the "AR fixed" or MBVAR models. We have already seen that, in terms of RMSFEs, the forecasting performance of the three models tends to be quite similar. However, if we go back to Table 5, we can notice that the DSGE model does a much better job at capturing the direction of the real exchange rate movements over longer horizons, except for Australia where all three models perform similarly. This suggests that the macroeconomic mechanisms embedded in the New Open Economy Macroeconomics (NOEM) framework, by allowing for time and country variation in the speed of mean reversion of the real exchange rate, tend to improve the quality of the forecasts for this variable. This result is remarkable considering that the DSGE model is subject to estimation error and is more sensitive to spurious in-sample dynamics than the calibrated "AR fixed" benchmark.

Naturally, there are also other, more practical considerations that may affect the choice between these three competing forecasting methods. The strength of the "AR fixed" and, to some extent, the MBVAR model is their simplicity and tractability. The comparative advantage of the DSGE model is that it is able to provide a consistent macroeconomic explanation of adjustment to the equilibrium, not only for the real exchange rate, but also for a wider set of economic variables.

Nominal exchange rate forecasting

The exchange rate disconnect puzzle typically refers more to our inability to forecast nominal rather than real exchange rates. It is hence entirely natural to consider whether the predictive ability that we have identified earlier for the real exchange rate applies to its subcomponents and in particular to the nominal exchange rate. In what follows we evaluate only the three best performing models from the previous section and the RW. The baseline DSGE and MBVAR can be labelled as "fully

[9] For instance, this is the case for the US dollar real exchange rate forecast elaborated with data ending in the third quarter of 2013. Even though the DSGE model interprets the real exchange rate as undervalued by 5.6%, it predicts a further depreciation by 3.5% over a six-year ahead horizon. The reason is that, according to the model, this depreciation was required to repay the US net foreign debt, which had been accumulated by persistent current account deficits in the past.

consistent" since they generate a complete set of forecasts for all variables, including the nominal exchange rate and relative prices. This is not the case for the "AR fixed" model since it does not predict any other variable than the real exchange rate. Hence, to extract a set of forecasts for the nominal exchange rate from this method, we have to take a shortcut. The simplest way to do this is to assume that the real exchange rate adjustment occurs entirely through changes in the nominal exchange rate and not in relative prices.

Table 7 presents the outcome of this exercise in terms of RMSFEs. We find that in this case the "fully consistent" DSGE model cannot beat, and in some cases even loses by some margin against, the RW. The "fully consistent" MBVAR model fares well only at longer horizons, where it generally outperforms the RW. Notwithstanding its ad-hoc nature, the "AR fixed" model beats the RW overwhelmingly for three out of five countries as before. This latter finding already hints that the nominal exchange rate, rather than relative prices, is the major channel through which the real exchange rate adjustment process operates in flexible exchange rate economies.

Figure 6 shows the recursive forecasts generated by the DSGE model for the nominal and the real exchange rates, relative price levels as well as their subcomponents, i.e. domestic and foreign prices. It is clear that the quality of DSGE model-based predictions of relative prices is far from satisfactory. This failure partly reflects the low accuracy of forecasts for foreign prices, which are modeled together with foreign output and interest rates in a trivariate VAR system. This is per se not sufficient evidence against the DSGE model, since the foreign block is not a core part of the theoretical model (Wickens, 2014). A first inspection points, however, to a more fundamental problem given the low quality of domestic price forecasts, especially in Australia, the UK and the US.

This matches the literature's finding that DSGE models estimated using data covering the period with high inflation do not forecast this variable well unless the prior for its steady state value is tightly centered around a mean significantly below the historical average, or long-term inflation expectations are used as an additional observable variable (Del Negro and Schorfheide, 2013). A similar remark was made by Wright (2013) in the context of BVAR models. The flat prior distribution for steady-state inflation at home and abroad in our benchmark DSGE model appears to be the underlying source of the problem in our forecasting context.[10]

[10]We also do not have sufficiently long time series on inflation expectations for countries included

This explanation is confirmed by Figure 7, which plots the recursive forecasts for the same variables as before for the MBVAR model. Note that in this model the steady-state prior for inflation (both domestic and foreign) is tightly centered at the same low annual value of 2%. As a result, the quality of relative price forecasts obtained from the MBVAR is much better, which explains its relatively good performance in terms of the nominal exchange rate. The lesson that we draw is that it is better to anchor domestic and foreign inflation at levels consistent with the more recent past (or at levels consistent with long-term inflation expectations) rather than to rely on the whole sample.

A formal way to show that the problem goes beyond the modeling of the foreign block is to generate a set of exchange rate forecasts that are conditional on the realised foreign variables ("conditional DSGE"). An interesting finding is that the exchange rate forecasts generated in this way tend to be more accurate in all cases except for the US dollar. However, for three currencies the improvement is not enough to make a real difference. The exchange rate disconnect result is overturned only in the case of the euro. This analysis suggests that the difficulty of the DSGE model to forecast exchange rates vis-à-vis the RW is mostly explained by its difficulties to forecast inflation domestically and abroad rather than an intrinsic inability to forecast price competitiveness trends.

To see this even more clearly, we evaluate the performance of what we label the "partially consistent" DSGE and MBVAR models, in which we assume that all the adjustment in the real exchange rate is channeled through changes in the nominal exchange rate as in the "AR fixed" model. With this hypothesis we clearly depart from the general-equilibrium analysis. It is, however, revealing that the forecasts produced in this way are either indistinguishable from or strongly preferable to those given by the RW at medium to long-term horizons. In contrast, we do not gain much over the "fully consistent" MBVAR model if we consider its "partially consistent" version. The clear message that emerges from this analysis is that, if mean reversion is a feature of the real exchange rate, there are ways to exploit it to beat the RW also in the context of nominal exchange rate forecasting.

in our sample.

5 Conclusions

The real exchange rate is a difficult variable to forecast as we all know. In this paper we have argued that, in order to deliver real exchange rate forecasts of good quality, models must fulfill two principles. First, they should be conservative, i.e. they must not attempt to replicate out of sample the high volatility of exchange rates observed in-sample. Second, they have to deliver real exchange rate forecasts that are mean reverting. Given that the baseline DSGE model conforms with both principles, it becomes intuitive why its performance is almost comparable to that of the RW in the short run and even better over the medium run.

The main message of our paper is that the ability of DSGE models to forecast real exchange rates, highlighted for the euro area by Adolfson et al. (2007b) and Christoffel et al. (2011), should not be overplayed as other models that conform with the same principles perform equally well. We have shown that there is indeed not an appreciable difference in the forecasting performance of the DSGE, "AR fixed" and MBVAR models at medium-term horizons. Therefore, additional arguments are required to make a convincing case in favor of our baseline DSGE. Its strength is that it provides a fully consistent story. It accounts in particular for the feedback effect of the real exchange rate on the current account, which ensures that the constraint of external sustainability is binding. These rich dynamics are also conducive to real exchange rate forecasts that capture better the direction of change in this variable. The DSGE model also provides precise estimates of how the exchange rate reacts to different shocks and a model-consistent definition of the equilibrium exchange rate. Its main weakness is that it is costly in analytical terms and prone to estimation error. On the other hand, DSGE models cannot be replaced by their competitors that easily since they also fail along some important dimensions. The "AR fixed" method is easy to implement, but provides no insights into what drives real exchange rate fluctuations or explanations of how the adjustment unfolds. The MBVAR model guarantees consistency for a multiple set of variables and richer dynamics, but similarly offers limited economic insights.

In this paper, we have also highlighted that the recent advances by the profession are not yet a "game changer"; the exchange rate disconnect puzzle, highlighted over thirty years ago by Meese and Rogoff (1983), is withstanding the scrutiny of time. We have investigated in particular the reasons why a DSGE model fails to beat the RW and concluded that this is due to its poor prediction of the relative price adjustment between the domestic economy and foreign trading partners. We have also shown that this failure cannot be attributed solely to the way the foreign block is specified, but depends also on the inflation dynamics implied by the model. Paying particular attention to the way economic models describe the exchange rate pass-through to inflation could be a priority for future research that may lead to further improvements in exchange rate forecasting. Finishing on a positive note, we have shown that, provided that real exchange rates are mean reverting, there are various ways to beat the RW in nominal exchange rate forecasting for horizons greater than one or two years.

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Table 1: Variable description and data sources

	Variable description	Source
$\tilde{ca}$	Current account balance-to-GDP ratio	MEI (bpbltt01) AWM (CAN_YEN)
$\tilde{e}$	Nominal exchange rate against the USD, quarterly average	MEI (ccusma02) AWM (EXR)
$\tilde{p}$	CPI index, seasonal adjustment with TRAMO-Seats	MEI (cpaltt01) AWM (HICPSA)
$\tilde{q}$	Effective real exchange rate, calculated using $\tilde{p}$ and $\tilde{e}$	
gdp	GDP at constant prices	IFS (bvrzfq) AWM (YER)
pop	Population, converted from annual data by cubic match last	AMECO
$\tilde{y}$	GDP per capita, calculated using gdp and pop	
$\tilde{i}$	Short term nominal money market rate	IFS (b00zfq)

Notes: MEI – OECD Main Economic Indicators, IFS – IMF International Financial Statistics, AMECO – European Commission AMECO database, AWM – ECB Area Wide Model database. The time-series tickers are shown in brackets. External sector variables are calculated as weighted averages using weights described in Section 3.

Table 2: Root mean squared forecast error (RMSFE) for the real exchange rate

	H=1	H=2	H=4	H=8	H=12	H=24
	United States					
AR fixed	0.99	0.98	0.94	0.89	0.87	0.73**
DBVAR	1.04	1.15	1.13	1.19	1.13	1.28
LBVAR	1.01	1.09	1.15	1.29*	1.36*	1.03
MBVAR	0.99	1.02	0.96	0.86	0.77	0.68**
DSGE with trend	1.12*	1.15	1.22	1.21	1.21	1.01
DSGE without trend	1.03	1.02	1.00	0.92	0.83	0.66***
	Euro Area					
AR fixed	1.00	1.00	0.97	0.92	0.87	0.76**
DBVAR	1.05	1.12*	1.20***	1.30***	1.36***	1.36**
LBVAR	1.05	1.12	1.22	1.31	1.25	0.93
MBVAR	1.02	1.05	1.07	1.01	0.93	0.75**
DSGE with trend	0.99	0.98	0.98	1.01	1.00	0.90
DSGE without trend	0.99	0.98	0.96	0.95	0.91	0.77**
	United Kingdom					
AR fixed	1.00	0.98	0.95	0.88**	0.86**	0.83***
DBVAR	1.06	1.18	1.23**	1.31**	1.43***	1.82***
LBVAR	1.12**	1.21**	1.24*	1.14	1.13	1.23*
MBVAR	1.02	1.06	1.00	0.89	0.86*	0.82**
DSGE with trend	1.01	0.98	0.94	0.84**	0.80***	0.86**
DSGE without trend	1.02	0.99	0.94	0.84	0.78**	0.67***
	Canada					
AR fixed	1.01	1.00	1.00	1.03	1.02	0.80
DBVAR	0.99	1.07*	1.15*	1.31*	1.41**	1.61**
LBVAR	1.04	1.09*	1.13**	1.09	1.03	1.10
MBVAR	0.99	1.03	1.05	1.07	1.07	0.88
DSGE with trend	1.02	1.02	1.03	1.08	1.08	1.04
DSGE without trend	1.03	1.03	1.04	1.08	1.05	0.79
	Australia					
AR fixed	1.01	1.00	1.00	1.02	1.03	0.88
DBVAR	1.03	1.10	1.12	1.05	0.98	1.18
LBVAR	1.02	1.06	1.09	1.07	1.07	1.22**
MBVAR	1.04*	1.08*	1.10*	1.07	1.06	0.92
DSGE with trend	1.07*	1.10	1.18**	1.34***	1.46***	1.55***
DSGE without trend	1.03	1.03	1.04	1.10	1.14	1.01

Notes: The table shows the ratios of the RMSFE from a given model in comparison to the RW benchmark so that values below unity indicate that forecasts from the model are more accurate than from this benchmark. Asterisks ***, ** and * denote, respectively, the 1%, 5% and 10% significance levels of the two-tailed Diebold-Mariano test, where the long-run variance is calculated with the Newey-West method.

Table 3: Forecast efficiency test for the real exchange rate

	$H=1$				$H=24$			
	$\hat{\alpha}$	$\hat{\beta}$	R^2	p-val.	$\hat{\alpha}$	$\hat{\beta}$	R^2	p-val.
	United States							
AR fixed	0.00	0.69	0.01	0.91	1.37	2.05	0.63	0.00
DBVAR	-0.29	0.42	0.04	0.05	1.68	0.31	0.17	0.00
LBVAR	-0.11	0.44	0.02	0.38	2.23	0.38	0.07	0.05
MBVAR	-0.23	0.61	0.04	0.45	1.76	1.31	0.56	0.00
DSGE with trend	0.15	-0.21	0.00	0.00	-24.89	1.87	0.48	0.00
DSGE without trend	-0.10	0.09	0.00	0.12	-4.79	2.64	0.89	0.00
	Euro Area							
AR fixed	-0.23	0.49	0.01	0.54	-5.96	1.88	0.59	0.00
DBVAR	-0.13	0.20	0.01	0.05	-15.77	-1.42	0.56	0.00
LBVAR	-0.05	0.22	0.01	0.02	-3.09	0.57	0.06	0.40
MBVAR	-0.22	0.42	0.03	0.12	-6.72	1.52	0.56	0.00
DSGE with trend	0.07	1.06	0.02	0.99	3.77	1.46	0.10	0.74
DSGE without trend	-0.15	0.99	0.02	0.94	0.43	2.35	0.56	0.00
	United Kingdom							
AR fixed	0.11	0.64	0.01	0.75	4.80	1.80	0.43	0.02
DBVAR	0.10	0.31	0.03	0.01	-4.68	-0.85	0.23	0.00
LBVAR	-0.13	-0.34	0.02	0.00	5.29	-0.16	0.00	0.00
MBVAR	0.12	0.42	0.02	0.09	4.39	1.39	0.35	0.11
DSGE with trend	-0.02	0.34	0.01	0.30	6.65	1.96	0.45	0.00
DSGE without trend	-0.08	0.33	0.01	0.08	-0.74	1.50	0.52	0.15
	Canada							
AR fixed	-0.21	0.22	0.00	0.30	-7.59	0.44	0.06	0.00
DBVAR	-0.29	0.57	0.05	0.00	-10.47	-0.09	0.01	0.00
LBVAR	-0.27	0.01	0.00	0.01	-13.64	0.69	0.15	0.00
MBVAR	-0.10	0.61	0.03	0.44	-10.84	0.07	0.00	0.00
DSGE with trend	-0.26	0.19	0.00	0.14	-11.83	0.50	0.03	0.00
DSGE without trend	-0.20	0.15	0.00	0.03	-7.12	0.44	0.05	0.03
	Australia							
AR fixed	-0.46	0.29	0.00	0.39	-14.56	0.06	0.00	0.02
DBVAR	-0.46	0.18	0.00	0.00	-16.52	0.31	0.04	0.00
LBVAR	-0.50	0.34	0.01	0.04	-21.77	0.88	0.10	0.00
MBVAR	-0.50	-0.26	0.00	0.00	-13.93	0.25	0.01	0.02
DSGE with trend	-0.57	0.05	0.00	0.01	-11.62	-0.24	0.01	0.00
DSGE without trend	-0.55	0.10	0.00	0.10	-15.31	-0.11	0.00	0.00

Notes: The table presents the outcome of the efficiency test regression given by (1) and the p-val. of the Wald χ^2 test that the null $\alpha = 0$ and $\beta = 1$. All statistics are corrected for heteroskedasticity and autocorrelation of the residuals with the Newey-West method.

Table 4: Comparison of one-quarter ahead forecasts and realizations for the real exchange rate

	US	EA	UK	CAN	AUS
	Correct sign (%)				
AR fixed	47.4	46.1	57.9	48.7	47.4
DBVAR	65.8***	64.5**	52.6	46.1	51.3
LBVAR	53.9	63.2**	47.4	47.4	47.4
MBVAR	59.2	52.6	55.3	59.2	47.4
DSGE with trend	42.1	53.9	56.6	50.0	43.4
DSGE without trend	52.6	48.7	53.9	53.9	47.4
	of which underprediction (%)				
AR fixed	83.3	94.3	81.8	81.1	83.3
DBVAR	66.0	81.6	75.0	82.9	84.6
LBVAR	80.5	75.0	72.2	77.8	77.8
MBVAR	80.0	82.5	73.8	88.9	80.6
DSGE with trend	68.8	97.6	76.7	81.6	78.8
DSGE without trend	82.5	97.3	70.7	82.9	80.6
	Correlation				
AR fixed	12%	9%	11%	5%	6%
DBVAR	21%	8%	17%	22%	6%
LBVAR	15%	9%	-13%	0%	11%
MBVAR	21%	16%	15%	18%	-6%
DSGE with trend	-5%	14%	8%	5%	1%
DSGE without trend	2%	16%	9%	4%	2%
	Relative volatility				
AR fixed	19%	20%	21%	31%	25%
DBVAR	53%	39%	51%	37%	35%
LBVAR	34%	42%	45%	30%	31%
MBVAR	34%	35%	42%	31%	23%
DSGE with trend	47%	14%	28%	28%	37%
DSGE without trend	26%	15%	36%	38%	25%

Notes: The figures in the first rows represent the fraction of forecasts that correctly predict the sign of the change in the real exchange rates. Asterisks ***, ** and * denote the rejection of the null of the goodness-of-fit χ^2 test (Pesaran and Timmermann, 1992), stating that the fraction of correct sign forecast is 50%, at the 1%, 5% and 10% significance levels. The relative volatility is calculated as the ratio of the average absolute forecasted change in the real exchange rate to the average absolute realized change in this variable.

Table 5: Comparison of 24-quarter-ahead forecasts and realizations for the real exchange rate

	US	EA	UK	CAN	AUS
	Correct sign (%)				
AR fixed	69.8***	69.8***	86.8***	64.2**	54.7
DBVAR	75.5***	52.8	15.1***	47.2	52.8
LBVAR	50.9	52.8	34.0**	47.2	37.7*
MBVAR	75.5***	67.9***	83.0***	64.2**	45.3
DSGE with trend	73.6***	75.5***	79.2***	50.9	22.6***
DSGE without trend	92.5***	79.2***	96.2***	69.8***	43.4
	of which underprediction (%)				
AR fixed	91.9	89.2	80.4	73.5	82.8
DBVAR	37.5	50.0	75.0	40.0	78.6
LBVAR	44.4	60.7	88.9	76.0	85.0
MBVAR	85.0	88.9	70.5	82.4	87.5
DSGE with trend	23.1	82.5	85.7	88.9	83.3
DSGE without trend	91.8	81.0	66.7	75.7	100.0
	Correlation				
AR fixed	79%	77%	65%	25%	3%
DBVAR	42%	-75%	-48%	-9%	21%
LBVAR	27%	24%	-5%	39%	31%
MBVAR	75%	75%	59%	3%	12%
DSGE with trend	69%	31%	67%	17%	-11%
DSGE without trend	94%	75%	72%	23%	-5%
	Relative volatility				
AR fixed	37%	40%	31%	68%	48%
DBVAR	135%	66%	106%	92%	49%
LBVAR	79%	45%	44%	44%	38%
MBVAR	56%	49%	38%	57%	37%
DSGE with trend	106%	46%	27%	25%	68%
DSGE without trend	40%	32%	60%	70%	36%

Notes: The figures in the first rows represent the fraction of forecasts that correctly predict the sign of the change in the real exchange rates. Asterisks ***, ** and * denote the rejection of the null of the goodness-of-fit χ^2 test (Pesaran and Timmermann, 1992), stating that the fraction of correct sign forecast is 50%, at the 1%, 5% and 10% significance levels. The relative volatility is calculated as the ratio of the average absolute forecasted change in the real exchange rate to the average absolute realized change in this variable.

Table 6: Pace of mean reversion for the real exchange rate

	H=1	H=2	H=4	H=8	H=12	H=24
	United States					
AR fixed	5.0	9.8	18.6	33.7	46.0	70.8
DBVAR	0.4	-0.1	-3.2	-9.4	-16.3	-41.0
LBVAR	4.8	11.2	27.0	62.7	95.7	131.6
MBVAR	2.6	6.1	14.5	34.6	54.7	93.1
DSGE with trend	2.5	4.8	8.4	12.2	13.4	12.1
DSGE without trend	2.9	5.8	10.8	17.8	22.7	33.8
Actuals	1.0	3.8	8.8	23.0	22.4	123.1
Actuals, frequency of mean reversion	48.7	52.0	52.1	50.7	47.7	69.8
	Euro Area					
AR fixed	5.0	9.7	18.5	33.7	46.0	70.8
DBVAR	-4.2	-8.1	-15.8	-29.9	-42.4	-74.5
LBVAR	2.8	6.8	16.1	30.3	38.6	45.5
MBVAR	6.3	13.9	29.0	52.9	67.7	84.5
DSGE with trend	1.8	3.2	5.3	7.7	8.3	5.7
DSGE without trend	3.4	6.5	12.0	20.7	27.4	41.4
Actuals	0.5	2.5	6.9	20.1	31.3	122.2
Actuals, frequency of mean reversion	46.1	45.3	47.9	52.2	56.9	69.8
	United Kingdom					
AR fixed	5.0	9.8	18.6	33.7	46.0	70.8
DBVAR	-7.2	-12.9	-22.3	-37.0	-50.2	-86.2
LBVAR	0.9	2.9	7.1	16.1	26.1	39.3
MBVAR	7.1	16.0	32.0	55.1	69.2	84.4
DSGE with trend	6.1	11.9	21.6	34.8	42.6	50.5
DSGE without trend	7.9	15.5	28.5	47.5	60.3	80.4
Actuals	4.1	10.9	26.1	65.4	94.4	169.2
Actuals, frequency of mean reversion	60.5	60.0	71.2	75.4	81.5	88.7
	Canada					
AR fixed	5.0	9.7	18.5	33.7	46.0	70.8
DBVAR	-2.7	-5.7	-12.5	-26.2	-37.9	-67.8
LBVAR	-2.4	-5.0	-10.0	-16.5	-18.3	-30.3
MBVAR	2.4	6.0	14.5	30.6	42.4	60.1
DSGE with trend	4.3	8.5	15.6	24.8	30.0	34.1
DSGE without trend	5.9	11.9	22.6	38.9	50.4	70.2
Actuals	0.8	3.8	8.5	14.0	24.4	59.2
Actuals, frequency	48.7	48.0	56.2	52.2	50.8	64.2
	Australia					
AR fixed	5.0	9.7	18.5	33.7	46.0	70.7
DBVAR	-4.4	-8.2	-15.1	-26.2	-37.0	-72.4
LBVAR	-2.8	-5.5	-9.7	-14.6	-17.6	-30.3
MBVAR	2.8	6.2	13.7	27.0	37.0	54.2
DSGE with trend	2.5	5.1	9.6	17.0	22.5	31.4
DSGE without trend	3.7	7.4	14.3	26.3	36.3	57.1
Actuals	1.7	4.2	5.2	1.3	-0.4	20.4
Actuals, frequency of mean reversion	47.4	44.0	50.7	43.5	41.5	52.8

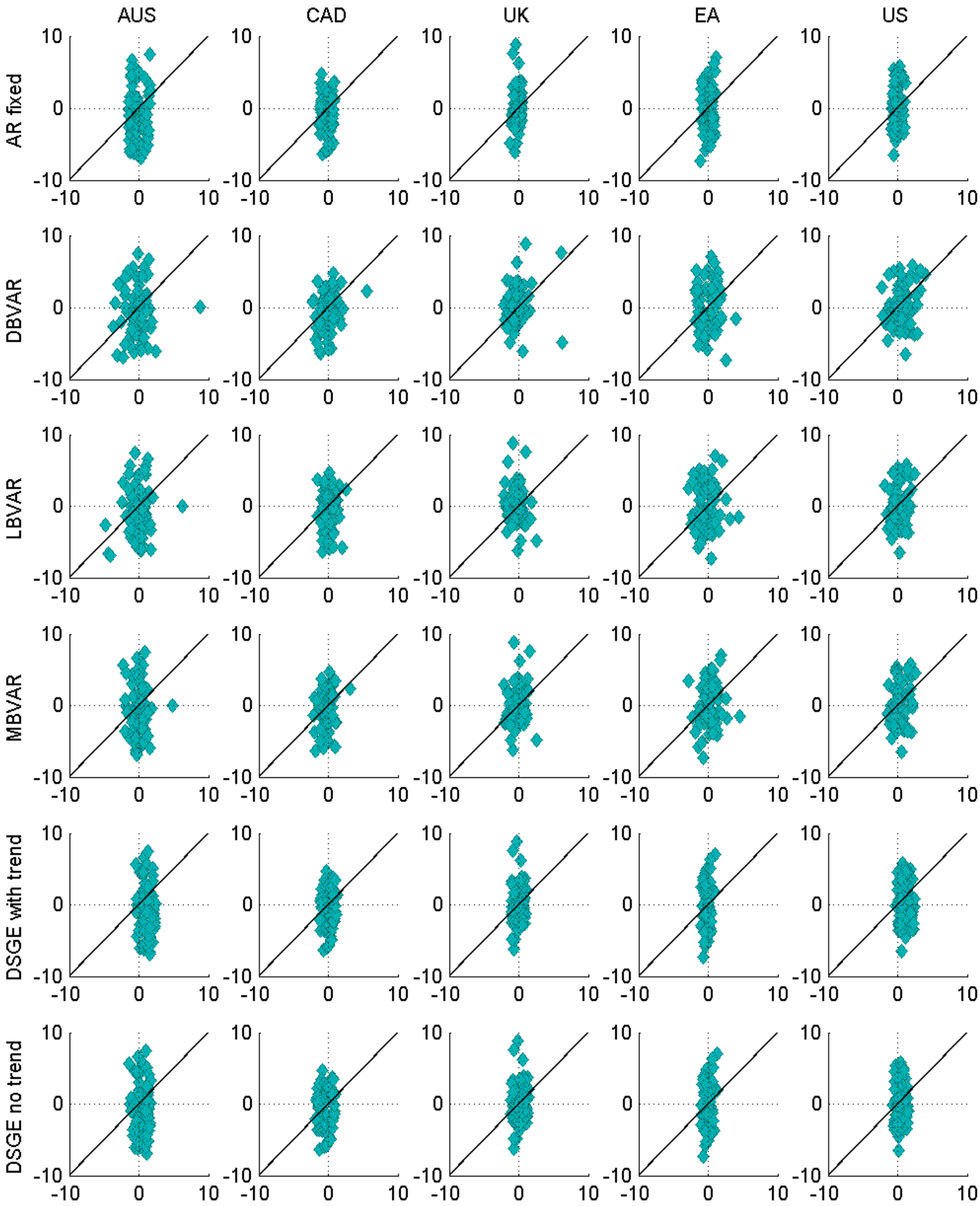
Notes: The table shows the weighted pace at which the forecasts or actuals revert to the recursive sample means (see formula (2) in the text). Negative numbers denote mean divergence.

Table 7: RMSFE for the nominal exchange rate

	H=1	H=2	H=4	H=8	H=12	H=24
	United States					
AR fixed	1.00	0.98	0.95	0.90	0.87	0.76**
MBVAR (partially consistent)	0.98	1.01	0.94	0.82	0.72*	0.70**
DSGE (partially consistent)	1.02	1.00	0.96	0.86	0.76*	0.67***
MBVAR (fully consistent)	1.00	1.04	1.00	0.86	0.69*	0.51***
DSGE (fully consistent)	1.03	1.03	1.05	1.01	0.97	0.78
DSGE (conditional)	1.06*	1.05	1.05	1.01	0.98	0.90
	Euro Area					
AR fixed	1.01	1.00	0.98	0.93	0.89	0.77**
MBVAR (partially consistent)	1.02	1.06	1.09	1.04	0.96	0.76*
DSGE (partially consistent)	0.99	0.98	0.97	0.96	0.93	0.78**
MBVAR (fully consistent)	1.04	1.08	1.13	1.10	1.04	0.82
DSGE (fully consistent)	1.01	1.01	1.01	1.00	0.98	0.89
DSGE (conditional)	0.95	0.94	0.95	0.92	0.88	0.66***
	United Kingdom					
AR fixed	1.01	1.01	0.99	0.94	0.93	0.88**
MBVAR (partially consistent)	1.04	1.08	1.05	0.97	0.96	0.89*
DSGE (partially consistent)	1.03	1.02	0.98	0.90	0.87	0.77***
MBVAR (fully consistent)	1.02	1.06	1.02	0.98	0.98	0.99
DSGE (fully consistent)	1.06**	1.05*	1.05	1.04	1.08	1.11**
DSGE (conditional)	1.03	1.03	1.04	1.03	1.06	1.16**
	Canada					
AR fixed	1.01	1.00	1.00	1.03	1.01	0.79
MBVAR (partially consistent)	0.98	1.02	1.04	1.05	1.04	0.86
DSGE (partially consistent)	1.02	1.03	1.03	1.07	1.02	0.77
MBVAR (fully consistent)	0.98	1.03	1.04	1.00	0.98	0.81
DSGE (fully consistent)	1.04	1.06	1.12*	1.21*	1.21	0.86
DSGE (conditional)	0.98	1.00	1.03	1.05	1.00	0.73**
	Australia					
AR fixed	1.01	1.00	1.01	1.05	1.09	1.01
MBVAR (partially consistent)	1.04*	1.08**	1.11*	1.09	1.09	0.98
DSGE (partially consistent)	1.02	1.02	1.03	1.09	1.14*	1.07
MBVAR (fully consistent)	1.05**	1.10**	1.14**	1.13*	1.13	1.06
DSGE (fully consistent)	1.06*	1.08*	1.14*	1.28**	1.38**	1.37**
DSGE (conditional)	0.98	1.00	1.06	1.16*	1.22*	1.25

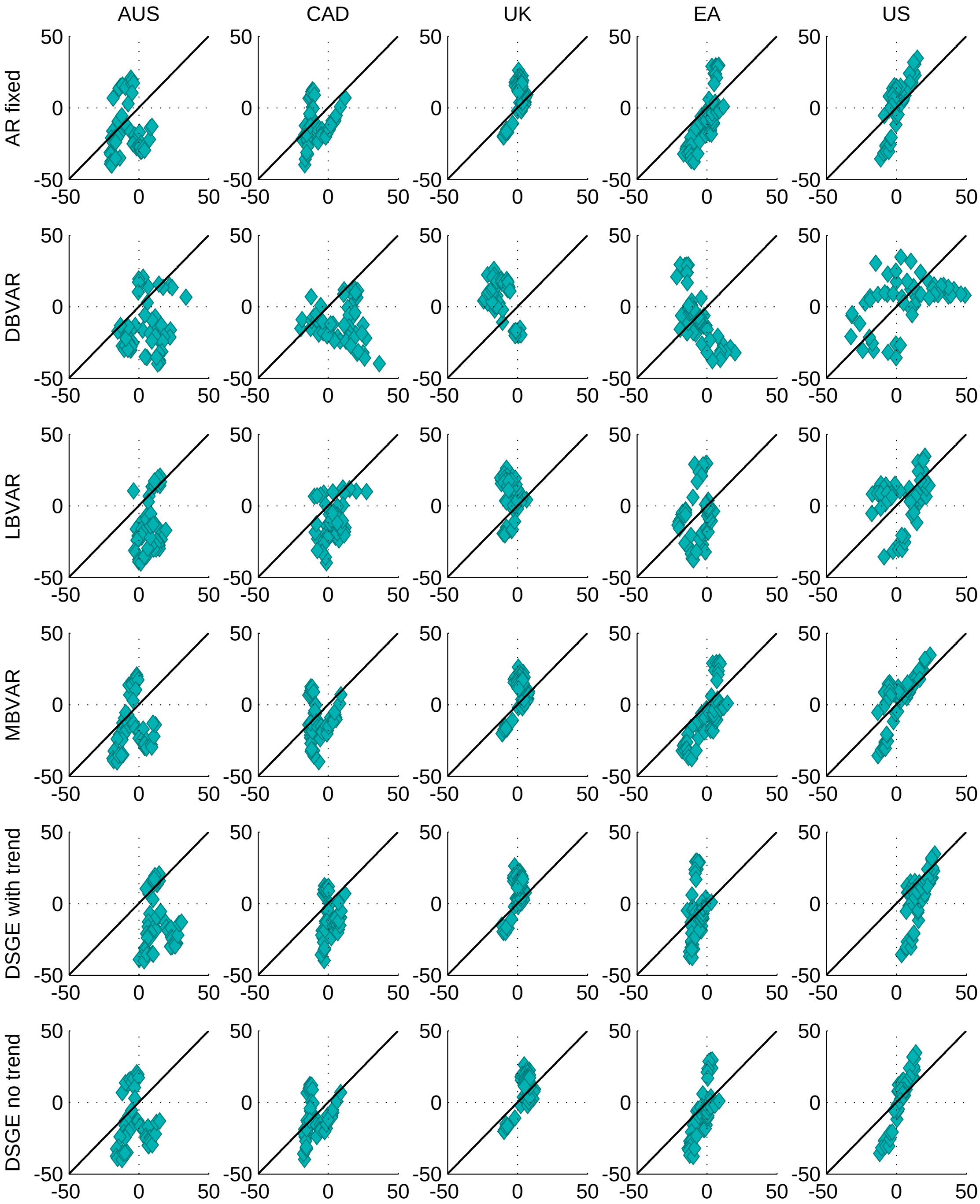
Notes: The table shows the ratios of the RMSFE from a given model in comparison to the RW benchmark so that values below unity indicate that forecasts from the model are more accurate than from this benchmark. Asterisks ***, ** and * denote, respectively, the 1%, 5% and 10% significance levels of the two-tailed Diebold-Mariano test, where the long-run variance is calculated with the Newey-West method.

Figure 1: Realizations and forecasts at one-quarter horizon for real exchange rates



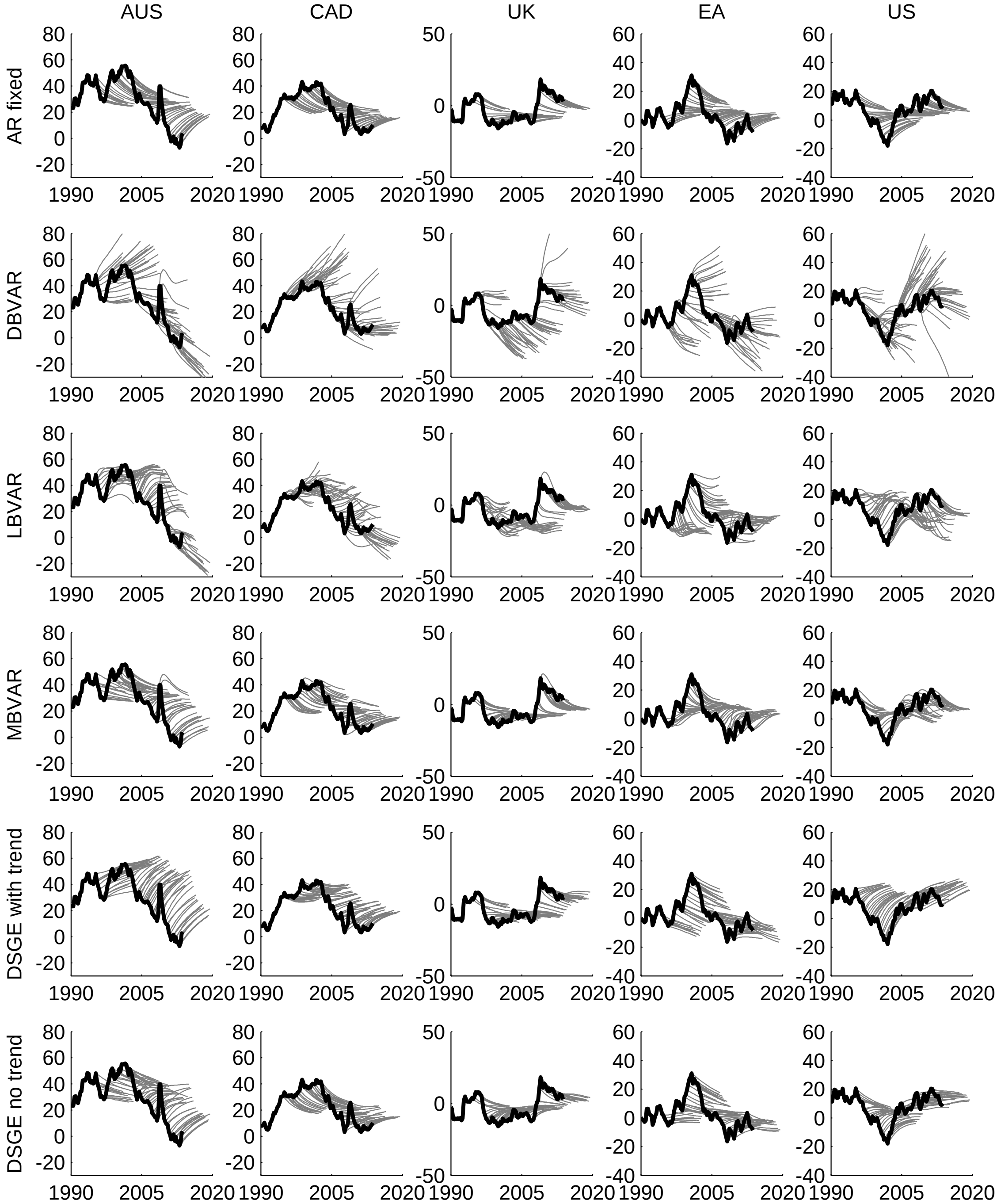
Note: The forecast values are on the horizontal axis, whereas the realizations are on the vertical one.

Figure 2: Realizations and forecasts at six-year horizon for real exchange rates



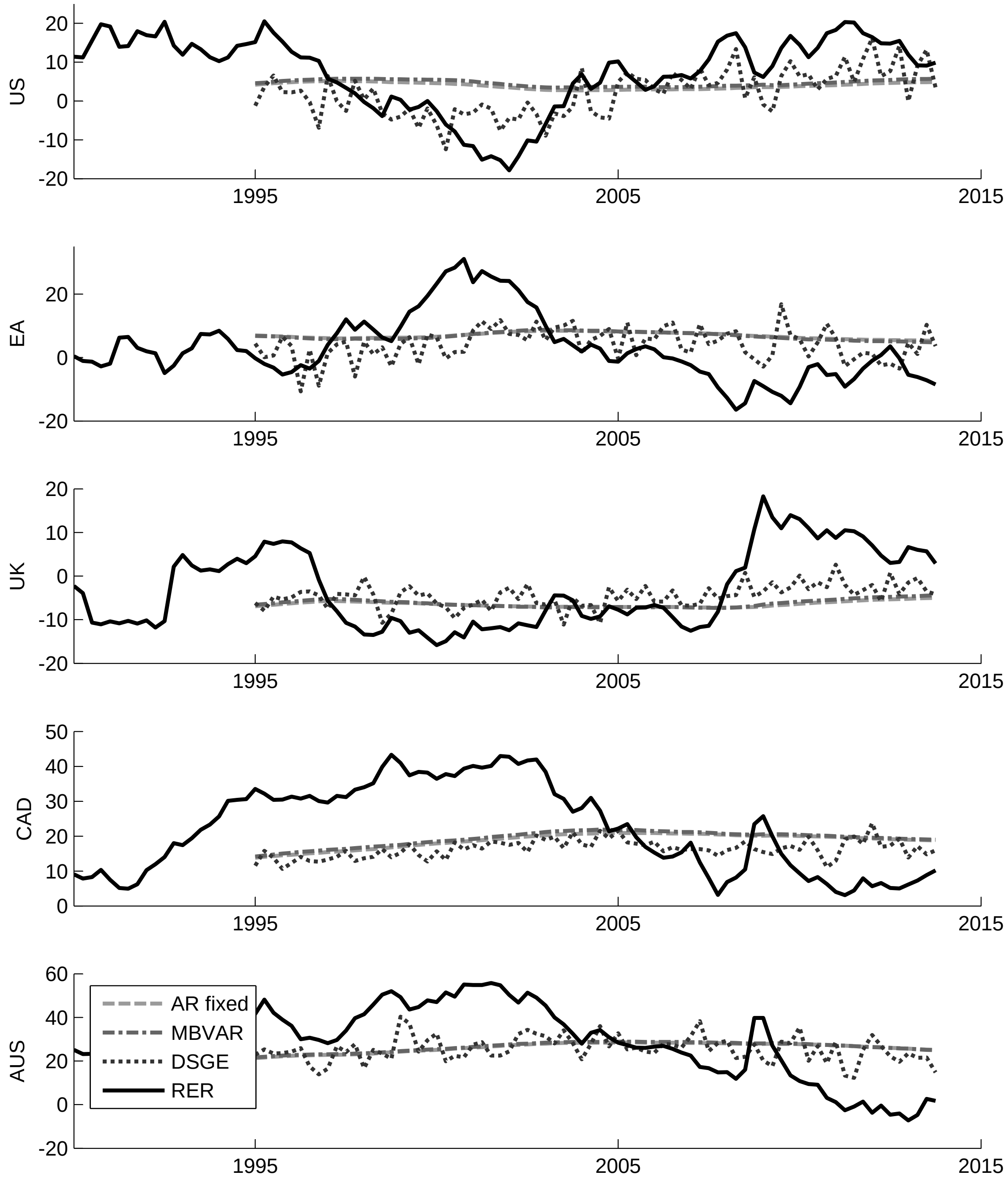
Note: The forecast values are on the horizontal axis, whereas the realizations are on the vertical one.

Figure 3: Recursive real exchange rate forecasts



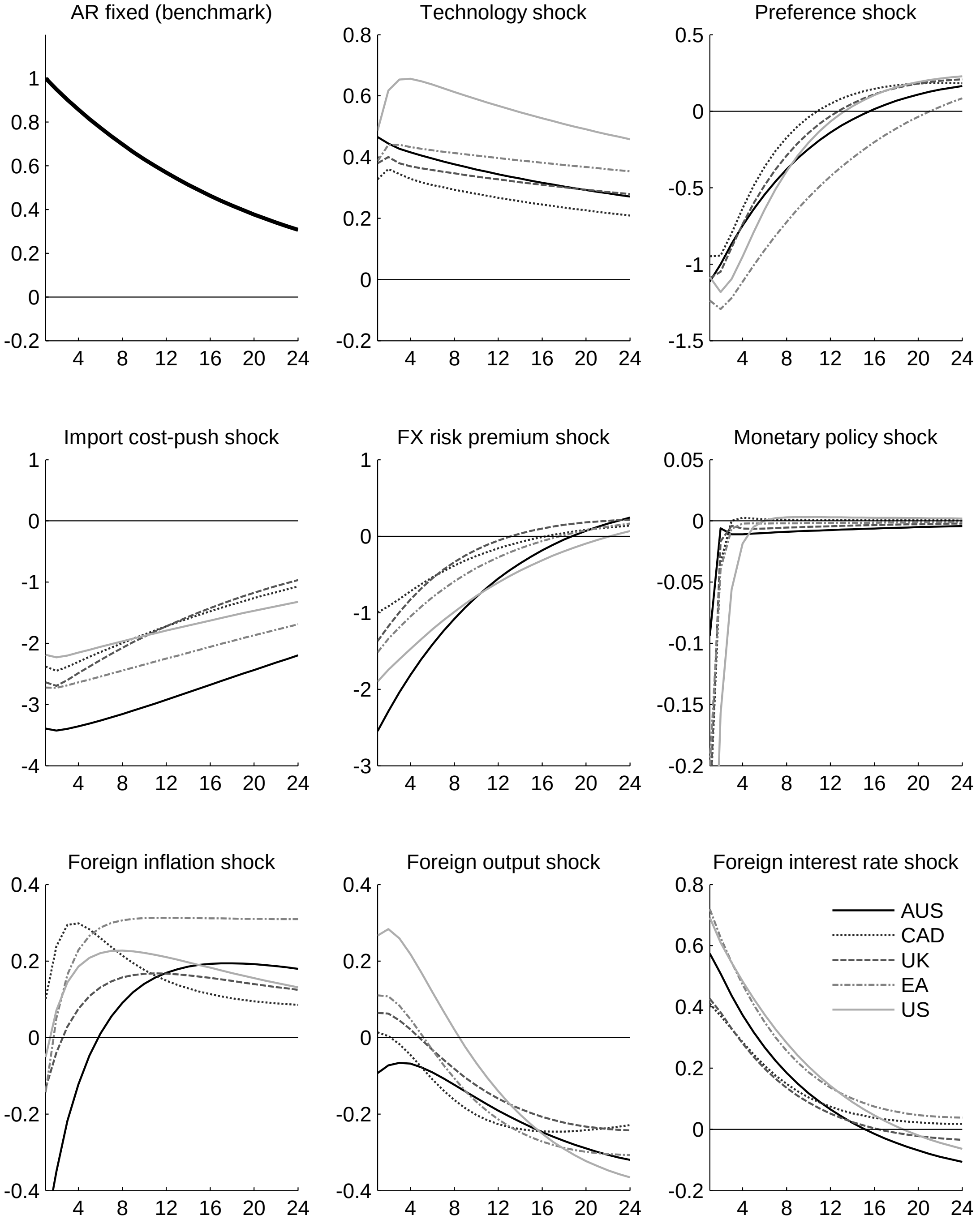
Notes: For each currency the scale on the axes is kept constant to be comparable across the models.

Figure 4: Recursive mean (AR fixed) and steady state (MBVAR and DSGE) for real exchange rates



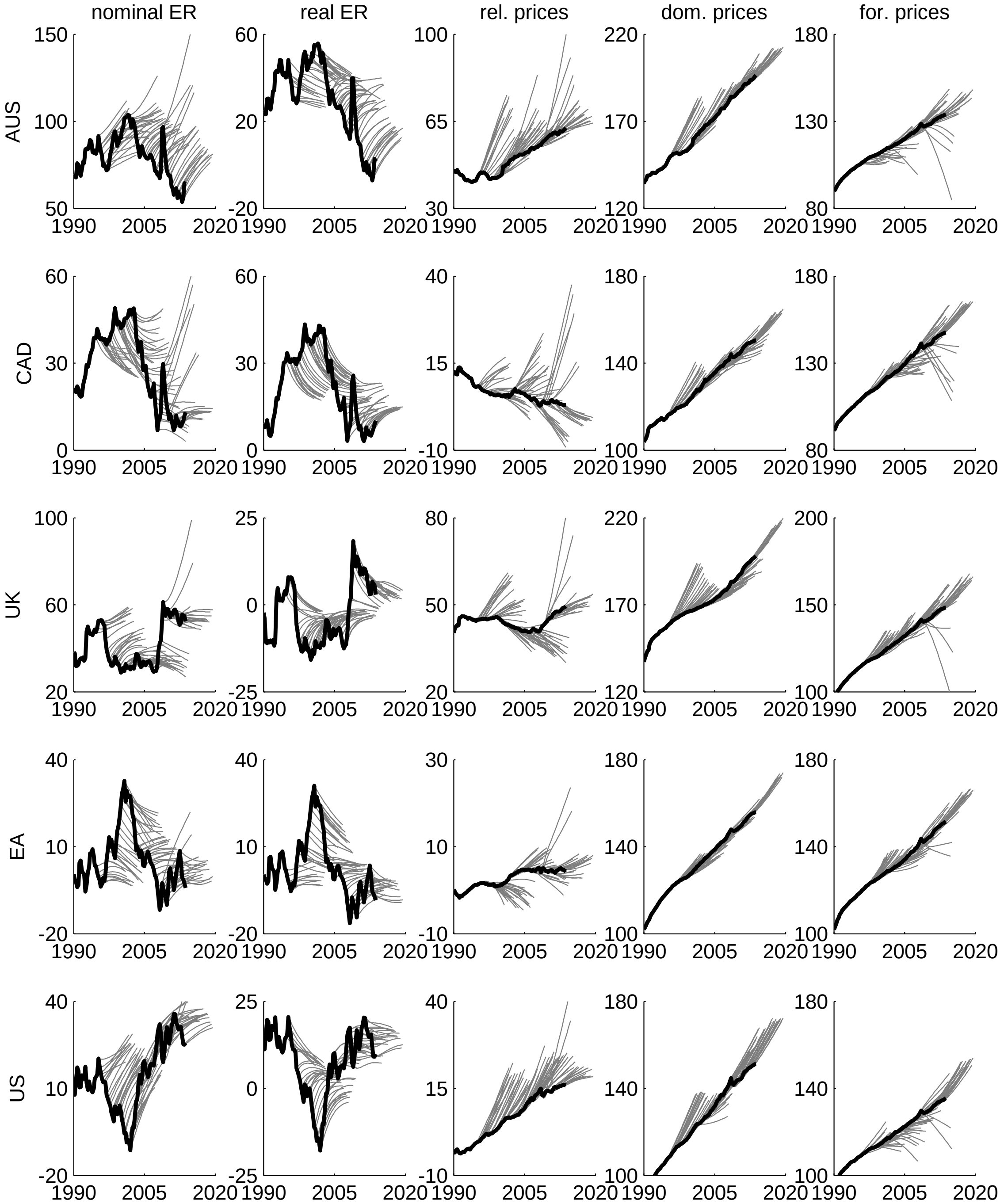
Note: The steady-state-values of the real exchange rate in the MBVAR and DSGE models are calculated using the posterior means of parameters from full sample estimations.

Figure 5: Speed of reversion to equilibrium for different shocks



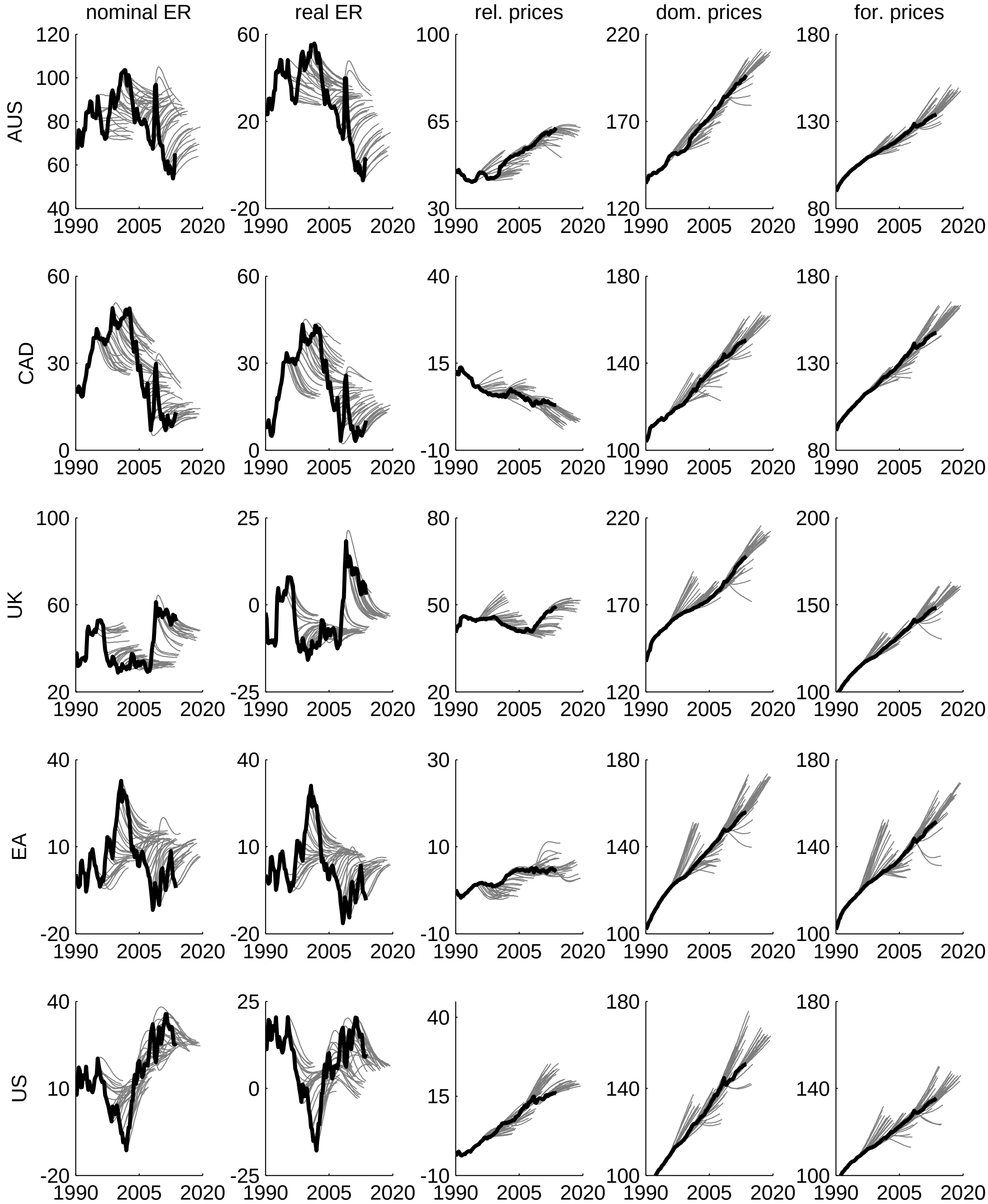
Notes: The impulse response functions are calculated as means based on 200,000 draws from the posterior distribution of parameters for full sample estimations. The dynamic adjustment for the "AR fixed" model is given for a comparison.

Figure 6: Recursive forecasts from DSGE model without trend



Note: Relative prices are defined as a log difference between the CPI indices at home and abroad.

Figure 7: Recursive forecasts from MBVAR



Note: Relative prices are defined as a log difference between the CPI indices at home and abroad.

Appendix

A Log-linearized equations of the DSGE model

Consumption Euler equation

$$c_t - hc_{t-1} = E_t c_{t+1} - hc_t - \frac{1-h}{\sigma}(i_t - E_t\pi_{t+1} - g_t + E_t g_{t+1})$$

Market clearing

$$y_t = (1-\alpha)c_t + \alpha\eta(2-\alpha)s_t + \eta\alpha\psi_{F,t} + \alpha y_t^*$$

Phillips curve for domestic goods

$$\pi_{H,t} - \delta_H \pi_{H,t-1} = \beta(E_t\pi_{H,t+1} - \delta_H\pi_{H,t}) + \frac{(1-\theta_H)(1-\beta\theta_H)}{\theta_H} mc_t$$

Marginal cost

$$mc_t = \varphi y_t - (1+\varphi)z_t + \alpha s_t + \frac{\sigma}{1-h}(c_t - hc_{t-1})$$

Phillips curve for imported goods

$$\pi_{F,t} - \delta_F \pi_{F,t-1} = \beta(E_t\pi_{F,t+1} - \delta_F\pi_{F,t}) + \frac{(1-\theta_F)(1-\beta\theta_F)}{\theta_F} \psi_{F,t} + cp_t$$

Law of one price gap

$$\psi_{F,t} = q_t - (1-\alpha)s_t$$

Consumer price inflation

$$\pi_t = (1-\alpha)\pi_{H,t} + \alpha\pi_{F,t}$$

Uncovered interest rate parity

$$(i_t - E_t\pi_{t+1}) - (i_t^* - E_t\pi_{t+1}^*) = E_t q_{t+1} - q_t - \chi a_t - \phi_t$$

Nominal exchange rate dynamics

$$\Delta e_t = q_t - q_{t-1} - \pi_t^* + \pi_t$$

Terms of trade dynamics

$$s_t - s_{t-1} = \pi_{F,t} - \pi_{H,t}$$

Current account

$$ca_t = -\alpha(s_t + \psi_{F,t}) + y_t - c_t$$

Net foreign assets

$$a_t = \frac{1}{\beta} a_{t-1} + ca_t$$

Interest rate rule

$$i_t = \rho_i i_{t-1} + (1-\rho_i)(\psi_\pi \pi_t + \psi_y y_t + \psi_{\Delta y} \Delta y_t + \psi_e \Delta e_t) + \sigma_m \varepsilon_{m,t}$$

Shock processes

$$z_t = \rho_z z_{t-1} + \sigma_z \varepsilon_{z,t}$$

$$g_t = \rho_g g_{t-1} + \sigma_g \varepsilon_{g,t}$$

$$cp_t = \rho_{cp} cp_{t-1} + \sigma_{cp} \varepsilon_{cp,t}$$

$$\phi_t = \rho_\phi \phi_{t-1} + \sigma_\phi \varepsilon_{\phi,t}$$

$$\pi_t^* = \rho_{\pi^*} \pi_{t-1}^* + \rho_{\pi^* y^*} y_{t-1}^* + \rho_{\pi^* i^*} i_{t-1}^* + \rho_{2\pi^*} \pi_{t-2}^* + \rho_{2\pi^* y^*} y_{t-2}^* + \rho_{2\pi^* i^*} i_{t-2}^* + \sigma_{\pi^*} \varepsilon_{\pi^*,t}$$

$$y_t^* = \rho_{y^* \pi^*} \pi_{t-1}^* + \rho_{y^*} y_{t-1}^* + \rho_{y^* i^*} i_{t-1}^* + \rho_{2y^* \pi^*} \pi_{t-2}^* + \rho_{2y^*} y_{t-2}^* + \rho_{2y^* i^*} i_{t-2}^* + \sigma_{y^*} \varepsilon_{y^*,t}$$

$$i_t^* = \rho_{i^* \pi^*} \pi_{t-1}^* + \rho_{i^* y^*} y_{t-1}^* + \rho_{i^*} i_{t-1}^* + \rho_{2i^* \pi^*} \pi_{t-2}^* + \rho_{2i^* y^*} y_{t-2}^* + \rho_{2i^*} i_{t-2}^* + \sigma_{i^*} \varepsilon_{i^*,t}$$

B Measurement equations used to estimate the DSGE model

Unlike Justiniano and Preston (2010), we do not demean the data prior to estimation. Instead, we do it within the estimation procedure by including intercepts μ in the measurement equations listed below. The only exception is the real exchange rate, for which our baseline specification features no intercept and hence imposes mean reversion on this variable.

$$\tilde{y}_t - \tilde{y}_{t-1} = \mu_y + y_t - y_{t-1}$$

$$\tilde{p}_t - \tilde{p}_{t-1} = \mu_\pi + \pi_t$$

$$\tilde{i}_t = \mu_i + i_t$$

$$\tilde{q}_t - \tilde{q}_{t-1} = q_t - q_{t-1}$$

$$\tilde{ca}_t = \mu_{ca} + ca_t$$

$$\tilde{y}^*_t - \tilde{y}^*_{t-1} = \mu^*_y + y^*_t - y^*_{t-1}$$

$$\tilde{p}^*_t - \tilde{p}^*_{t-1} = \mu^*_\pi + \pi^*_t$$

$$\tilde{i}^*_t = \mu^*_i + i^*_t$$

C Calibration and estimation details

Our calibration and estimation follows very closely Justiniano and Preston (2010). In particular, we calibrate β to 0.99, χ to 0.01 and fix the openness parameter α using the average GDP shares of exports and imports, corrected for the import content of exports estimated by the OECD. This gives α equal to 0.14 for Australia, 0.19 for Canada, 0.13 for the euro area, 0.19 for the United Kingdom and 0.09 for the United States. The remaining parameters are estimated using Bayesian methods implemented in Dynare. Unlike Justiniano and Preston (2010), we do not demean the data before estimation but include a set of intercepts in some of the measurement equations presented in the previous section. The prior distributions for these intercepts are assumed to be uniform and hence uninformative. The prior assumptions for the remaining parameters are identical to those used by Justiniano and Preston (2010). The posterior distributions are approximated with 200,000 draws obtained from four Markov Monte Carlo chains generated with the Metropolis-Hastings algorithm after burning in the initial 50,000 draws. All these calculations were done using Dynare, version 4.4.3. Detailed estimation results are available from the authors upon request.

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